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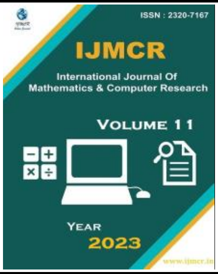

Convective Cattaneo-Christov Heat Flux and Heat Generation Effect on Mhd Williamson Fluid Flow Over an Exponentially Stretching Surface with Thermal Radiation

Rekha K.[1], S. B. Sathyanarayana [2], Sushma V. Jakati [3], Asha C. S.[4] ,Achala L. Nargund[5]
[1,4,5] P. G. Dept of Mathematics and Research Centre in Applied Mathematics
[1,3,4,5] M E S College of Arts, Commerce and Science, Malleswaram, Bangalore-560003.
[2] Department of Mathematics, Vijaya College, R V Road, Basavanagudi, Bangalore-560004.

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Corresponding Author:
Achala L. Nargund

ABSTRACT
The two-dimensional steady incompressible MHD boundary layer flow of convective Williamson fluid flow of the Cattaneo-Christov heat flux type over an exponentially stretching surface in the presence of heat generation and thermal radiation is considered. The nonlinear governing partial differential equations are reduced to ordinary differential equations by using a similarity transformation. The Homotopy Analysis Method (HAM) is applied to solve the reduced equations and the effects of importance of fluid parameters are explained through graphs. Also, the HAM solutions were compared with the numerical solutions and good agreement was observed.

KEYWORDS: Williamson fluid, Cattaneo-Christov heat flux, MHD, thermal radiation, HAM.
Mathematics Subject Classification: 76

1 INTRODUCTION

Williamson [1] proposed a fluid model known as Williamson fluid model to know the flow behaviour of pseudoplastic fluids. The heat transfer in viscoelastic flow caused by an exponentially stretched sheet is described by the Cattaneo-Christov heat flux model. Using the Oldroyds' upper-convected derivative, Christov proposed the frame-indifferent generalisation of Cattaneo's law from which he derived a single temperature governing equation [2]. The upper-convected Maxwell model and Cattaneo–Christov heat flux model is used to investigate heat transfer and boundary layer flow of a viscoelastic fluid above a stretching plate with velocity slip boundary by Han et al. [3]. A modified version of Fourier's law known as the Cattaneo-Christov heat flow model, is used to investigate the phenomenon of heat transport by Khan et al.[4]. The two-dimensional Oldroyd-B fluid over a stretching surface with gyrotactic microorganism using Cattaneo-Christov heat flux is investigated by Bashir et al. [5]. Recently Bilal et al. [6] analyzed the two-dimensional incompressible flow of Williamson nanofluid over an exponentially stretching surface with Cattaneo- Christov heat flux model and reported that fluid velocity decreases in relation to the Williamson fluid model parameter.

The steady flow of nanofluid with gyrotactic microorganism over a Riga plate using Cattaneo- Christov heat flux model is studied by Faizan et al. [7]. Alharbi et al. [8] used Tiwari-Das model and the Cattaneo-Christov model to examine the impact of Marangoni convection and volume fraction during heat transfer and divulged that Marangoni convection shortens skin friction. Mahabaleshwar et al. [9] compared the Cattaneo-Christov heat flux concept in the flow of two viscoelastic fluids to explore the heat transfer individualities with changing thermal conductivity using the characteristics of the Appell hypergeometric function. Shahzad et al. [10] explore the effect of gyrotactic microorganisms and convective thermal boundary conditions on the Darcy-Forchheimer in a micropolar nanofluid flow between two coaxial, parallel, and radially expanding disks and observed that the stretching ratio parameter of the disks accelerates the axial and micro rotational velocities of the nanofluid.

Reddy et al. [11] studied the two-dimensional viscous flow of Cassson nanofluid with Cattaneo–Christov model and used Spectral Homotopy Analysis Method (SHAM) to get the numerical solution. Ahmad et al. [12] investigated the Cattaneo-Christov heat flux model of second-grade nanofluid flow over a stretching sheet and used HAM to get analytic solution. Ahmed et al. [13] analyzed the Maxwell nanofluid flow in three-dimensional porous medium with the idea of Cattaneo-Christov and Buongiorno models and the resultant non-linear ordinary differential equations are solved using Homotopy Analysis Method. Akinbo et al. [14] discussed the effect of viscous Walters B fluid flow of Cattaneo-Christov model over an exponentially stretching sheet and the results are compared using HAM and Galerkin Weighted Residual method. Amjad et al.[15] investigated the Cattaneo-Christov double diffusion (CCDD) heat flux model for Williamson nanofluid over an exponentially stretching surface with variable thermal conductivity.

Prasad et al. [16] observed that the distribution of temperature and concentration is decreased as a result of the Cattanneo-Christov heat flux model. The non-Newtonian fluid behavior of Casson fluid model over an Exponentially Stretching Sheet with Heat Source and Sink is studied by Prakash et al. [17]. Shehzad et al. [18, 19] analyzed Cattaneo-Christov model for both third-grade fluid flow over an exponentially stretching sheet and for Darcy-Forchheimer flow of an Oldroyd-B fluid over a moving sheet. The influence of Cattaneo-Christov model of Williamson fluid over a permeable sheet is studied using HAM by Ray [20]. The flow of Williamson Sutterby nanofluid in Darcy–Forchheimer sponge medium with Cattaneo-Christov heat flux is discussed by Yahya et al. [21]. The flow of Williamson and Casson fluid flows over a penetrable extending Sheet in a permeable medium using Runge-Kutta Fehlberg method together with Shooting method is studied by Mangathai et al. [22].

Jamshed et al. [23] applied Keller-box method in the analysis of engine oil-based Williamson hybrid nanofluid flow. Hayat et al. [24] surveyed the flow of thixotropic fluid over a stretching surface of Cattaneo-Christov heat flux model. The bioconvection nanofluid flow with temperature-dependent variable viscosity by applying HAM is studied by Mondal et al. [25]. The MHD three-dimensional flow of Maxwell fluid over a bi-directional stretching surface with Cattaneo-Christov heat flux model by using HAM is analyzed by Rubab et al. [26]. Salahuddin et al. [27] analyzed the thermal relaxation time effect of MHD Williamson fluid flow using Cattaneeo-Christov heat flux model over a stretching sheet. Kumar et al. [28] utilized Cattaneo-Christov model for the flow over a wedge and a cone and the results are compared using R-K method and Newton's method. Bhatti et al. [29] discussed the flow of Williamson nanofluid under the influence of thermal diffusion and thermal radiation over a porous stretching and shrinking sheet and the results are obtained using Successive linearization method (SLM) and Chebyshev spectral collocation method (CSC).

In this paper, we are analyzing Williamson fluid model in a boundary layer over an exponentially stretching surface in the presence of magnetic field, heat source and thermal radiation. The distribution of the paper is first section contains Introduction, second section contains Mathematical formulation, third section is Homotopy Analysis solution of the problem, fourth section is result and discussion and fifth section consists of graphs and tables.

2 MATHEMATICAL FORMULATION

The two-dimensional steady heat and mass transfer of incompressible Williamson fluid flow of uniform magnetic field over an exponentially stretching sheet is considered. By considering u and v as velocity components along x and y directions and the fundamental equations for this model will be [4].

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\sqrt{2}\nu\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u+g\beta(T-T_\infty)+g\beta^*(C-C_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\lambda_1\left[u^2\left(\frac{\partial^2 T}{\partial x^2}\right)+2uv\left(\frac{\partial^2 T}{\partial x\partial y}\right)+v^2\left(\frac{\partial^2 T}{\partial y^2}\right)\right]-\lambda_1\left[\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\left(\frac{\partial T}{\partial x}\right)+\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)\left(\frac{\partial T}{\partial y}\right)\right]+\frac{Q_0}{\rho c_p}(T-T_\infty)+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_p}\left(\frac{\partial q_r}{\partial y}\right), \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+D_T\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

with boundary conditions,

$$u=U_w(x)=U_0 exp\left(\frac{x}{l}\right), v=0, -k\frac{\partial T}{\partial y}=h_f(T_w-T), C=C_w \text{ at } y=0;$$

$$u\to 0, T\to T_\infty, C\to C_\infty \text{ as } y\to\infty. \tag{5}$$

Where ν, the kinematic viscosity, k, thermal conductivity, Γ, time rate constant, ρ, density of the fluid, σ, the electrical conductivity, g, gravitational force, B_0, uniform magnetic field, β, thermal expansion coefficient, β^*, concentration expansion coefficient, T, the temperature of the fluid, C, the concentration of the fluid, c_p, specific heat, λ_1, thermal relaxation time, Q_0, heat source, D_m, mass diffusion coefficient, D_T, thermal diffusion coefficient, h_f, convective heat transfer coefficient, l, characteristic length, radiative heat flux $q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$, where σ^*, Stefan Boltzmann constant and k^*, absorption coefficient. When temperature difference is small, q_r can be linearized by expanding T^4 into Taylor's series about T_∞, takes the form after neglecting higher order terms by $T^4\cong 4T_\infty^3T-3T_\infty^4$.

Here we consider similarity transformations as follows,

$$\eta=y\sqrt{\frac{U_0}{2\nu l}}exp\left(\frac{x}{2l}\right), u=U_0 exp\left(\frac{x}{l}\right)f'(\eta), v=-\sqrt{\frac{\nu U_0}{2l}}exp\left(\frac{x}{2l}\right)[f(\eta)+\eta f'(\eta)],$$

$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}. \quad (6)$$

Equations (2) - (5) will get reduced to (7) - (10) by using (6)

$$f''' + ff'' - 2(f')^2 + \lambda f''f''' - Mf' + Gr\theta + Gm\phi = 0, \quad (7)$$

$$\theta'' - \frac{1}{2}Pr\gamma f^2\theta'' + R\theta'' + \frac{1}{2}Pr\gamma ff'\theta' + Prf\theta' + 2QPr\theta + PrEc(f'')^2 = 0, \quad (8)$$

$$\phi'' + Scf\phi' + SoSc\theta'' = 0. \quad (9)$$

$$f = 0, f' = 1, \theta' = -\gamma_B[1-\theta], \phi = 1, \; at \;\; \eta = 0,$$
$$f' \to 0, \theta \to 0, \phi \to 0, \; as \;\; \eta \to \infty. \quad (10)$$

Where $M = \frac{2\sigma B_0^2 l}{\rho U_w}$; Magnetic field parameter, $\lambda = \Gamma \frac{U_w^{\frac{3}{2}}}{\sqrt{\nu l}}$; Williamson fluid parameter, $Pr = \frac{\nu\rho c_p}{k}$; Prandtl number, $Gr = \frac{2g\beta(T_w-T_\infty)l}{U_w^2}$; Grashof number, $Gm = \frac{2g\beta^*(C_w-C_\infty)l}{U_w^2}$; modified Grashof number, $\gamma = \frac{\lambda_1 U_w}{l}$; Thermal relaxation time parameter, $\gamma_B = \frac{h_f}{k}\sqrt{\frac{2\nu l}{U_0}}$; Biot number, $R = \frac{16}{3}\frac{\sigma^* T_\infty^3}{kk^*}$; Radiation parameter, $Q = \frac{Q_0}{\rho c_p U_w}$; Heat source parameter $Ec = \frac{U_w^2}{c_p(T_w-T_\infty)}$; Eckert number, $Sc = \frac{\nu}{Dm}$; Schmidt number, $So = \frac{Dt(T_w-T_\infty)}{\nu(C_w-C_\infty)}$; Soret number.

3. HOMOTOPY ANALYSIS SOLUTION

Shijun Liao (1992) [31, 32, 33, 34, 35, 36, 37, 38, 39, 40] explained Homotopy Analysis Method (HAM) to solve non-linear differential equations analytically. Using this method [41, 42] we solve coupled nonlinear equations of this problem. The steps of the method are,

$$N[f(\eta)] = f''' + ff'' - 2(f')^2 + \lambda f''f''' - Mf' + Gr\theta + Gm\phi, \quad (11)$$

$$N[\theta(\eta)] = \theta'' - \frac{1}{2}Pr\gamma f^2\theta'' + R\theta'' + \frac{1}{2}Pr\gamma ff'\theta' + Prf\theta' + 2QPr\theta + PrEc(f'')^2, \quad (12)$$

$$N[\phi(\eta)] = \phi'' + Scf\phi' + SoSc\theta''. \quad (13)$$

Linear operators considered are as follows,

$$L(f) = \frac{\partial^3 f}{\partial\eta^3} + \frac{\partial^2 f}{\partial\eta^2}, \quad (14)$$

$$L(\theta) = \frac{\partial^2\theta}{\partial\eta^2} + \frac{\partial\theta}{\partial\eta}, \quad (15)$$

$$L(\phi) = \frac{\partial^2\phi}{\partial\eta^2} + \frac{\partial\phi}{\partial\eta}, \quad (16)$$

which gives initial approximations as,

$$f_0 = 1 - e^{-\eta}, \quad (17)$$

$$\theta_0 = \left(\frac{\gamma_B}{1+\gamma_B}\right)e^{-\eta}, \quad (18)$$

$$\phi_0 = e^{-\eta}. \quad (19)$$

The nonlinear equations for approximate solutions are,

$$(1-p)L[f(\eta,p) - f_0(\eta)] = hp\left[\frac{\partial^3 f}{\partial\eta^3} + f\frac{\partial^2 f}{\partial\eta^2} - 2\left(\frac{\partial f}{\partial\eta}\right)^2 + \lambda\frac{\partial^2 f}{\partial\eta^2}\frac{\partial^3 f}{\partial\eta^3} - M\frac{\partial f}{\partial\eta} + Gr\theta + Gm\phi\right], \quad (20)$$

$$(1-p)L[\theta(\eta,p) - \theta_0(\eta)] = hp[\frac{\partial^2\theta}{\partial\eta^2} - \frac{1}{2}Pr\gamma f^2\frac{\partial^2\theta}{\partial\eta^2} + R\frac{\partial^2\theta}{\partial\eta^2} + \frac{1}{2}Pr\gamma f\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\eta} + Prf\frac{\partial\theta}{\partial\eta} + 2QPr\theta + PrEc\left(\frac{\partial^2 f}{\partial\eta^2}\right)^2], \quad (21)$$

$$(1-p)L[\phi(\eta,p) - \phi_0(\eta)] = hp[\frac{\partial^2\phi}{\partial\eta^2} + Scf\frac{\partial\phi}{\partial\eta} + SoSc\frac{\partial^2\theta}{\partial\eta^2}], \quad (22)$$

with following boundary conditions,

$$f(0,p) = 0, f_\eta(0,p) = 1, f_\eta(\infty,p) = 0, \quad (23)$$

$$\theta_\eta(0,p) = -\gamma_B[1-\theta(0,p)], \theta(\infty,p) = 0, \quad (24)$$

$$\phi(0,p) = 1, \phi(\infty,p) = 0. \quad (25)$$

Varying the values of p from 0 to 1 we get the solution from first approximation to required solution. Using Maclaurin's series expansion and applying Leibnitz theorem we get the series solution. The convergence of the series solution is derived by calculating the convergence parameter h.

$$L[f_m - \chi_m f_{m-1}] = hr_m(\eta), \quad (26)$$

$$L[\theta_m - \chi_m\theta_{m-1}] = hs_m(\eta), \quad (27)$$

$$L[\phi_m - \chi_m\phi_{m-1}] = ht_m(\eta), \quad (28)$$

$$\text{where} \quad \chi_m = \begin{cases} 0, & \text{when } m \le 1 \\ 1, & \text{when } m > 1 \end{cases} \quad \text{and} \quad (29)$$

$$r_m(\eta) = f'''_{m-1}(\eta) + \sum_{k=0}^{m-1} f_{m-1-k}(\eta)f''_k(\eta) - 2\sum_{k=0}^{m-1} f'_{m-1-k}(\eta)f'_k(\eta) - Mf'_{m-1}(\eta)$$
$$+\lambda\sum_{k=0}^{m-1} f''_{m-1-k}(\eta)f'''_k(\eta) + Gr\theta_{m-1}(\eta) +$$

$Gm\phi_{m-1}(\eta),$ (30)

$$s_m(\eta) = \theta''_{m-1}(\eta) - \frac{1}{2}Pr\gamma\sum_{k=0}^{m-1} f_{m-1-k}(\eta)\sum_{j=0}^{k} f_{k-j}(\eta)\theta''_j(\eta) + R\theta''_{m-1}(\eta) + \frac{1}{2}Pr\gamma\sum_{k=0}^{m-1} f_{m-1-k}(\eta)\sum_{j=0}^{k} f'_{k-j}(\eta)\theta'_j(\eta) + Pr\sum_{k=0}^{m-1} f_{m-1-k}(\eta)\theta'_k(\eta) + 2QPr\theta_{m-1}(\eta) + PrEc\sum_{k=0}^{m-1} f''_{m-1-k}(\eta)f''_k(\eta), \quad (31)$$

$$t_m(\eta) = \phi''_{m-1}(\eta) + Sc\sum_{k=0}^{m-1} f_{m-1-k}(\eta)\phi'_k(\eta) + SoSc\theta''_{m-1}(\eta), \quad (32)$$

with boundary conditions,

$$f_m(0) = 0, f'_m(0) = 0, f'_m(\infty) = 0, \quad (33)$$

$$\theta'_m(0) - \gamma_B\theta_m(0) = 0, \theta_m(\infty) = 0, \quad (34)$$

$$\phi_m(0) = 0, \phi_m(\infty) = 0. \quad (35)$$

The required solution is

$$f = f_0 + f_1 + f_2 + \ldots, \quad (36)$$

$$\theta = \theta_0 + \theta_1 + \theta_2 + \ldots, \quad (37)$$

$$\phi = \phi_0 + \phi_1 + \phi_2 + \ldots. \quad (38)$$

Soving equtions (26)-(28) by using MATHEMATICA we get

$$f_1 = -\frac{-h+4Gmh-4hM-h\lambda-h\gamma_B+4Gmh\gamma_B+4Grh\gamma_B-4hM\gamma_B-h\lambda\gamma_B}{4(1+\gamma_B)} + \frac{1}{4(1+\gamma_B)}(e^{-2\eta})(-2e^{\eta}(h + 2Gmh - 2hM + h\lambda + h\gamma_B + 2Gmh\gamma_B + 2Grh\gamma_B - 2hM\gamma_B + h\lambda\gamma_B) + h((1 + \lambda)(1 + \gamma_B) + 4e^{\eta}(Gr\gamma_B + Gm(1 + \gamma_B) - M(1 + \gamma_B))(2 + \eta))), \ldots \quad (39)$$

$$\theta_1 = \frac{1}{4(1+\gamma_B)}(e^{-2\eta})(hPr(2 + \gamma)\gamma_B + 2EchPr(1 + \gamma_B) - 2e^{\eta}(\frac{1}{2}(2EchPr - 4h\gamma_B + 6hPr\gamma_B + 2EchPr\gamma_B - 8hPrQ\gamma_B - 4hR\gamma_B + 3hPr\gamma\gamma_B) + h(2(1 + R) + Pr(-2 + 4Q - \gamma))\gamma_B(1 + \eta))), \ldots \quad (40)$$

$$\phi_1 = \frac{1}{2(1+\gamma_B)}(e^{-2\eta})(hSc(1 + \gamma_B) - 2e^{\eta}(\frac{1}{2}(-2h + 3hSc - 2h\gamma_B + 3hSc\gamma_B - 2hSoSc\gamma_B) + h(1 + \gamma_B + Sc(-1 + (-1 + So)\gamma_B))(1 + \eta))), \ldots \quad (41)$$

4. RESULTS AND DISCUSSION

The semi-analytical solutions of non-dimensional equations (7)-(9) are obtained by homotopy Analysis Method and numerical solution is obtained by Runge-Kutta method. The calculations were done using MATHEMATICA to derive the Williamson fluid's flow, heat, and mass transfer characteristics. The auxiliary non-zero parameters h_f, h_θ and h_ϕ have significant influence on convergence of HAM solution. In figure 1 it can be seen convergence range for velocity profile is $-1.5 < h_f < 0.5$, for temperature profile is $-1.5 < h_\theta < 1.2$ and for concentration profile is $-2.5 < h_\phi < 0.5$.

Figures 2 - 13 delineates the effect of various parameters like Magnetic parameter M, Williamson parameter λ, Grashof number Gr, modified Grashof number Gm, Prandtl number Pr, Thermal relaxation time parameter γ, Heat source parameter Q, Biot number γ_B, Radiation parameter R, Eckert number Ec, Schmidt number Sc and Soret number So on velocity, temperature and concentration profiles. Figure 2 illustrates that the velocity of Williamson fluid drastically decreases as the effect of magnetic field increases. This is due to the Lorentz force which is dragging force in the presence of magnetic field. A similar pattern is observed in the case of Williamson parameter due to fluid relaxation time, shown in figure 3. From figures 4 and 5 we have observed that velocity profile increases with increase in Grashof number and modified Grashof number.

From figure 6, it is observed temperature profile decreases with increase in Prandtl number. The effect of Thermal relaxation parameter on the temperature field is shown in figure 7 and observed that temperature is seen to fall as a result of the relaxation parameter. In the absence of Thermal relaxation parameter, the Cattaneo-Christov heat flux model can be reduced to the Fourier's law of heat conduction. Additionally, Cattaneo-Christov heat flux model has a lower temperature than the Fourier's model. The temperature profile is clearly a function of the Heat source parameter in an increasing manner shown in figure 8. It is observed that the increase in values of Biot number, Radiation parameter and Eckert number increases the temperature profile seen in figures 9, 10 and 11.

The concentration profile increases with the increase in Williamson parameter and Soret number shown in figures 12 and 14, where as decreases with increase in Schmidt number and modified Grashof number shown in figures 13 and 15. Figures 16, 17 and 18 presents Domb-Sykes plots of velocity, temperature and concentration through which we get the radius of convergence as 0.71145, 0.12205 and 1.78374 respectively.

We have compared HAM solution with numerical solution for velocity, temperature and concentration profiles shown in figures 19, 20 and 21 and observed good agreement. We have also compared our solutions with the well known results of Elbashbeshy [30] shown in the table 1.

5. GRAPHS AND TABLES

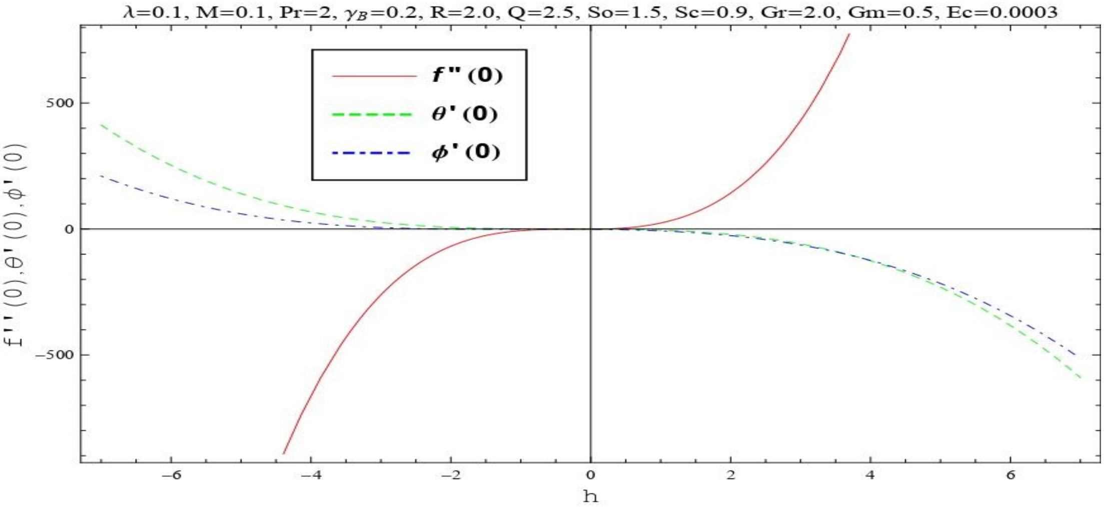

Figure 1: h-curves for the functions f, θ and ϕ

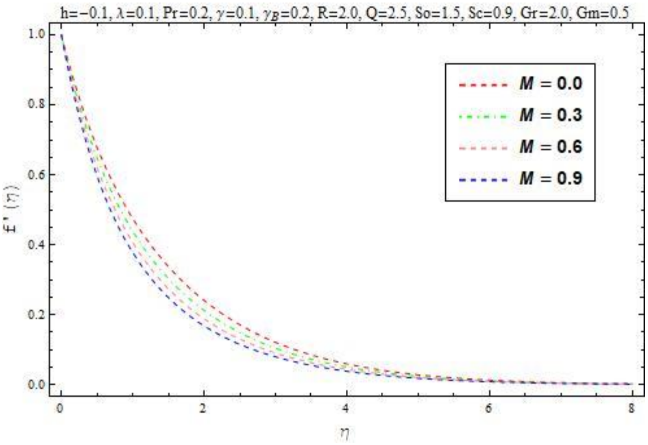

Figure 2: Effect of Magnetic parameter M on velocity profile

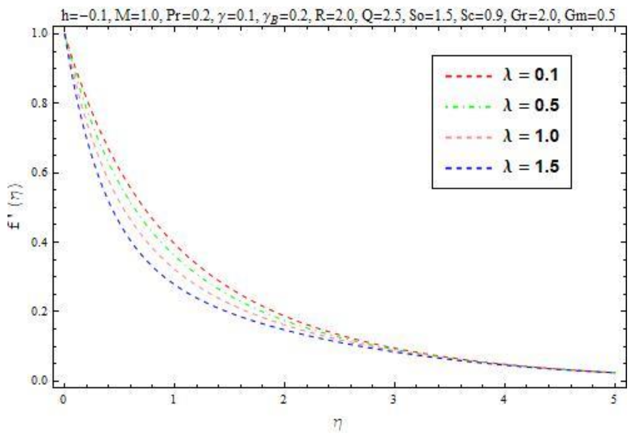

Figure 3: Effect of Williamson parameter λ on velocity profile

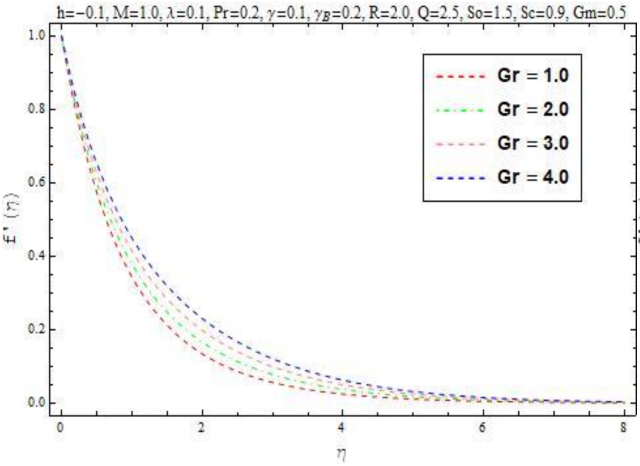

Figure 4: Effect of Grashof number Gr velocity profile

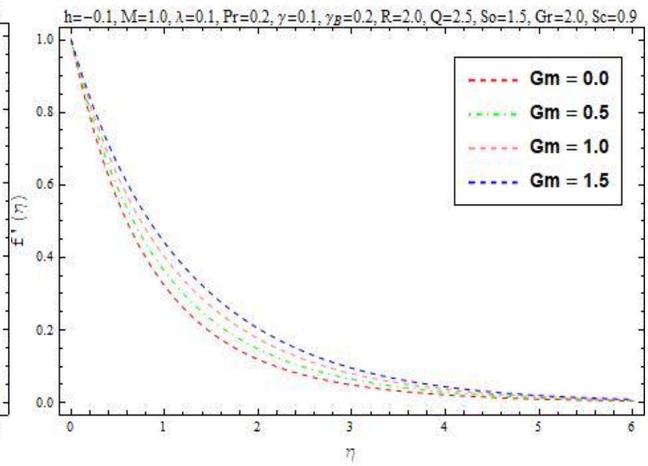

Figure 5: Effect of modified Grashof number on Gm on velocity profile

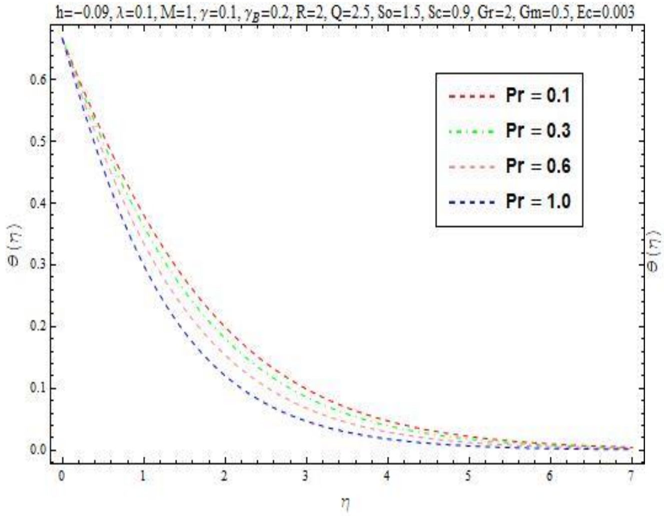

Figure 6: Effect of Prandtl number Pr on temperature profile

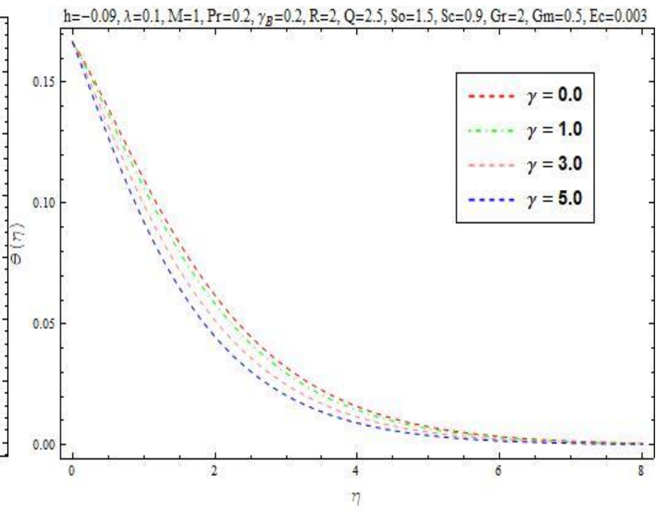

Figure 7: Effect of Thermal relaxation time parameter γ on temperature profile

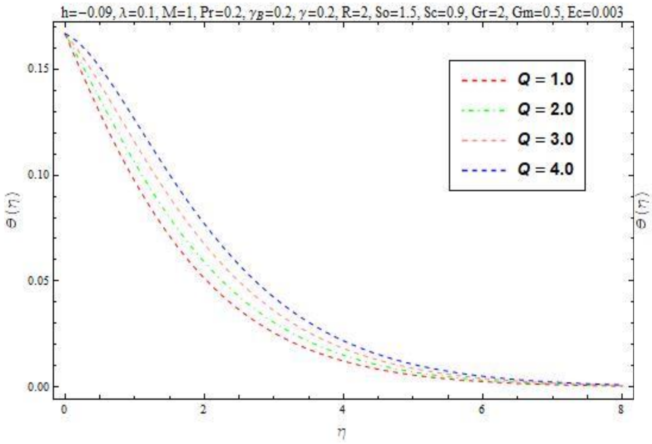

Figure 8: Effect of Heat source parameter Q on temperature profile

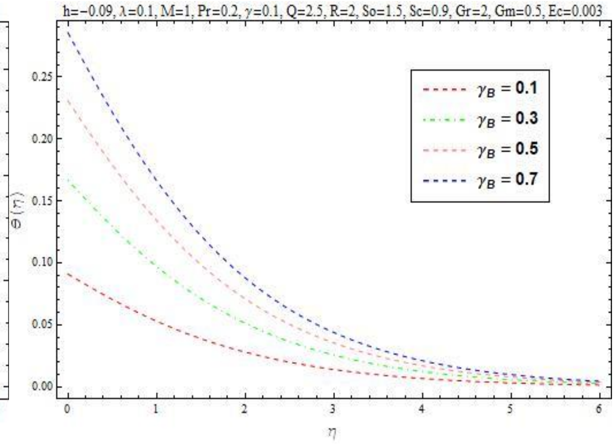

Figure 9: Effect of Biot number γ_B on temperature profile

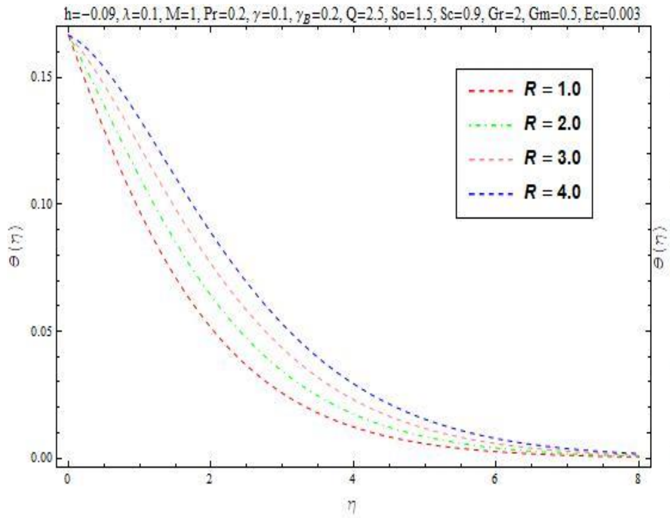

Figure 10: Effect of Radiation parameter R on temperature profile

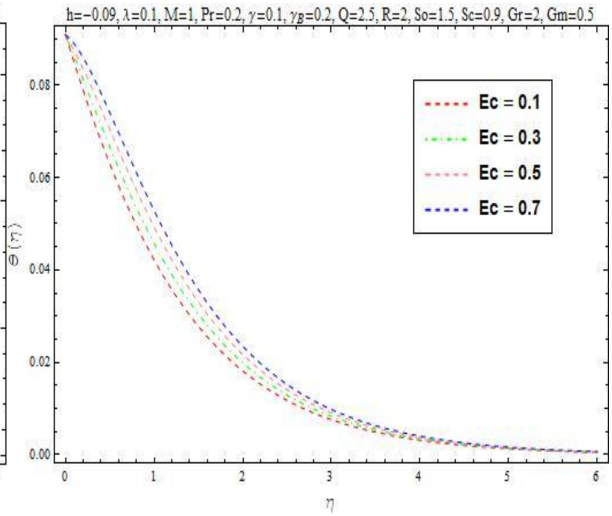

Figure 11: Effect of Eckert number Ec on temperature profile

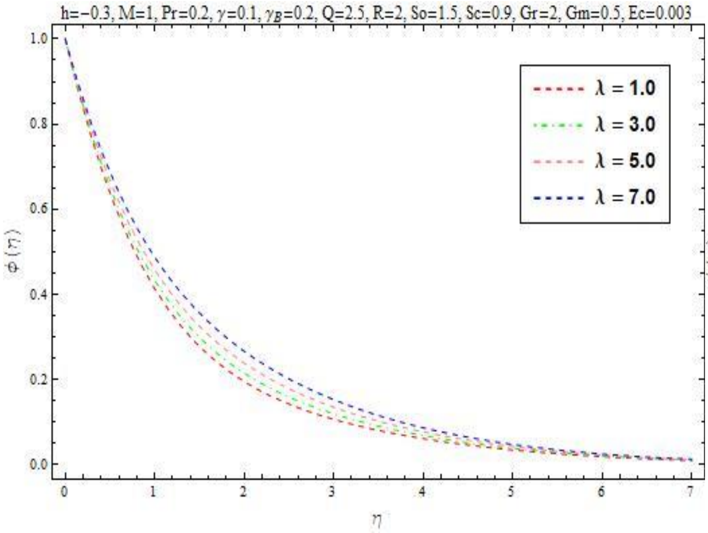

Figure 12: Effect of Williamson parameter λ on concentration profile

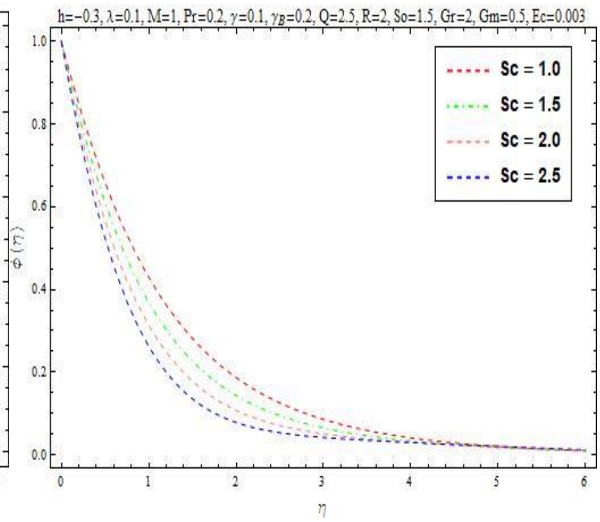

Figure 13: Effect of Schmidt number Sc on concentration profile

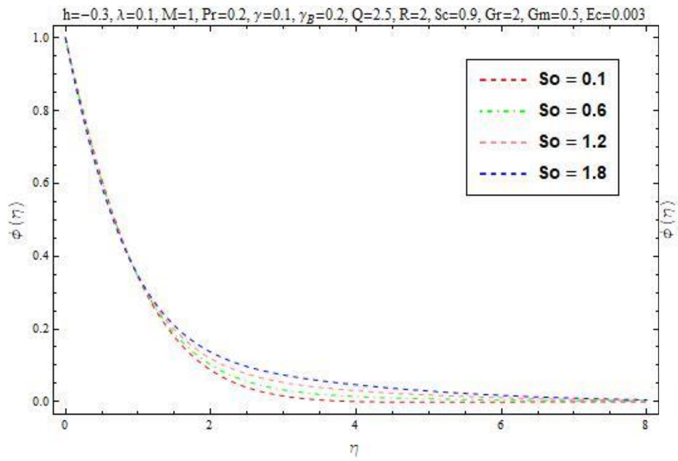

Figure 14: Effect of Soret number So concentration

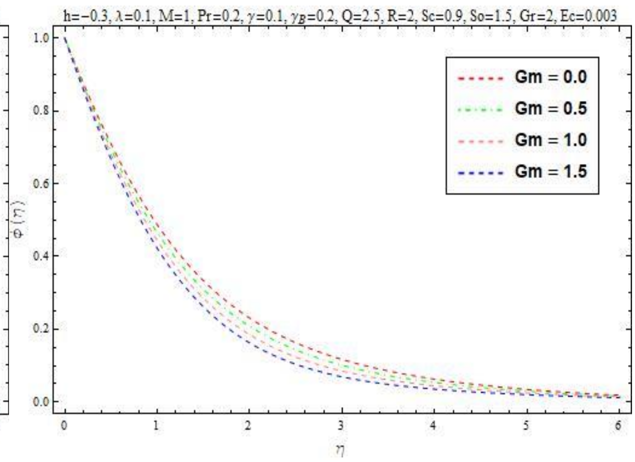

Figure 15: Effect of modified Grashof number on Gm on concentration profile

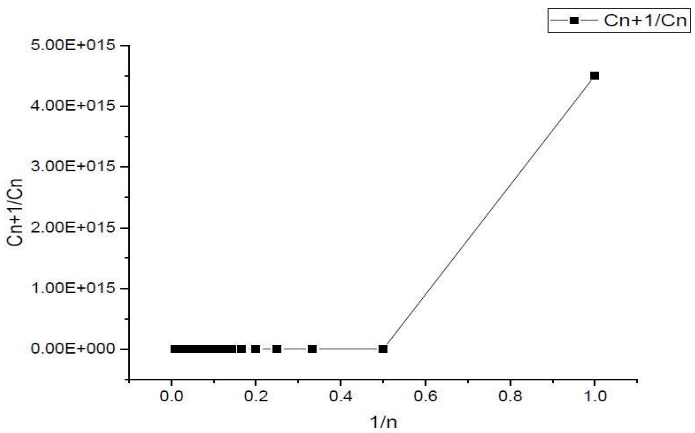

Figure 16: Domb-Sykes velocity plot

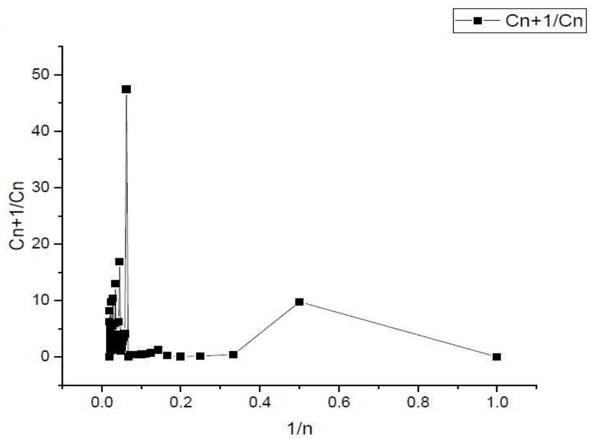

Figure 17: Domb-Sykes temperature plot

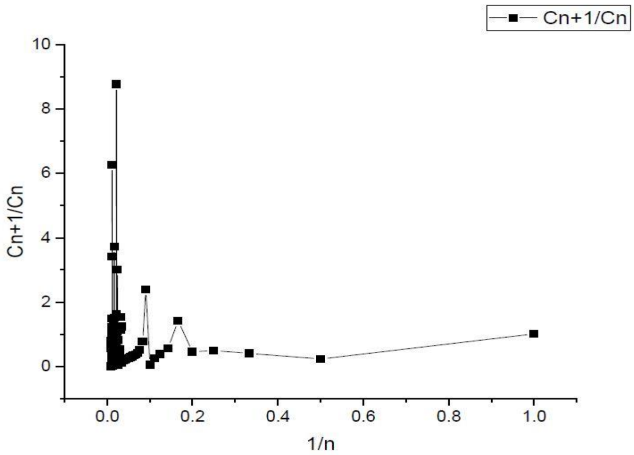

Figure 18: Domb-Sykes concentration plot

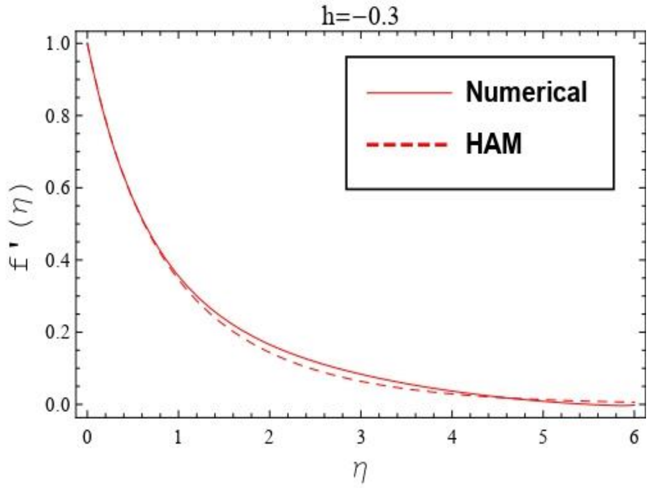

Figure 19: Comparison of HAM and numerical solution for Magnetic parameter $M = 0$ on velocity profile

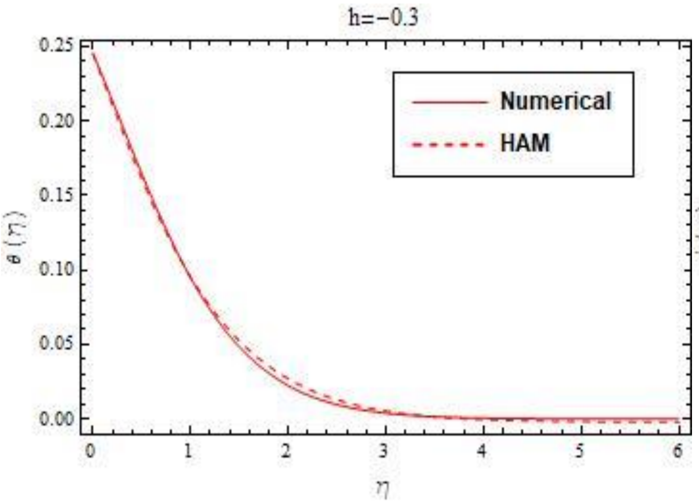

Figure 19: Comparison of HAM and numerical solution for Magnetic parameter $M = 0$ on temperature profile

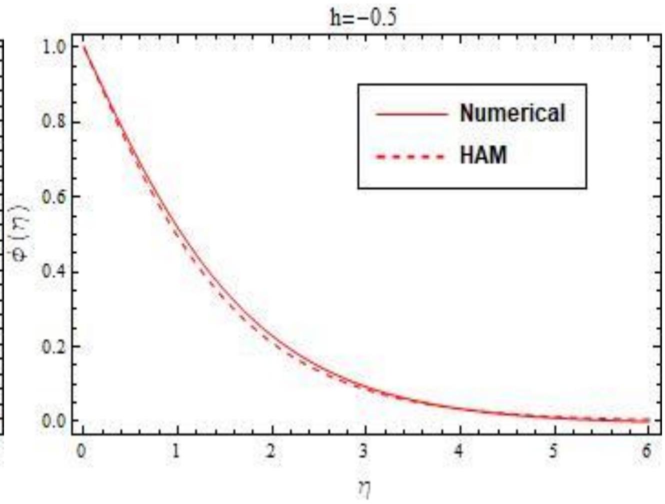

Figure 20: Comparison of HAM and numerical solution for Magnetic parameter $Sc = 1$ on concentration profile

Table 1: Comparision of $-f''(0)$ when $\lambda = M = Gr = Gm = 0$ and $h = -0.5$

$-f''(0)$		
Elbashbeshy [30]	Present work (HAM)	Present work (R K Method)
1.28181	1.28182	1.28182

Conflict of interest

The authors declare that there is no conflict of interest.

REFERENCES

1. Willamson, R. V., The flow of Pseudoplastic materials, *Industrial and Engineering Chemistry*, Vol. 21, No. 11, 1929, 1108-1111.
2. Christov, C. I. ,On Frame Indifferent Formulation of the Maxwell-Cattaneo Model of Finite-Speed Heat Conduction, *Mechanics Research Communications*, Vol. 36, No. 4, 2009, 481–486.
3. Han, S., Zheng, L., Li, C. and Zhang, X., Coupled Flow and Heat Transfer in Viscoelastic Fluid With Cattaneo-Christov Heat Flux Model,*Applied Mathematics Letters*, Vol. 38, 2014, 87–93.
4. Khan, M. S., Rahman, M. M., Arifuzzaman, S. M., Biswas, P. and Karim, I., Williamson Fluid Flow Behaviour Of MHD Convectiveradiative Cattaneo–Christov Heat Flux type Over A Linearly Stretched-Surface With Heat Generation and Thermal-Diffusion, *Frontiers in Heat and Mass Transfer*, Vol. 9, No. 15, 2017.
5. Bashir, S., Ramzan M., Ghazwani, H. A. S., Nisar, K. S., Saleel, C. A., and Abdelrahman A., Magnetic

Dipole and Thermophoretic Particle Deposition Impact on Bioconvective Oldroyd-B Fluid Flow over a Stretching Surface with Cattaneo–Christov Heat Flux, *Nanomaterials*, Vol. 12, 2022.

6. Bilal, S., Shah, M. I., Khan, N. Z., Akgul, A. A., and Nisar, K. S., Onset about non-isothermal flow of Williamson liquid over exponential surface by computing numerical simulation in perspective of Cattaneo Christov heat flux theory, *Alexandria Engineering Journal*, Vol. 61, 2022, 6139–6150.
7. Faizan, M., Ali, F., Loganathan, K., Zaib, A., Reddy, C. A., Abdelsalam, S. I., Entropy Analysis of Sutterby Nanofluid Flow over a Riga Sheet with Gyrotactic Microorganisms and Cattaneo–Christov Double Diffusion, *Mathematics*, Vol. 10, 2022, 3157.
8. Alharbi, K. A. M., Alshahrani, M. N., Ullah, N., Khan, N. M., Marek, K., Mousa, A. A. A. and Ali, S., Cattaneo–Christov heat flow model for copper–water nanofluid heat transfer under Marangoni convection and slip conditions, *Scientific Reports*, Vol. 12, 2022, 5260.
9. Mahabaleshwar, U. S., Sneha, K. N. and Hatami, M., Effect of Cattaneo_Christov approximation for viscoelastic fluid with carbon nanotubes on flow and heat transfer, *Scientific Reports*, Vol. 12, 2022, 9485.
10. Shahzad, A., Imran, M., Tahir, M., Khan, S. A., Akgul, A., Abdullaev, S., Park, C. and Zahran, H. Y., Brownian motion and thermophoretic diffusion impact on Darcy-Forchheimer flow of bioconvective micropolar nanofluid between double disks with Cattaneo-Christov heat flux, *Alexandria Engineering Journal*, 2022.
11. Reddy, K. V., Reddy, G. V. R. and Hari Krishna, Y., Effects of Cattaneo–Christov heat flux analysis on heat and mass transport of Casson nanoliquid past an accelerating penetrable plate with thermal radiationandSoret–Dufour mechanism, *Heat Transfer*, Vol. 50, 2021, 3458–3479.
12. Ahmad, M. W., McCash, L. B., Shah, Z. and Nawaz, R., Cattaneo-Christov Heat Flux Model for Second Grade Nanofluid Flow with Hall Eject through Entropy Generation over Stretchable Rotating Disk, *Coatings*, Vol. 10, 2020, 610.
13. Ahmed, A., Khan, M., Sarfraz, M., Ahmed, J., and Iqbal, Z., Forced convection in 3D Maxwell nanofluid flow via Cattaneo–Christov theory with Joule heating, *J Process Mechanical Engineering*, 2021, 1-11.
14. Akinbo, B. J. and Olajuwon, B. I.,Cattaneo-Christov Heat Flux And Heat Generation/Absorption Effect On Viscous Walters' B Fluid Through A Porous Medium With Chemical Reaction, *Journal of the Nigerian Mathematical Society*, Vol. 40, No. 3, 2021, 205-226.
15. Amjad, M., Ahmed, K., Akbar, T., Muhammad, T., Ahmed, T. and Alshomrani, A.S.,Numerical investigation of double diffusion heat flux model in Williamson nanofluid over an exponentially stretching surface with variable thermal conductivity, *Case Studies in Thermal Engineering*, Vol. 36, 2022, 102231
16. Prasad, K. V., Vaidya H., Vajravelu, K. and Ramanjini, V., Analytical Study of Cattaneo-Christov Heat Flux Model for Williamson-Nanofluid Flow Over a Slender Elastic Sheet with Variable Thickness, *Journal of Nanofluids*, Vol. 7, 2018, 583–594.
17. Prakash, J., Prasad, P. D., Kumar, G. V., Kumar, R. V. M. S. S. K. and Varma, S. V. K., Heat and Mass Transfer Hydromagnetic Radiative Casson Fluid Flow over an Exponentially Stretching Sheet with Heat Source or Sink, *International Journal of Engineering Science Invention*, Vol. 5, No. 7, 2016, 12-23.
18. Shehzad, S. A., Abbasi, F. M., Hayat, T. and Ahmad, B., Cattaneo-Christov heat flux model for third-grade fluid flow towards exponentially stretching sheet, *Applied Mathematics and Mechanics*, Vol. 37, No. 6, 2016, 761–768.
19. Shehzad, S. A., Abbasi, F. M., Hayat, T. and Alsaedi, A., Cattaneo-Christov heat flux model for Darcy-Forchheimer flow of an Oldroyd-B fluid with variable conductivity and non-linear convection, *Journal of Molecular Liquids*, 2016.
20. Ray A. K., Flow of Electrically Conducting Williamson Fluid with Cattaneo-Christov heat flux due to Permeable Sheet, *International Research Journal on Advanced Science Hub*, Vol. 2, No. 12, 2020, 17-22.
21. Yahya, A. U., Salamat, N., Habib, D., Ali, B., Hussain, S. and Abdal, S., Implication of Bio-convection and Cattaneo-Christov heat flux on Williamson Sutterby nanofluid transportation caused by a stretching surface with convective boundary, *Chinese Journal of Physics*, Vol. 73, 2021, 706–718.
22. Mangathai, P. and Sidhartha, B. R., Unsteady Mhd Williamson And Casson Nano Fluid Flow In The Presence Of Radiation And Viscous Dissipation, *Turkish Journal of Computer and Mathematics Education*, Vol.12, No.13, 2021, 1036-1051.
23. Jamshed, W., Nisar, K.S., Ibrahim, R. W., Mukhtar, K., Vijayakumar, V. and Ahmad, F., Computational frame work of Cattaneo-Christov heat flux effects on Engine Oil based Williamson hybrid nanofluids: A thermal case study, *Case Studies in Thermal*

Engineering, Vol. 26, 2021, 101179.

24. Hayat, T., Qayyum, S., Alsaedi, A. and Ahmad, B., Nonlinear convective flow with variable thermal conductivity and Cattaneo-Christov heat flux, *Neural Comput and Applic*, 2017.
25. Mondal, S. K., and Pal, D., Computational analysis of bioconvective flow of nanofluid containing gyrotactic microorganisms over a nonlinear stretching sheet with variable viscosity using HAM, *Journal of Computational Design and Engineering*, Vol. 7, No. 2, 2020, 251–267.
26. Rubab, K. and Mustafa, M., Cattaneo-Christov Heat Flux Model for MHD Three-Dimensional Flow of Maxwell Fluid over a Stretching Sheet, *PLOS ONE*, Vol. 11, No. 4, 2016, 1-16.
27. Salahuddin, T., Malik, M. Y., Hussain, A., Bilal, S. and Awais, M., MHD Flow of Cattanneo-Christov Heat Flux Model for Williamson Fluid Over a Stretching Sheet With Variable Thickness: Using Numerical Approach, *Journal of Magnetism and Magnetic Materials*, 2015, 1–14.
28. Kumar, K. A., Reddy J. V. R., Sugunamma V., and Sandeep N., Magnetohydrodynamic Cattaneo-Christov flow past a cone and a wedge with variable heat source/sink, *Alexandria Engineering Journal*, 2016, 1-9.
29. Bhatti, M. M. and Rashidi, M. M., Effects of Thermo-Diffusion and Thermal Radiation on Williamson Nanofluid over a Porous Shrinking/Stretching Sheet, 2016.
30. Elbashbeshy, E. M. A., Heat transfer over an exponentially stretching continuous surface with suction, *Arch. Mech.*, Vol. 53, No. 6, 2001, 643-651.
31. Liao, S. J., *Homotopy analysis method in nonlinear differential equations*, Springer, 2011.
32. Liao, S. J., *Advances in the homotopy analysis method*, World Scientific Publishing Co. Pte. Ltd., 2013.
33. Liao, S. J.,*Beyond Perturbation: Introduction to the Homotopy Analysis Method*, Chapman and Hall/CRC Press, Boca Raton, 2003.
34. Liao, S. J., On the homotopy analysis method for nonlinear problems, *Journal of Applied Mathematics and Computation*, Vol. 147, 2004, 499-513.
35. Liao, S. J., Comparision between the homotopy analysis method and homotopy perturbation method, *Applied Mathematics and Computations*, Vol. 169, 2005, 1186-1194.
36. Achala, L. N. and Sathyanarayana, S. B., Homotopy Analysis Method For Nonlinear Boundary Value Problems, *JP of Applied Mathematics*, Vol. 5, No. 1 & 2, 2012, 27-46.
37. Sathyanarayana, S. B. and Achala, L. N., Approximate Analytical Solution of Magnetohydrodynamics Compressible Boundary Layer flow with Pressure Gradient and suction/injection, *Journal of Advances in Physics*, Vol. 6, No. 3, 2014.
38. Sathyanarayana, S. B. and Achala, L. N., Approximate analytic Solution of Compressible Boundary Layer flow with an adverse Pressure Gradient by Homotopy Analysis Method, *Theoritical Mathematics & Applications*, Vol. 5, No. 1, 2015, 15-31.
39. Achala, L. N., Madusudhan, R. and Sathyanarayana, S. B., Semi Analytic Approximate Solution Of Nonlinear Partial Differential Equations, *International Journal of Mathematics and Computer Research*, Vol. 4, No. 6, 2016, 1418-1428.
40. Sushma,V. J., Raju B. T., Achala, L. N. and Sathyanarayana, S. B., MHD boundary layer flow of nanofluid over a moving surface in presence of thermal radiation by homotopy analysis method, *Global Journal of Engineering Science and Researches*, 2019, 435-443.
41. Rekha, K., Asha, C. S. and Achala, L. N., Williamson Nanofluid Flow over a Moving Surface on Boundary Layer by Homotopy Analysis Method, *International Journal of Mechanical Engineering*, Vol. 7, No. 2, 2022, 1723–1736.
42. Rekha, K., Asha, C. S. and Achala, L. N., Magnetohydrodynamic Boundary Layer Flow of Williamson Nanofluid over a Moving Surface in the presence of Gyrotactic Microorganism, *International Journal of Mechanics and Thermodynamics*, Vol. 13, No. 1, 2022, 15-31.