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Bianchi Type I Magnetized Dark Energy Cosmological Model in Bimetric Theory of Gravitation

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ABSTRACT

We investigated Bianchi type-I magnetized dark energy cosmological model in bimetric theory of gravitation. The paper explores the geometrical and physical properties of the model in both power law and exponential law. All the physical parameters of both the laws are decreasing with time. It is found that expansion with steady speed in power law model while in exponential model, our universe has accelerating contraction. Other geometrical and physical properties are also discussed.

KEYWORDS: Bimetric, Gravitation, Dark Energy, Dark Matter.

1. INTRODUCTION

Presently our universe appears that the cosmos is composed of 4.9% baryonic (ordinary) matter, 26.8% dark matter, and 68.3% dark energy in the form of energy densities.[1-4] Recently, various cosmological theories including dark matter as well as dark energy have been investigated. Due to the development of the dark energy problem, the constraints of general gravitation have once more become apparent. The dark energy will explain how the universe is clearly expanding at an acceleration rate. Dark energy has been suggested to be responsible for of the universe's observed acceleration in expansion. The dark energy is speculative type of energy acting appallingly which allows all of space and will in general speed up the extension of the cosmos. This sped up extension of the cosmos was affirmed by the perceptions like Supernovae type I,[5-7] LSS,[8] SDS Survey,[9] WMA Probe,[10-13] BAO[14] and CMBR.[2] Cosmologists presented a number of contenders for dark energy based on these senses and their theories about it.[15-17] The cosmological steady up-and-comer experiences two notable issues to be specific the tweaking and the grandiose incidental problems.[18] Positive energy thickness and negative tension are combined in the pith dynamical dark energy model. The basic scenario is further inspired by the tracker field.[19,20] The Chaplygin gas model, which is seen as the combined effect of dark matter along with dark energy, is another option to the dynamical dark energy model. [21-26] Investigating the importance of attractiveness in the cosmos is important, given that the attractive field exists in the space between universes and plays very important role in cosmology. Melvin propose that matter was severely ionised during the creation of the cosmos also, because of the expansion of the cosmos, this matter is once more flawlessly united with the attractive field and builds unbiased matter.[27] Therefore, it seems sense that there would be an attracting field throughout the cosmos. The incredible result of general gravitation did not stop any other explanations for the universe's expanding pace and the existence of dark matter from being put forth. In actuality, a short while after Einstein's theory was published, numerous elective speculations were arisen telling the best way to expand, how to connect together with other theories and how to consolidate. Rosen's bimetric theory of gravity is one of the other alternatives, which is free of singularities as well as black hole which where appear in Big Bag theory of cosmological model. The theory is consistent with the most recent observational evidence on GR, and it substantially agrees with the covariance and equivalence principles of GR[28,29]. Therefore, Rosen's bimetric theory of gravitation is significant and is dependent on two matrices namely fundamental metric and flat metric.

$$ds^2 = g_{ij}dx^i dx^j \tag{1}$$

$$dn^2 = \gamma_{ij}dx^i dx^j \tag{2}$$

The Rosen's field equations are

$$N_i^j - \frac{1}{2}N\delta_i^j = -8\pi k T_i^j \tag{3}$$

$N_i^j = \frac{1}{2}\gamma^{pr}\left(g^{sj}g_{si|p}\right)_{|r}$. Here $(\,|\,)$ denotes γ–covariant differentiation and T_i^j is the energy–momentum tensor.

$N = N_i^i$, $k = \sqrt{g/\gamma}$ together with $g = determinant(g_{ij})$ and $\gamma = determinant(\gamma_{ij})$.

Numerous specialists have fostered the hypothesis and examined numerous models of the cosmos, in biometric theory of gravitation and concentrated on their way of behaving geometrically as well as physically.[30-40]

In this paper, we will focus on the Bianchi type-I magnetised dark energy cosmological model in the bimetric theory of gravitation since it has a great deal more fidelity to the physics of spatial homogeneous cosmology.

According to theories of gravitation, the main issue with the advancement of the cosmos is focusing on the singularity's situation and knowing its nature. Rosen [28,29] joined the γ_{ij} along with g_{ij} to keep away from the singularities in bimetric theory of gravitation. Any hypothesis should be released from singularity since its presence suggests the breakdown of the rules propounded by the theory. This has led to the development of the bimetric theory of gravity, which is now free of singularities. Many researchers like Borkar, Karade, Kruskal, Penrose and so forth in GR are focus on it. There are two type of singularity one is removable and other is non removable.[41-45] The nonremovable singularity problem has attracted the attention of experts in several gravitational theories. In order to depict the gravitational field around large things with singularities, Schwarzschild (1916) deduced an alternative arrangement for Einstein's vacuum spacetime field equation.[45] There is only one singularity at r = 0 in bimetric gravity for static circularly symmetric spacetime in the case of emptiness, and it would disappear in the unlikely event that the molecule represented by a matter tensor differs from zero in a specific area.[28,29]

In this work,Bianchi model type-I magnetized dark energy cosmological model in bimetric theory of gravitation by solving Rosen’s equations and Investigate the exponential as well as power law expansions of this model.

2. THE METRIC AND ROSEN’S FIELD EQUATIONS

We consider Bianchi Type I metric in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2 \quad (4)$$

where A, B and C are functions of $t -$ alone.

The flat metric corresponding to metric (4) is

$$d\eta^2 = -dt^2 + dx^2 + dy^2 + dz^2 \quad (5)$$

The energy momentum tensor T_i^j for a cloud of massive string and perfect fluid distribution with electromagnetic field is taken as

$$T_i^j = (\rho + p)v_i v^j + p g_i^j - \lambda x_i x^j + E_i^j \quad (6)$$

with

$$v^i v_i = -x^i x_i = -1 \quad (7)$$

$$v^i x_i = 0 \quad (8)$$

In this model, λ is the string density, p is the isotropic pressure, ρ is the proper energy density for a cloud string with particles attached to them, $v^i$is the four-velocity of the particle and$x^i$is a unit space like vector denoting the direction of string, and it is

$$x^i = (\frac{1}{A}, 0, 0, 0)$$

The electromagnetic field E_{ij} is given as

$$E_{ij} = \bar{\mu}\left[|h|^2\left(v_i v_j + \tfrac{1}{2} g_{ij}\right) - h_i h_j\right] \quad (9)$$

The four velocity vector $v_i$is given by

$$g_{ij} v_i v_j = -1 \quad (10)$$

and$\bar{\mu}$is the magnetic permeability and h_i is the magnetic flux vector defined by

$$h_1 = \frac{\sqrt{-g}}{2\bar{\mu}} \in_{ijkl} F^{kl} v^j \quad (11)$$

where F_{kl} is the electromagnetic field tensor and ϵ_{ijkl} is the Levi Civita tensor density.

Assume the comoving coordinates and hence we have $v^1 = v^2 = v^3 = 0, v^4 = 1$

Further we assume that the magnetic field is along x–axis, so that

$$h_1 \neq 0,\ \ h_2 = h_3 = h_4 = 0$$

The set of Maxwell’s equation

$$F_{[ij,k]} = 0 \quad (12)$$

yield

$$F_{23} = constant = H\ (say)$$

Due to the assumption of infinite electrical conductivity we have

$$F_{14} = F_{24} = F_{34} = 0$$

The only non–vanishing component of $F_{ij}$is F_{23}. So that

$$h_1 = \frac{AH}{\bar{\mu}BC} \quad (13)$$

Since $|h^2| = h_i h^i = h_1 h^1 = g^{11}(h_1)^2 = \frac{H^2}{\bar{\mu}^2 B^2 C^2}$

(14)

From equation (9), we obtain

$$-E_1^1 = E_2^2 = E_3^3 = -\frac{H^2}{2\bar{\mu}B^2C^2} \quad (15)$$

If the particle density of the configuration is denoted by ρ_p, then we have

$\rho = \rho_p + \lambda$ Equation (6) of energy momentum tensor yield

$$T_1^1 = \left(p - \lambda - \frac{H^2}{2\bar{\mu}B^2C^2}\right),\ T_2^2 = T_3^3 = \left(p + \frac{H^2}{2\bar{\mu}B^2C^2}\right), T_4^4 = \left(-\rho - \frac{H^2}{2\bar{\mu}B^2C^2}\right) \quad (16)$$

The Rosen’s field equations (3) for the metric (4) and (5) with the help of (16) gives

$$-\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{{A_4}^2}{A^2} - \frac{{B_4}^2}{B^2} - \frac{{C_4}^2}{C^2} = 16\pi ABC\left(-p + \lambda + \frac{H^2}{2\bar{\mu}B^2C^2}\right) \quad (17)$$

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{{A_4}^2}{A^2} + \frac{{B_4}^2}{B^2} - \frac{{C_4}^2}{C^2} = 16\pi ABC\left(-p - \frac{H^2}{2\bar{\mu}B^2C^2}\right) \quad (18)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{{A_4}^2}{A^2} - \frac{{B_4}^2}{B^2} + \frac{{C_4}^2}{C^2} = 16\pi ABC\left(-p - \frac{H^2}{2\bar{\mu}B^2C^2}\right) \quad (19)$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{C_{44}}{C}-\frac{{A_4}^2}{A^2}-\frac{{B_4}^2}{B^2}-\frac{{C_4}^2}{C^2}=16\pi ABC\left(\rho+\frac{H^2}{2\bar{\mu}B^2C^2}\right) \quad (20)$$

where $A_4=\frac{dA}{dt},\ B_4=\frac{dB}{dt},\ C_4=\frac{dC}{dt}\quad etc.$

3. SOLUTION OF FIELD EQUATIONS

The equations (17)-(20) are the arrangement of four equations with six unknowns A, B, C, λ, ρ and p. Thus initially the system is undetermined, so we want two additional conditions to achieve the total solution of the system. Hence, we guessed that expansion θ is proportional to share σ.

$$A=(BC)^n \quad \text{and} \quad H=la^{-m} \quad (21)$$

For this model, the spatial volume scale factor V can be obtained by $V^3=ABC$ (22)

The average scale factor a is as follows

$$a=(ABC)^{\frac{1}{3}} \quad (23)$$

Hubble parameter H as

$$H=\frac{1}{3}\left(H_x+H_y+H_z\right) \quad 24)$$

where, $H_x=\frac{A_4}{A}$, $H_y=\frac{B_4}{B}$ and $H_z=\frac{C_4}{C}$ are, respectively, the directional for the Hubble parameter H in the x, y, and z directions.

Assume that the condition, $C=\alpha$ (25)

The deceleration parameter q is given by

$$q=\left(-\frac{aa_{44}}{{a_4}^2}\right)=(m-1) \text{ for } m\neq 0 \quad (26)$$

Above equation gives steady value of deceleration parameter q. The sign of q indicates whether or not the model accelerates. When m > 1, the universe decelerates, but when m < 1, the cosmos speeds up.

We determine the value of the average scale factor a from equation (22) as

$$a=(mlt+c_1)^{\frac{1}{m}} \quad \text{for } m\neq 0 \quad (27)$$

where, c_1 is the integration constant.

We have,

$$a=c_2e^{lt} \quad \text{for } m=0 \quad (28)$$

where, c_2 is the integration constant. Hence the There are two forms of cosmic expansion according to the law of variation of the Hubble parameter: one is Power law expansion shown by equation (26) and other is Exponential law expansion shown by equation (28).

Case-I) Power Law Model

The average scale factor a in power law has been determined as follows

$$a=(mlt+c_1)^{\frac{1}{m}} \quad \text{for} \quad m\neq 0$$

Using equation (21), (23), (25) and (26), The values of scale factors A and B are determined as follows

$$A=(mlt+c_1)^{\frac{3n}{m(n+1)}} \quad (29)$$

$$B=\frac{1}{\alpha}(mlt+c_1)^{\frac{3}{m(n+1)}} \quad (30)$$

Where $m\neq 0$ and $l, c_2, n>0$, They are all constants.

As a result, an appropriate metric for power law is

$$ds^2=-dt^2+(mlt+c_1)^{\frac{6n}{m(n+1)}}dx^2+\frac{1}{\alpha^2}(mlt+c_1)^{\frac{6}{m(n+1)}}dy^2+\alpha^2dz^2 \quad (31)$$

This is the LRS Bianchi type-I magnetized dark energy cosmological model in bimetric theory of gravitation in power law expansion.

Case-II) Exponential Law Model

The average scale factor a in Exponential law has been determined as follows

$$a=c_2e^{lt} \quad \text{for } m=0$$

Using equation (21), (23), (25) and (27), The values of scale factors A and B are determined as follows

$$A(t)=(c_2e^{lt})^{\frac{3n}{(n+1)}} \quad (31)$$

$$B(t)=\frac{1}{\alpha}(c_2e^{lt})^{\frac{3}{(n+1)}} \quad (32)$$

Where $m\neq 0$ and $l, c_2, n>0$, They are all constants.

As a result, an appropriate metric for exponential law is

$$ds^2=-dt^2+(c_2e^{lt})^{\frac{3n}{(n+1)}}dx^2+\frac{1}{\alpha^2}(c_2e^{lt})^{\frac{3}{(n+1)}}dy^2+\alpha^2dz^2 \quad (33)$$

This is the LRS Bianchi type-I magnetized dark energy cosmological model in bimetric theory of gravitation in exponential law expansion.

4. GEOMETRICAL AND PHYSICAL SIGNIFICANCE

4.1 Power Law Model

The power law model in equation (30) contains scale factors A and B that rise over time and finally infinite values. Subsequently Early on, the model becomes flat; later, the scale factor diverges to infinity.

Physical quantities that are important to cosmology include some of the following: the pressure (P), the string tension density (λ), the matter energy density (ρ), the particle density (ρ_p), the Hubble parameter (H), scalar expansion (θ), and the shear (σ) have been calculated for the power law model (30) as follows

$$P=\frac{3mnl^2}{16\pi(n+1)}(mlt+c_1)^{\left(-\frac{3}{m}-2\right)}-\frac{H^2}{2\bar{\mu}}(mlt+c_1)^{-\frac{12}{m(n+1)}} \quad (34)$$

$$\lambda=\frac{6m(n-1)l^2}{16\pi(n+1)}(mlt+c_1)^{\left(-\frac{3}{m}-2\right)}-\frac{H^2}{\bar{\mu}}(mlt+c_1)^{-\frac{12}{m(n+1)}} \quad (35)$$

$$\rho=\frac{-3ml^2}{16\pi}(mlt+c_1)^{\left(-\frac{3}{m}-2\right)}-\frac{H^2}{2\bar{\mu}}(mlt+c_1)^{-\frac{12}{m(n+1)}} \quad (36)$$

$$\rho_P=\frac{-3m(2n+1)l^2}{16\pi(n+1)}(mlt+c_1)^{\left(-\frac{3}{m}-2\right)}+\frac{H^2}{2\bar{\mu}}(mlt+c_1)^{-\frac{12}{m(n+1)}} \quad (37)$$

$$H=l(mlt+c_1)^{-1} \quad (38)$$

$$\theta=3H=3l(mlt+c_1)^{-1} \quad (39)$$

$$\sigma^2=3l^2\left[\frac{(n-1)^2}{(n+1)^2}\right](mlt+c_1)^{-2} \quad (40)$$

Now, we have

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3}\left[\frac{(n-1)^2}{(n+2)^2}\right] \quad (42)$$

$$q = -\frac{aa_{44}}{{a_4}^2} = (m-1) \quad (43)$$

It is observed that in power law expansion, all the physical parameters are inversely proportional to time and all are decreases with increase in time. At the beginning, the expansion of the universe attains high value, whereas at late epoch of time, the expansion converges to minimum value.

The deceleration parameter has importance in dark energy cosmological model. Our universe is expanded when the deceleration parameter shows that for value m = 1 results in expansion that is constant in speed, neither accelerating nor decelerating.

4.2 Exponential Law Model

The model of exponential law given in equation (33) contains scale factors A and B that increases time and finally infinite values. Subsequently Early on, the model becomes flat; later, the scale factor diverges to infinity.

Physical quantities that are important in terms of cosmology which include some of the following: the pressure (P), the string tension density (λ), the matter energy density (ρ), the particle density (ρ_p), the mean Hubble parameter (H), scalar expansion (θ), and the shear (σ) have been calculated for the exponential law model (33) as follows

$$P = -\frac{H^2}{2\bar{\mu}}\left(c_2 e^{lt}\right)^{-\frac{12}{(n+1)}} \quad (44)$$

$$\lambda = -\frac{H^2}{\bar{\mu}}\left(c_2 e^{lt}\right)^{-\frac{12}{(n+1)}} \quad (45)$$

$$\rho = -\frac{H^2}{2\bar{\mu}}\left(c_2 e^{lt}\right)^{-\frac{12}{(n+1)}} \quad (46)$$

$$\rho_P = \frac{H^2}{2\bar{\mu}}\left(c_2 e^{lt}\right)^{-\frac{12}{(n+1)}} \quad (47)$$

$$H = l \quad (48)$$

$$\theta = 3H = 3l \quad (49)$$

$$\sigma^2 = 3l^2\left[\frac{(n-1)^2}{(n+2)^2}\right] \quad (50)$$

$$\frac{\sigma^2}{\theta^2} = \frac{(n-1)^2}{3(n+2)^2} \quad (51)$$

$$q = -\frac{aa_{44}}{{a_4}^2} = -1 \quad (52)$$

It is observed that in exponential law expansion, the physical parameters all are inversely proportional to time and decreases with increase in time. At the beginning, all these quantities attain maximum value, whereas at late epoch of time, the expansion converges to minimum value.

The decelerating parameter leads to expansion of the universe which means our universe in exponential model has accelerating contraction.

5. CONCLUSION

In this paper, we examine Bianchi type-I magnetized dark energy cosmological model in bimetric theory of gravitation by solving Rosen's field equations. We investigate the geometrical and physical characteristics of model in terms of (i) power law and (ii) exponential law.

The pressure and string tension density are diminishing functions of time in the power law model. Additionally, the particle density decreases with time. The Hubble parameter has an inversely related with time and the scalar expansion decreases with time. At the beginning, the expansion attains maximum value, whereas at late epoch of time, the expansion converges to minimum value. The deceleration parameter shows that our universe for value $m = 1$ results in expansion that is constant in speed, neither accelerating nor decelerating. In exponential law model, The pressure, string tension density, and particle density all decrease over time. Other physical properties are also discussed.

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