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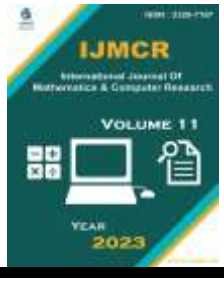

Some Contribution to Product of Strong Neutrosophic Graphs

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ABSTRACT

In this work we developed the α- cut worthy(Level) Graphs of Homomorphic, Box dot, Star Product of Strong Neutrosophic Graphs. To explore some propositions, theorems and Examples.

KEYWORDS: Strong Neutrosophic Graphs, α- cut worthy(Level) Graphs, Homomorphic, Box dot, Star Product of Strong Neutrosophic Graphs.

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1. INTRODUCTION

Graph Theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects, a graph in this context is made up of vertices which are connected by edges. Although graph theory is one of the younger branches of mathematics, it is fundamental to a number of applied fields, including operations research, computer science, and social network analysis. Rosenfeld introduced fuzzy graph in 1975. The operations of Cartesian product, compositions of fuzzy graphs were defined by J.N. Mordeson and C.S. Peng [1]. Developed the degree of a node in some fuzzy graphs, A. Nagoorgani and K. Radha [3]. The degree of a Node in fuzzy graphs using these operations was discussed by A. Nagoorgani and K. Radha. F.smarandache Single Valued Neotrosophic Graphs and three regions: Truth (or) acceptance (T), rejection (F), and (neutrality) indeterminacy (I) degrees both to Nodes and Lines. In this chapter we discuss the basic concepts of α- cut worthy (Level) Graphs of Homomorphic, Box dot, Star Product of Strong Neutrosophic Graphs. To explore some propositions, theorems and examples of Neutrosophic Graphs by level graphs.

2. PRELIMINARIES

Definition 2.1.

A Neutrosophic Graph is of the form G = < N, L > Where,

(i) $N = \{a_i, a_j, a_k, \ldots a_z\}$ such that $\lambda_T : N \to [a, b]$, $\lambda_I : N \to [a, b]$ and $\lambda_F : N \to [a, b]$ denote the degree of membership, degree of membership and non- membership of the element $a_i \in N$, respectively, with a = 0 and b = 1.

$0 \le \lambda_T(a_i) + \lambda_I(a_i) + \lambda_F(a_i) \le 3$ for every $a_i \in N$, (i = 1, 2,…n)

(ii) $L \subseteq NxN$ Where $\eta_T : NxN \to [a, b]$, $\eta_I : NxN \to [a, b]$, and $\eta_F : NxN \to [a, b]$, are such that

$\eta_T(a_ia_j) \le \min[\lambda_T(a_i), \lambda_T(a_j),]$

$\eta_I(a_ia_j) \le \min[\lambda_I(a_i), \lambda_I(a_j)]$,

$\eta_F(a_ia_j) \le \max[\lambda_F(a_i), \lambda_F(a_j)]$, and

$0 \le \eta_T(a_ia_j) + \eta_I(a_ia_j) + \eta_F(a_ia_j) \le 3$, for every $(a_ia_j) \in L$ (i,j = 1,2,…,n)

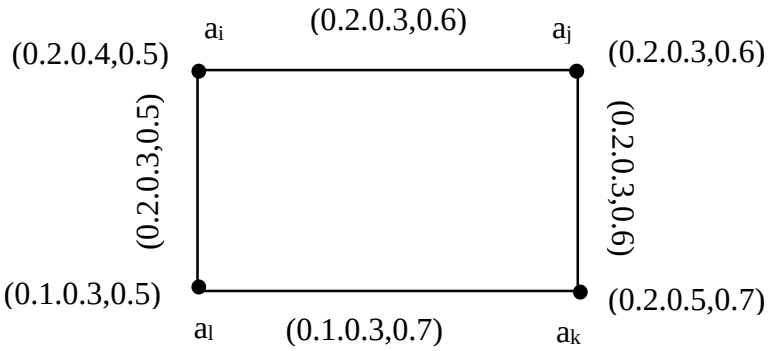

Figure 1: Neutrosophic Graph

Definition 2.2.

A Neutrosophic Graph NG = < N, L > with the triplet ($\lambda_T, \lambda_I, \lambda_F$) and (η_T, η_I, η_F) is called Strong Neutrosophic Graph if

$$\eta_T(a_ia_j) = \min[\lambda_T(a_i), \lambda_T(a_j)]$$
$$\eta_I(a_ia_j) = \min[\lambda_I(a_i), \lambda_I(a_j)]$$
$$\eta_F(a_ia_j) = \max[\lambda_F(a_i), \lambda_F(a_j)], \text{ for all } (a_ia_j) \in L.$$

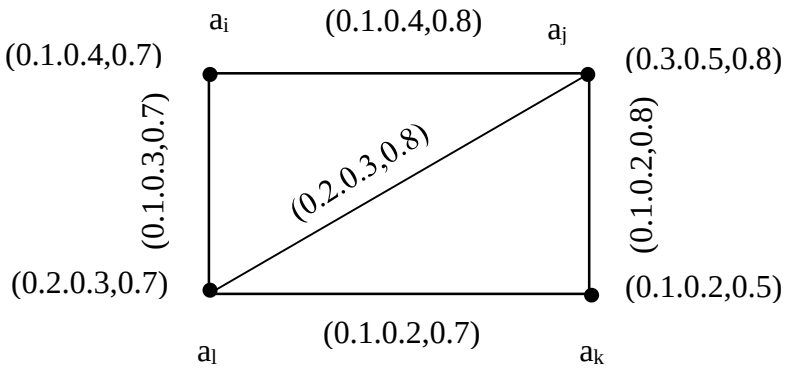

Figure 2: StrongNeutrosophic Graph

3. MAIN RESULT

Homomorphic product of Strong Neutrosophic Graphs

Definition 1. Let $(SNG)_{1_H} = (\lambda_1, \eta_I)$ and $(SNG)_{2_H} = (\lambda_2, \eta_2)$ be two Strong Neutrosophic Graphs corresponding to the crisp graph $((SNG)_{1_H})^* = (N_1, L_1)$ and $((SNG)_{2_H})^* = (N_2, L_2)$. Then the Homomorphic Product of Strong Neutrosophic Graphs is defined $(SNG)_{1_H}$ *and* $(SNG)_{2_H}$ is a pair of functions $(\lambda_1 \Diamond \lambda_2, \eta_1 \Diamond \eta_2,)$ with underlying node set

$\lambda_1 \Diamond \lambda_2 = \{(a_i,b_i): a_i \in N_1 \text{ and } b_i \in N_2\}$ and underlying line set

$\eta_1 \Diamond \eta_2 = \{((a_i,b_i)(a_j,b_j)): a_i = a_j, b_ib_j \in L_2 \text{ or } a_ia_j \in L_1, b_ib_j \notin L_2\}$ with

$\lambda_1 \Diamond \lambda_2)(a_i, b_i) = \min((\lambda_1)_T(a_i), (\lambda_2)_T(b_i)), \text{where } a_i \in N_1 \text{ and } b_i \in N_2.$

$(\eta_1 \Diamond \eta_2)((a_i, b_i)(a_j, b_j)) = \min((\lambda_1)_T(a_i), (\eta_2)_T(b_ib_j)), \text{if } a_i = a_j \text{ and } b_ib_j \in L_2.$

$(\eta_1 \Diamond \eta_2)((a_i, b_i)(a_j, b_j)) = \min((\eta_1)_T(a_i, a_j), (\lambda_2)_T(b_i), (\lambda_2)_T(b_j)), \text{if } a_ia_j \in L_1 \text{ and } b_ib_j \notin L_2.$

Boxdot Product of Strong Neutrosophic Graph:

Definition 2. Let $(SNG)_{1_B} = (\lambda_1, \eta_I)$ and $(SNG)_{2_B} = (\lambda_2, \eta_2)$ be two Strong Neutrosophic Graphs corresponding to the crisp graph $((SNG)_{1_B})^* = (N_1, L_1)$ and $((SNG)_{2_B})^* = (N_2, L_2)$. Then the Box dot Product of Strong Neutrosophic Graphs is defined $(SNG)_{1_B}$ *and* $(SNG)_{2_B}$ is a pair of functions

$(\lambda_1 \boxdot \lambda_2, \eta_1 \boxdot \eta_2,)$ with underlying node set

$\lambda_1 \boxdot \lambda_2 = \{(a_i,b_i): a_i \in N_1 \text{ and } b_i \in N_2\}$ and underlying line set

$\eta_1 \boxdot \eta_2 = \{((a_i,b_i)(a_j,b_j)): a_i = a_j, b_ib_j \notin L_2 \text{ or } a_ia_j \in L_1, b_ib_j \notin L_2\}$ with

$(\lambda_1 \boxdot \lambda_2)_T(a_i,b_i) = \min((\lambda_1)_T(a_i),(\lambda_2)_T(b_i)$, where $a_i \in N_1$ and $b_i \in N_2$.

$(\eta_1 \boxdot \eta_2)_T((a_i,b_i)(a_j,b_j)) = \min((\lambda_1)_T(a_i),(\lambda_2)_T(b_i),(\lambda_2)_T(b_j)$, if $a_i = a_j$ and $b_ib_j \notin L_2$.

$(\eta_1 \boxdot \eta_2)_T((a_i,b_i)(a_j,b_j)) = \min(\eta_1(a_i,a_j)_T,\lambda_2(b_i),\lambda_2(b_j)$ if $a_ia_j \in L_1$ and $b_ib_j \notin L_2$.

Star Product of Strong Neutrosophic Graph:

Definition 3. Let $(SNG)_{1S} = (\lambda_1,\eta_1)$ and $(SNG)_{2S} = (\lambda_2,\eta_2)$ be two Strong Neutrosophic Graphs corresponding to the crisp graph $((SNG)_{1S})^* = (N_1,L_1)$ and $((SNG)_{2S})^* = (N_2,L_2)$. Then the Star Product of Strong Neutrosophic Graphs is define$(SNG)_{1S}$ *and* $(SNG)_{2S}$ is a pair of functions

$(\lambda_1 * \lambda_2, \eta_1 * \eta_2,)$ with underlying node set

$\lambda_1 * \lambda_2 = \{(a_i,b_i): a_i \in N_1 \text{ and } b_i \in N_2\}$ and underlying line set

$\eta_1 * \eta_2 = \{((a_i,b_i)(a_j,b_j)): a_i = a_j, b_ib_j \notin L_2 \text{ or } a_ia_j \in L_1, b_ib_j \in L_2\}$ with

$(\lambda_1 * \lambda_2)_T(a_i,b_i) = \min((\lambda_1)_T(a_i),(\lambda_2)_T(b_i))$, where $a_i \in N_1$ and $b_i \in N_2$.

$(\eta_1 * \eta_2)_T((a_i,b_i)(a_j,b_j)) = \min((\lambda_1)_T(a_i),(\lambda_2)_T(b_i),(\lambda_2)_T(b_j)$, if $a_i = a_j$ and $b_ib_j \notin L_2$.

$(\eta_1 * \eta_2)_T((a_i,b_i)(a_j,b_j)) = \min((\eta_1)_T(a_i,a_j),(\eta_2)_T(b_ib_j))$ if $a_ia_j \in L_1$ and $b_ib_j \notin L_2$.

and also for the indeterminancy, falsity.

Definition 4.

Let $\lambda = \{< c,\lambda_T(c),\lambda_I(c),\lambda_F(c)>, c \in N\}$, The α- cut worthy set of a Neutrosophic set λ of the set N is the crisp set λ_α is given by

$\lambda_\alpha = \{c \in N$:either $(\lambda_T(c) \geq \alpha, \lambda_I(c) \geq \alpha$ and $\lambda_F(c) \leq 1-\alpha\}$. where $\alpha \in [0,1]$.

Let $\eta = \{<ab,\eta_T(cc'),\eta_I(cc'),\eta_F(cc')>\}$, The α- cut worthy set of a Neutrosophic set η of the set $L \subseteq N \times N$ is the crisp set η_α is given by

$\eta_\alpha = \{ cc' \in L$: either $(\eta_T(cc') \geq \alpha, \eta_I(cc') \geq \alpha$ and $\eta_F(cc') \leq 1-\alpha\}$.

where $\alpha \in [0,1]$.

Example: 1.

In the Neutrosophic Graph NG = (λ,η) on non-empty set

N = {a_i, a_j, a_k} as shown in Figure 4.2.

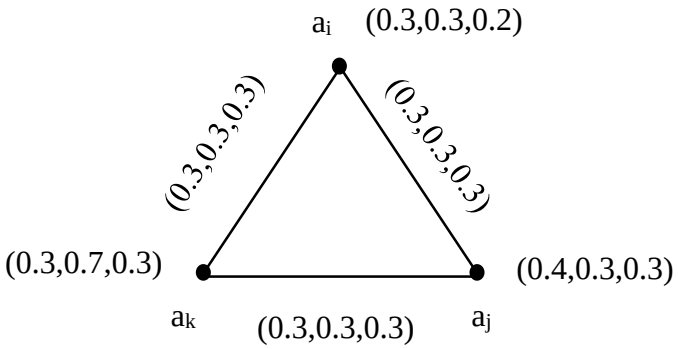

Figure 3: Neutrosophic Graph NG = (λ, η)

Let α = 0.3. We have $\lambda_{0.3} = \{a_i, a_j, a_k\}$, $\eta_{0.3} = \{a_ia_j, a_ja_k, a_ka_i\}$. Clearly, the 0.3 - cut worthy graph $G_{0.3} = (\lambda_{0.3}, \eta_{0.3})$is a crisp graph $G^* = (N, L)$.

Proposition 1. The cut worthy graph $NG_\alpha = (\lambda_\alpha, \eta_\alpha)$ is a crisp graph.

Theorem: 1.

If any two Homomorphic Product of Strong Neutrosophic Graphs implies a Strong Neutrosophic Graphs

Let SNG_1: (λ_1,η_1) and SNG_2: (λ_2,η_2) be two Neutrosophic Graphs corresponding to the crisp graph $(SNG_1)^*$: (N_1,L_1) and $(SNG_2)^*$: (N_2,L_2) respectively. Then SNG = (λ,η) is the Homomorphic Product of Strong Neutrosophic Graphs SNG_1 and SNG_2 for each $\alpha \in [0,1]$. The α- cut worthy graph SNG_α is the Homomorphic Product of Strong Neutrosophic Graphs $(SNG_1)_\alpha$ and

$(SNG_2)_\alpha$.

Proof:

Let NG = (λ,η) be the Homomorphic Product of Strong Neutrosophic Graphs SNG_1 and SNG_2 for each $\alpha \in [0, 1]$, if $(a_i,b_i) \in \lambda_\alpha$.

$$\min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} = (\lambda)_T(a_i, b_i) \geq \alpha$$
$$\min\{(\lambda_1)_I(a_i), (\lambda_2)_I(b_i)\} = (\lambda)_I(a_i, b) \geq \alpha$$
$$\max\{(\lambda_1)_F(a_i), (\lambda_2)_F(b_i)\} = (\lambda)_F(a_i, b_i) \leq 1 - \alpha$$
$$\text{if } a_i \in N_1 \text{ and } b_i \in N_2$$

so, $(a_i) \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$. i.e,$(a_i,b_i) \in (\lambda_1)_\alpha \Diamond (\lambda_2)_\alpha$. Therefore $\lambda_\alpha \subseteq (\lambda_1)_\alpha \Diamond (\lambda_2)_\alpha$.

Let $(a_i,b_i) \in (\lambda_1)_\alpha \Diamond (\lambda_2)_\alpha$.. then $a_i \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$. It follows that,

$\min(\lambda_1)_T(a_i), (\lambda_2)_T(b_i) \geq \alpha, \min(\lambda_1)_I(a_i), (\lambda_2)_I(b_i) \geq \alpha,$

$\max(\lambda_1)_F(a_i), (\lambda_2)_F(b_i) \leq 1 - \alpha.$

Since (λ,η) is the Homomorphic Product of SNG_1 and SNG_2.

$(\lambda)_T(a_i, b_i) \geq \alpha$, $(\lambda)_I(a_i, b_i) \geq \alpha$, $(\lambda)_F(a_i, b_i) \leq 1 - \alpha$. i.e., $(a_i) \in (\lambda_1)_\alpha$.

Therefore $(\lambda_1)_\alpha \Diamond (\lambda_2)_\alpha \subseteq \lambda_\alpha$ and so $(\lambda_1)_\alpha \Diamond (\lambda_2)_\alpha = \lambda_\alpha$

To prove η_α= L, L is a Line set of the Homomorphic Strong Neutrosophic graph

$(SNG_1)_\alpha \Diamond (SNG_2)_\alpha \; \forall \; \alpha \in [0, 1]$. Then $(a_i, b_i)(a_j, b_j)) \in \eta_\alpha$.

Then$(a_i, b_i)(a_j, b_j)) \geq \alpha$, $(a_i, b_i)(a_j, b_j)) \geq \alpha$, $(a_i, b_i)(a_j, b_j)) \leq 1 - \alpha$.

Since (λ,η) is the Homomorphic Product of NG_1 and NG_2.

$$\eta_T(a_i, b_i)(a_j, b_j)) = \min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T(b_i, b_j)\} \geq \alpha$$
$$\eta_T(a_i, b_i)(a_j, b_j)) = \min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T(b_i, b_j)\} \geq \alpha$$
$$\eta_F(a_i, b_i)(a_j, b)) = \min\{(\eta_1)_F(a_i, a_j), (\eta_2)_F(b_i, b_j)\} \leq 1 - \alpha$$
$$\text{if } a_i a_j \in L_1 \text{ and } b_i b_j \in L.$$

Similarly, to the node set

$(\eta_1 \Diamond \eta_2))_T((a_i, b_i)(a_j, b_j)) = \min\{((\lambda_1)_T(a_i), (\lambda_1)_T(a_j), (\lambda_2)_T(a_i)), (\lambda_2)_T(a_j)\},$

$$\text{if } a_1a_2 \notin L_1 \text{and} b_1b_2 \notin L_2.$$

Similarly, the results also apply for the intermediate and falsity values.

Conversely,

Suppose that NG_α: $(\lambda_\alpha, \eta_\alpha)$is the Homomorphic Product of Neutrosophic Graphs

$((SNG_1)_\alpha = ((\lambda_1)_\alpha, (\eta_1)_\alpha)$ and $(SNG_2)_\alpha = ((\lambda_2)_\alpha, (\eta_2)_\alpha)$ for each $\alpha \in [0, 1]$.

$$\min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha, \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha$$

$\max\{(\lambda_1)_F(a_i), (\lambda_2)_F(b_i)\} \leq 1 - \alpha.$ if $a_i \in N_1$ and $b_i \in N_2$.

$(a_i) \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$., by hypothesis $(a_i, b_i) \in (\lambda)_\alpha$

$$(\lambda)_T(a_i, b_i) \geq \alpha = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\}$$
$$(\lambda)_I(a_i, b_i) \geq \alpha = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\}$$
$$(\lambda)_F(a_i, b_i) \leq 1 - \alpha = \max\{(\lambda_1)_F(a_i), (\lambda_2)_F(b_i)\}$$

Take $(\lambda)_T(a_i, b_i) = \beta, (\lambda)_I(a_i, b_i) = \beta, (\lambda)_F(a_i, b_i) = 1 - \beta$, then $(a_i,b_i) \in (\lambda)_\beta$

Since $(\lambda_\beta, \eta_\beta)$ is the Homomorphic Product of Neutrosophic Graphs

$(SNG_1)_\beta = ((\lambda_1)_\beta, (\eta_1)_\beta)$ and $(SNG_2)_\beta = ((\lambda_2)_\beta, (\eta_2)_\beta)$

Then $(a_i) \in (\lambda_1)_\beta$ and $b_i \in (\lambda_2)_\beta$.

Hence , $(\lambda_1)_T(a_i) \geq \beta$,, $(\lambda_1)_I(a_i) \geq \beta$,, $(\lambda_1)_F(a_i) \leq 1 - \beta$, and

$(\lambda_2)_T(b_i) \geq \beta, (\lambda_2)_I(b_i) \geq \beta$,, $(\lambda_2)_F(b_i) \leq 1 - \beta$,

It follows that

$(\lambda)_T(a_i, b_i) = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha$, for all $(a_i, b_i) \in N_1 \Diamond N_2$

$$\text{if } a_i \in N_1 \text{ and } b_i \in N_2$$

Hence , $(\lambda_1)_T(a_i, a_j) \geq \alpha, (\lambda_1)_I(a_i, a_j) \geq \alpha, (\lambda_1)_F(a_i, a_j) \leq 1 - \alpha$ and

$(\lambda_2)_T(b_i, b_j) \geq \alpha, (\lambda_2)_I(b_i, b_j) \geq \alpha, (\lambda_2)_F(b_i, b_j) \leq 1 - \alpha,$

$(\eta_1)_T(a_i, a_j) \geq \beta, (\eta_1)_I(a_i, a_j) \geq \beta, (\eta_1)_F(a_i, a_j) \leq 1 - \beta$, and

$(\eta_2)_T(b_i, b_j) \geq \beta, (\eta_2)_I(b_i, b_j) \geq \beta, (\eta_2)_F(b_i, b_j) \leq 1 - \beta,$

$\min\{(\eta_1)_T(a_i, a_j), (\eta_2)_T\} = \alpha, \min\{(\eta_1)_I(a_i, a_j), (\eta_2)_I(b_i, b_j)\} = \alpha$

$\max\{(\eta_1)_F(a_i, a_j), (\eta_2)_F(b_i, b_j)\} = 1 - \alpha$

$\min\{(\eta_1)_T(a_i,a_j),(\eta_2)_T(b_i,b_j)\}=\beta, \min\{(\eta_1)_I(a_i,a_j),(\eta_2)_I(b_i,b_j)\}=\beta \quad \max\{(\eta_1)_F(a_i,a_j),(\eta_2)_F\}=1-\beta$

$$\eta_T(a_i,b_i)(a_j,b_j))\geq\alpha = \min\{(\eta_1)_T(a_i,a_j),(\eta_2)_T(b_i,b_j)\}$$
$$\eta_I(a_i,b_i)(a_j,b_j))\geq\alpha = \min\{(\eta_1)_I(a_i,a_j),(\eta_2)_I(b_i,b_j)\}$$
$$\eta_F(a_i,n_i)(a_j,b_j))\leq 1-\alpha = \max\{(\eta_1)_F(a_i,a_j),(\eta_2)_F(b_i,b_j)\}.$$
$$\text{if } a_ia_j\in L_1 \text{ and } b_ib_j\in L_2$$
$$\eta_T(a_i,b_i)(a_j,b_j))\geq\beta = \min\{(\eta_1)_T(a_i,a_j),(\eta_2)_T(b_i,b_j)\}$$
$$\eta_I(a_i,b_i)a_j,b_j))\geq\beta = \min\{(\eta_1)_I(a_i,a_j),(\eta_2)_I(b_i,b_j)\}$$
$$\eta_F(a_i,b_i)(a_j,b_j))\leq 1-\beta = \max\{(\eta_1)_F(a_i,a_j),(\eta_2)_F(b_i,b_j)\}.$$
$$\text{if } a_ia_j\in L_1 \text{ and } b_ib_j\in L_2$$

$$\eta_T(a_i,b_i)(a_j,b_j)) = \min\{(\eta_1)_T(a_i,a_j),(\eta_2)_T(b_i,b_j)\}$$
$$\eta_I(a_i,b_i)(a_j,b_j)) = \min\{(\eta_1)_I(a_i,a_j),(\eta_2)_I(b_i,b_j)\}$$
$$\eta_F(a_i,b_i)(a_j,b_j)) = \max\{(\eta_1)_F(a_i,a_j),(\eta_2)_F(b_i,b_j)\}$$
$$\text{if } a_ia_j\in L_1 \text{ and } b_ib_j\in L_2.$$

Theorem: 2.

If any two Bot dot Product of Strong Neutrosophic Graphs implies a Strong Neutrosophic Graphs

Let $SNG_1:(\lambda_1,\eta_1)$ and $SNG_2:(\lambda_2,\eta_2)$ be two Strong Neutrosophic Graphs corresponding to the crisp graph $(SNG_1)^*:(N_1,L_1)$ and $(SNG_2)^*:(N_2,L_2)$ respectively. Then SNG = (λ,η) is the bot dot Product of Strong Neutrosophic Graphs SNG_1 and SNG_2 for each $\alpha\in[0,1]$. The α- cut worthy graph SNG_α is the box dot Product of Strong Neutrosophic Graphs $(SNG_1)_\alpha$ and $(SNG_2)_\alpha$.

Proof:

Let SNG = (λ,η) be the Box dot Product of Strong Neutrosophic Graphs SNG_1 and SNG_2 for each $\alpha\in[0,1]$, if $(a_i,b_i)\in\lambda_{\alpha.}$

$$\min\{(\lambda_1)_T(a_i),(\lambda_2)_T(b_i)\}=(\lambda)_T(a_i,b_i)\geq\alpha$$
$$\min\{(\lambda_1)_I(a_i),(\lambda_2)_I(b_i)\}=(\lambda)_I(a_i,b)\geq\alpha$$
$$\max\{(\lambda_1)_F(a_i),(\lambda_2)_F(b_i)\}=(\lambda)_F(a_i,b_i)\leq 1-\alpha$$
$$\text{if } a_i\in N_1 \text{ and } b_i\in N_2$$

so, $(a_i)\in(\lambda_1)_\alpha$ and $b_i\in(\lambda_2)_{\alpha.}$ i.e,$(a_i,b_i)\in(\lambda_1)_\alpha\boxdot(\lambda_2)_{\alpha.}$ Therefore $\lambda_\alpha\subseteq(\lambda_1)_\alpha\Diamond(\lambda_2)_{\alpha.}$

Let $(a_i,b_i)\in(\lambda_1)_\alpha\boxdot(\lambda_2)_{\alpha.}$. then $a_i\in(\lambda_1)_\alpha$ and $b_i\in(\lambda_2)_\alpha$. It follows that,

$\min(\lambda_1)_T(a_i),(\lambda_2)_T(b_i)\geq\alpha, \min(\lambda_1)_I(a_i),(\lambda_2)_I(b_i)\geq\alpha,$

$\max(\lambda_1)_F(a_i),(\lambda_2)_F(b_i)\leq 1-\alpha.$

Since (λ,η) is the Box dot Product of SNG_1 and $SNG_{2.}$ $(\lambda)_T(a_i,b_i)\geq\alpha$, $(\lambda)_I(a_i,b_i)\geq\alpha$, $(\lambda)_F(a_i,b_i)\leq 1-\alpha$. i.e., $(a_i)\in(\lambda_1)_{\alpha.}$

Therefore $(\lambda_1)_\alpha\boxdot(\lambda_2)_\alpha\subseteq\lambda_\alpha$ and so $(\lambda_1)_\alpha\Diamond(\lambda_2)_\alpha=\lambda_\alpha$

To prove $\eta_{\alpha.}$= L, L is a Line set of the Box dot Strong Neutrosophic Graph

$(NG_1)_\alpha\boxdot(NG_2)_\alpha\ \forall\ \alpha\in[0,1]$. Then $(a_i,b_i)(a_j,b_j))\in\eta_\alpha$.

Then$(a_i,b_i)(a_j,b_j))\geq\alpha$, $(a_i,b_i)(a_j,b_j))\geq\alpha$, $(a_i,b_i)(a_j,b_j))\leq 1-\alpha$.

Since (λ,η) is the Box dot Product of SNG_1 and SNG_2.

$$\eta_T(a_i,b_i)(a_j,b_j))=\min\{(\eta_1)_T(a_i,a_j),(\eta_2)_T(b_i,b_j)\}\geq\alpha$$
$$\eta_T(a_i,b_i)(a_j,b_j))=\min\{(\eta_1)_T(a_i,a_j),(\eta_2)_T(b_i,b_j)\}\geq\alpha$$
$$\eta_F(a_i,b_i)(a_j,b))=\min\{(\eta_1)_F(a_i,a_j),(\eta_2)_F(b_i,b_j)\}\leq 1-\alpha$$
$$\text{if } a_ia_j\in L_1 \text{ and } b_ib_j\in L.$$

Similarly, to the node set

$(\eta_1\boxdot\eta_2))_T((a_i,b_i)(a_j,b_j))=\min\{((\lambda_1)_T(a_i),(\lambda_1)_T(a_j),(\lambda_2)_T(a_i)),(\lambda_2)_T(a_j)\},$

$$\text{if } a_1a_2\notin L_1 \text{and} b_1b_2\notin L_{2.}$$

Similarly, the results also apply for the intermediate and falsity values.

Conversely,

Suppose that $SNG_\alpha:(\lambda_\alpha,\eta_\alpha)$ is the Box dot Product of Strong Neutrosophic Graphs

$((NG_1)_\alpha=((\lambda_1)_\alpha,(\eta_1)_\alpha)$ and $(NG_2)_\alpha=((\lambda_2)_\alpha,(\eta_2)_\alpha)$ for each $\alpha\in[0,1]$.

$$\min\{(\lambda_1)_T(a_i),(\lambda_2)_T(b_i)\} \geq \alpha, \min\{(\lambda_1)_T(a_i),(\lambda_2)_T(b_i)\} \geq \alpha$$

$\max\{(\lambda_1)_F(a_i),(\lambda_2)_F(b_i)\} \leq 1-\alpha$. if $a_i \in N_1$ and $b_i \in N_2$.

$(a_i) \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$., by hypothesis $(a_i,b_i) \in (\lambda)_\alpha$

$$(\lambda)_T(a_i,b_i) \geq \alpha = \min\{(\lambda_1)_T(a_i),(\lambda_2)_T(b_i)\}$$
$$(\lambda)_I(a_i,b_i) \geq \alpha = \min\{(\lambda_1)_T(a_i),(\lambda_2)_T(b_i)\}$$
$$(\lambda)_F(a_i,b_i) \leq 1-\alpha = \max\{(\lambda_1)_F(a_i),(\lambda_2)_F(b_i)\}$$

Take $(\lambda)_T(a_i,b_i)=\beta, (\lambda)_I(a_i,b_i)=\beta, (\lambda)_F(a_i,b_i)=1-\beta$, then $(a_i,b_i) \in (\lambda)_\beta$

Since $(\lambda_\beta,\eta_\beta)$ is the box dot Product of Strong Neutrosophic Graphs

$(SNG_1)_\beta = ((\lambda_1)_\beta,(\eta_1)_\beta)$ and $(SNG_2)_\beta = ((\lambda_2)_\beta,(\eta_2)_\beta)$

Then $(a_i) \in (\lambda_1)_\beta$ and $b_i \in (\lambda_2)_\beta$.

Hence , $(\lambda_1)_T(a_i) \geq \beta$,, $(\lambda_1)_I(a_i) \geq \beta$,, $(\lambda_1)_F(a_i) \leq 1-\beta$, and

$$(\lambda_2)_T(b_i) \geq \beta, (\lambda_2)_I(b_i) \geq \beta,, (\lambda_2)_F(b_i) \leq 1-\beta.$$

It follows that

$(\lambda)_T(a_i,b_i) = \min\{(\lambda_1)_T(a_i),(\lambda_2)_T(b_i)\} \geq \alpha$, for all $(a_i,b_i) \in N_1 \boxdot N_2$

if $a_i \in N_1$ and $b_i \in N_2$

Hence , $(\lambda_1)_T(a_i,a_j) \geq \alpha, (\lambda_1)_I(a_i,a_j) \geq \alpha, (\lambda_1)_F(a_i,a_j) \leq 1-\alpha$ and

$$(\lambda_2)_T(b_i,b_j) \geq \alpha, (\lambda_2)_I(b_i,b_j) \geq \alpha, (\lambda_2)_F(b_i,b_j) \leq 1-\alpha,$$
$$(\eta_1)_T(a_i,a_j) \geq \beta, (\eta_1)_I(a_i,a_j) \geq \beta, (\eta_1)_F(a_i,a_j) \leq 1-\beta, \text{ and}$$
$$(\eta_2)_T(b_i,b_j) \geq \beta, (\eta_2)_I(b_i,b_j) \geq \beta, (\eta_2)_F(b_i,b_j) \leq 1-\beta,$$
$$\min\{(\eta_1)_T(a_i,a_j),(\eta_2)_T\} = \alpha, \min\{(\eta_1)_I(a_i,a_j),(\eta_2)_I(b_i,b_j)\} = \alpha$$
$$\max\{(\eta_1)_F(a_i,a_j),(\eta_2)_F(b_i,b_j)\} = 1-\alpha$$
$$\min\{(\eta_1)_T(a_i,a_j),(\eta_2)_T(b_i,b_j)\} = \beta, \min\{(\eta_1)_I(a_i,a_j),(\eta_2)_I(b_i,b_j)\} = \beta \quad \max\{(\eta_1)_F(a_i,a_j),(\eta_2)_F\} = 1-\beta$$

$$\eta_T(a_i,b_i)(a_j,b_j)) \geq \alpha = \min\{(\eta_1)_T(a_i,a_j),(\eta_2)_T(b_i,b_j)\}$$
$$\eta_I(a_i,b_i)(a_j,b_j)) \geq \alpha = \min\{(\eta_1)_I(a_i,a_j),(\eta_2)_I(b_i,b_j)\}$$
$$\eta_F(a_i,n_i)(a_j,b_j)) \leq 1-\alpha = \max\{(\eta_1)_F(a_i,a_j),(\eta_2)_F(b_i,b_j)\}.$$

if $a_ia_j \in L_1$ and $b_ib_j \in L_2$

$$\eta_T(a_i,b_i)(a_j,b_j)) \geq \beta = \min\{(\eta_1)_T(a_i,a_j),(\eta_2)_T(b_i,b_j)\}$$
$$\eta_I(a_i,b_i)a_j,b_j)) \geq \beta = \min\{(\eta_1)_I(a_i,a_j),(\eta_2)_I(b_i,b_j)\}$$
$$\eta_F(a_i,b_i)(a_j,b_j)) \leq 1-\beta = \max\{(\eta_1)_F(a_i,a_j),(\eta_2)_F(b_i,b_j)\}.$$

if $a_ia_j \in L_1$ and $b_ib_j \in L_2$

$$\eta_T(a_i,b_i)(a_j,b_j)) = \min\{(\eta_1)_T(a_i,a_j),(\eta_2)_T(b_i,b_j)\}$$
$$\eta_I(a_i,b_i)(a_j,b_j)) = \min\{(\eta_1)_I(a_i,a_j),(\eta_2)_I(b_i,b_j)\}$$
$$\eta_F(a_i,b_i)(a_j,b_j)) = \max\{(\eta_1)_F(a_i,a_j),(\eta_2)_F(b_i,b_j)\}$$

if $a_ia_j \in L_1$ and $b_ib_j \in L_2$.

Theorem: 3.

If any two Star Product of Strong Neutrosophic Graphs implies a Strong Neutrosophic Graphs

Let $SNG_1:(\lambda_1,\eta_1)$ and $SNG_2:(\lambda_2,\eta_2)$ be two Strong Neutrosophic Graphs corresponding to the crisp graph $(SNG_1)^*:(N_1,L_1)$ and $(SNG_2)^*:(N_2,L_2)$ respectively. Then SNG = (λ,η) is the Star Product of Strong Neutrosophic Graphs SNG_1 and SNG_2 for each $\alpha \in [0,1]$. The α- cut worthy graph SNG_α is the Star Product of Strong Neutrosophic Graphs $(SNG_1)_\alpha$ and $(SNG_2)_\alpha$.

Proof:

Let SNG = (λ,η) be the Star Product of Strong Neutrosophic Graphs SNG_1 and SNG_2 for each $\alpha \in [0,1]$, if $(a_i,b_i) \in \lambda_\alpha$.

$$\min\{(\lambda_1)_T(a_i),(\lambda_2)_T(b_i)\} = (\lambda)_T(a_i,b_i) \geq \alpha$$
$$\min\{(\lambda_1)_I(a_i),(\lambda_2)_I(b_i)\} = (\lambda)_I(a_i,b) \geq \alpha$$
$$\max\{(\lambda_1)_F(a_i),(\lambda_2)_F(b_i)\} = (\lambda)_F(a_i,b_i) \leq 1-\alpha$$

if $a_i \in N_1$ and $b_i \in N_2$

so, $(a_i) \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$. i.e,$(a_i,b_i) \in (\lambda_1)_\alpha * (\lambda_2)_\alpha$. Therefore $\lambda_\alpha \subseteq (\lambda_1)_\alpha \Diamond (\lambda_2)_\alpha$.

Let $(a_i,b_i) \in (\lambda_1)_\alpha * (\lambda_2)_\alpha$.. then $a_i \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$. It follows that,

$\min(\lambda_1)_T(a_i), (\lambda_2)_T(b_i) \geq \alpha, \min(\lambda_1)_I(a_i), (\lambda_2)_I(b_i) \geq \alpha,$
$\max(\lambda_1)_F(a_i), (\lambda_2)_F(b_i) \leq 1-\alpha.$
Since (λ,η) is the Star Product of SNG_1 and SNG_2. $(\lambda)_T(a_i,b_i) \geq \alpha$, $(\lambda)_I(a_i,b_i) \geq \alpha$, $(\lambda)_F(a_i,b_i) \leq 1-\alpha$. i.e., $(a_i) \in (\lambda_1)_\alpha$.
Therefore $(\lambda_1)_\alpha * (\lambda_2)_\alpha \subseteq \lambda_\alpha$ and so $(\lambda_1)_\alpha \Diamond (\lambda_2)_\alpha = \lambda_\alpha$
To prove η_α.= L, L is a Line set of the Box dot Strong Neutrosophic Graph
$(SNG_1)_\alpha * (SNG_2)_\alpha \; \forall \; \alpha \in [0,1]$. Then $(a_i,b_i)(a_j,b_j)) \in \eta_\alpha$.
Then$(a_i,b_i)(a_j,b_j)) \geq \alpha$, $(a_i,b_i)(a_j,b_j)) \geq \alpha$, $(a_i,b_i)(a_j,b_j)) \leq 1-\alpha$.
Since (λ,η) is the Box dot Product of SNG_1 and SNG_2.

$$\eta_T(a_i,b_i)(a_j,b_j)) = \min\{(\eta_1)_T(a_i,a_j), (\eta_2)_T(b_i,b_j)\} \geq \alpha$$
$$\eta_T(a_i,b_i)(a_j,b_j)) = \min\{(\eta_1)_T(a_i,a_j), (\eta_2)_T(b_i,b_j)\} \geq \alpha$$
$$\eta_F(a_i,b_i)(a_j,b)) = \min\{(\eta_1)_F(a_i,a_j), (\eta_2)_F(b_i,b_j)\} \leq 1-\alpha$$
$$\text{if } a_ia_j \in L_1 \text{ and } b_ib_j \in L.$$

Similarly, to the node set
$(\eta_1 * \eta_2))_T((a_i,b_i)(a_j,b_j)) = \min\{((\lambda_1)_T(a_i),(\lambda_1)_T(a_j),(\lambda_2)_T(a_i)),(\lambda_2)_T(a_j)\}$,
$$\text{if } a_1a_2 \notin L_1 \text{and} b_1b_2 \notin L_2.$$
Similarly, the results also apply for the intermediate and falsity values.
Conversely,
Suppose that SNG_α: $(\lambda_\alpha,\eta_\alpha)$is the Box dot Product of Strong Neutrosophic Graphs
$((SNG_1)_\alpha = ((\lambda_1)_\alpha,(\eta_1)_\alpha)$ and $(SNG_2)_\alpha = ((\lambda_2)_\alpha,(\eta_2)_\alpha)$ for each $\alpha \in [0,1]$.
$$\min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha, \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha$$
$\max\{(\lambda_1)_F(a_i), (\lambda_2)_F(b_i)\} \leq 1-\alpha$. if $a_i \in N_1$ and $b_i \in N_2$.
$(a_i) \in (\lambda_1)_\alpha$ and $b_i \in (\lambda_2)_\alpha$., by hypothesis $(a_i,b_i) \in (\lambda)_\alpha$
$$(\lambda)_T(a_i,b_i) \geq \alpha = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\}$$
$$(\lambda)_I(a_i,b_i) \geq \alpha = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\}$$
$$(\lambda)_F(a_i,b_i) \leq 1-\alpha = \max\{(\lambda_1)_F(a_i), (\lambda_2)_F(b_i)\}$$
Take $(\lambda)_T(a_i,b_i) = \beta, (\lambda)_I(a_i,b_i) = \beta, (\lambda)_F(a_i,b_i) = 1-\beta$, then $(a_i,b_i) \in (\lambda)_\beta$
Since $(\lambda_\beta,\eta_\beta)$ is the box dot Product of Strong Neutrosophic Graphs
$(NG_1)_\beta = ((\lambda_1)_\beta,(\eta_1)_\beta)$ and $(NG_2)_\beta = ((\lambda_2)_\beta,(\eta_2)_\beta)$
Then $(a_i) \in (\lambda_1)_\beta$ and $b_i \in (\lambda_2)_\beta$.
Hence , $(\lambda_1)_T(a_i) \geq \beta$,, $(\lambda_1)_I(a_i) \geq \beta$,, $(\lambda_1)_F(a_i) \leq 1-\beta$, and
$(\lambda_2)_T(b_i) \geq \beta, (\lambda_2)_I(b_i) \geq \beta$,, $(\lambda_2)_F(b_i) \leq 1-\beta$,
It follows that
$(\lambda)_T(a_i,b_i) = \min\{(\lambda_1)_T(a_i), (\lambda_2)_T(b_i)\} \geq \alpha$, for all $(a_i,b_i) \in N_1 * N_2$
$$\text{if } a_i \in N_1 \text{ and } b_i \in N_2$$
Hence , $(\lambda_1)_T(a_i,a_j) \geq \alpha, (\lambda_1)_I(a_i,a_j) \geq \alpha, (\lambda_1)_F(a_i,a_j) \leq 1-\alpha$ and
$(\lambda_2)_T(b_i,b_j) \geq \alpha, (\lambda_2)_I(b_i,b_j) \geq \alpha, (\lambda_2)_F(b_i,b_j) \leq 1-\alpha$,
$(\eta_1)_T(a_i,a_j) \geq \beta, (\eta_1)_I(a_i,a_j) \geq \beta, (\eta_1)_F(a_i,a_j) \leq 1-\beta$, and
$(\eta_2)_T(b_i,b_j) \geq \beta, (\eta_2)_I(b_i,b_j) \geq \beta, (\eta_2)_F(b_i,b_j) \leq 1-\beta$,
$\min\{(\eta_1)_T(a_i,a_j), (\eta_2)_T\} = \alpha, \min\{(\eta_1)_I(a_i,a_j), (\eta_2)_I(b_i,b_j)\} = \alpha$
$\max\{(\eta_1)_F(a_i,a_j), (\eta_2)_F(b_i,b_j)\} = 1-\alpha$
$\min\{(\eta_1)_T(a_i,a_j), (\eta_2)_T(b_i,b_j)\} = \beta, \min\{(\eta_1)_I(a_i,a_j), (\eta_2)_I(b_i,b_j)\} = \beta$ $\max\{(\eta_1)_F(a_i,a_j),(\eta_2)_F\} = 1-\beta$

$$\eta_T(a_i,b_i)(a_j,b_j)) \geq \alpha = \min\{(\eta_1)_T(a_i,a_j), (\eta_2)_T(b_i,b_j)\}$$
$$\eta_I(a_i,b_i)(a_j,b_j)) \geq \alpha = \min\{(\eta_1)_I(a_i,a_j), (\eta_2)_I(b_i,b_j)\}$$
$$\eta_F(a_i,n_i)(a_j,b_j)) \leq 1-\alpha = \max\{(\eta_1)_F(a_i,a_j), (\eta_2)_F(b_i,b_j)\}.$$
$$\text{if } a_ia_j \in L_1 \text{ and } b_ib_j \in L_2$$
$$\eta_T(a_i,b_i)(a_j,b_j)) \geq \beta = \min\{(\eta_1)_T(a_i,a_j), (\eta_2)_T(b_i,b_j)\}$$
$$\eta_I(a_i,b_i)a_j,b_j)) \geq \beta = \min\{(\eta_1)_I(a_i,a_j), (\eta_2)_I(b_i,b_j)\}$$
$$\eta_F(a_i,b_i)(a_j,b_j)) \leq 1-\beta = \max\{(\eta_1)_F(a_i,a_j), (\eta_2)_F(b_i,b_j)\}.$$
$$\text{if } a_ia_j \in L_1 \text{ and } b_ib_j \in L_2$$
$$\eta_T(a_i,b_i)(a_j,b_j)) = \min\{(\eta_1)_T(a_i,a_j), (\eta_2)_T(b_i,b_j)\}$$

$$\eta_I(a_i, b_i)(a_j, b_j)) = \min\{(\eta_1)_I(a_i, a_j), (\eta_2)_I(b_i, b_j)\}$$
$$\eta_F(a_i, b_i)(a_j, b_j)) = \max\{(\eta_1)_F(a_i, a_j), (\eta_2)_F(b_i, b_j)\}$$
$$\text{if } a_i a_j \in L_1 \text{ and } b_i b_j \in L_2.$$

CONCLUSION

In this paper, we have found the α- cut worthy (Level) Graphs of Homomorphic, Box dot, Star Product of Strong Neutrosophic Graphs. To explore some propositions, theorems and examples of Strong Neutrosophic Graphs by α- cut worthy (Level) graphs.

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