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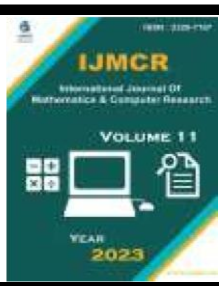

Multiplicative (Generalized)-(σ,τ)-Reverse Derivations in Prime Rings

M. Sri Kameswara Rao[1], K. Subbarayudu[2], A. Vidhyullatha[3], C. Jaya Subba Reddy[4]

[1]Research Scholar, Department of Mathematics, S.V.University, Tirupati-517502, A.P., India.
[2]Lecturer in Mathematics, Government Polytechnic, Simhadripuram, Andhra Pradesh, India.
[3]Department of Mathematics, SPW Degree & PG College TTD, Tirupati-517502, A.P., India.
[4]Department of Mathematics, S. V. University, Tirupati-517502, Andhra Pradesh, India.

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Corresponding author:
C. Jaya Subba Reddy

ABSTRACT

Let R be a prime ring, I be a non-zero left ideal of R and σ,τ be a anti-automorphisms of R. Suppose F and H are two multiplicative (generalized)-(σ,τ)-reverse derivation associated with the mappings d and h respectively, on R. In this paper, we proved the following identities in prime rings: If (i) $F(xy)+\sigma(xy)=0$; (ii) $F(xy)+\sigma(yx)=0$; (iii) $F(xy)+F(y)F(x)=0$; (iv) $F(xy)=\sigma(x)oH(y)$; (v) $F(xy)=[\sigma(x),H(y)]$; for all $x,y\in I$, where σ and τ are anti automorphisms on R.

KEYWORDS : Prime ring, (σ,τ)-reverse derivation, Multiplicative (Generalized) reverse derivation and Multiplicative (Generalized)-(σ,τ)-reverse derivation.

1. INTRODUCTION

In 1991, the concept of derivation was extended to generalized derivation by Bresar [3]. In 1991, Daif [5] introduced the concept of multiplicative derivation. In 1997, Daif and Tammam EL-Sagid [6] extended the concept of multiplicative derivation to multiplicative generalized derivation. In 2013, Dhara and Ali [7] introduced the concept of multiplicative (generalized)-derivation. In 1957, the concept of reverse derivation was first time introduced by Herstein [8]. Further Bresar and Vukman [4] studied the reverse derivations. In 2015, Aboubakr and Gonzalez [1] studied the relationship between generalized reverse derivation and generalized derivation on an ideal in semi prime rings. In 2017, Tiwari et.al [11] defined multiplicative (generalized) reverse derivation. In 2011, the concepts of (θ, φ)-reverse derivation and generalized (θ, φ)-reverse derivation has been introduced by Anwar Khaleel Faraj in [2]. In 2019, Nadeem ur Rehman et.al [10] proved some results on a note on multiplicative (generalized)- (α,β)-reverse derivations in prime rings. In 2022, Jaya Subba Reddy et.al [9] proved some results on a note on multiplicative (generalized)- (α,β)-reverse derivations on left ideals in prime rings. In this paper, we proved some results on multiplicative (generalized) (σ,τ)-reverse derivations in prime rings.

2. PRELIMINARIES

Throughout this paper R denote an associative ring with center Z. Recall that a ring R is prime if $xRy=\{0\}$ implies $x=0$ or $y=0$. For any $x,y\in R$, the symbol $[x,y]$ stands for the commutator $xy-yx$ and the symbol (xoy) denotes the anti-commutator $xy+yx$. A mapping $d{:}\,R\rightarrow R$ (not necessarily additive) is called a multiplicative derivation if $d(xy)=d(x)y+xd(y)$, for all $x,y\in R$. An additive mapping $d{:}\,R\rightarrow R$ is called a reverse derivation if $d(xy)=d(y)x+yd(x)$, for all $x,y\in R$. A mapping $d{:}\,R\rightarrow R$ (not necessarily additive) is called a multiplicative reverse derivation if $d(xy)=d(y)x+yd(x)$, for all $x,y\in R$. An additive mapping $d{:}\,R\rightarrow R$ is called a (σ,τ)-reverse derivation if $d(xy)=d(y)\sigma(x)+\tau(y)d(x)$, for all $x,y\in R$. A mapping $d{:}\,R\rightarrow R$ (not necessarily additive) is called a multiplicative (σ,τ)-reverse derivation if $d(xy)=d(y)\sigma(x)+\tau(y)d(x)$, for all $x,y\in R$. An additive mapping $F{:}\,R\rightarrow R$ is called a generalized reverse derivation, if there exists a reverse derivation $d{:}\,R\rightarrow R$ such that $F(xy)=F(y)x+yd(x)$, for all $x,y\in R$. A mapping $F{:}\,R\rightarrow R$ (not necessarily additive) is said to be a multiplicative (generalized)- reverse derivation of R, if there exists a map $d{:}\,R\rightarrow R$ (neither necessarily additive nor derivation) such that $F(xy)=F(y)x+yd(x)$, for all $x,y\in R$. A mapping $F{:}\,R\rightarrow R$ (not necessarily additive) is said to be a multiplicative (generalized)-(σ,τ)-derivation of R, if there exists a map $d: R \to R$ (neither necessarily additive nor derivation) such that $F(xy) = F(x)\sigma(y) + \tau(x)d(y)$, for all $x, y \in R$. A mapping $F: R \to R$ (not necessarily additive) is said to be a multiplicative (generalized)-(σ,τ)-reverse derivation of R, if there exists a map $d: R \to R$ (neither necessarily additive nor derivation) such that $F(xy) = F(y)\sigma(x) + \tau(y)d(x)$, for all $x, y \in R$. Throughout this paper, we shall make use of the basic commutator identities:

$xo(yz) = (xoy)z - y[x,z] = y(xoz) + [x,y]z$;

$(xy)oz = x(yoz) - [x,z]y = (xoz)y + x[y,z]$;

$[x,yz] = y[x,z] + [x,y]z$; $[xy,z] = [x,z]y + x[y,z]$;

$[xy,z]_{\sigma,\tau} = x[y,z]_{\sigma,\tau} + [\sigma(x),z]y$.

3. MAIN RESULTS

Theorem 3.1: Let R be a prime ring and I be a non-zero left ideal of R. Suppose F is a multiplicative (generalized)-(σ,τ)-reverse derivation on R associated with the map d on R. If $F(xy) + \sigma(xy) = 0$, for all $x, y \in I$, then $F(x) = -\sigma(x)$ for all $x \in I$ and $\tau(I)d(I) = (0)$.

Proof: We have $F(xy) + \sigma(xy) = 0$, for all $x, y \in I$. (3.1)

Replacing x by zx in equation (3.1), we obtain

$\big(F(xy) + \sigma(xy)\big)\sigma(z) + \tau(y)\tau(x)d(z) = 0$, for all $x, y, z \in I$.

Using equation (3.1), it reduces to $\tau(y)\tau(x)d(z) = 0$, for all $x, y, z \in I$. (3.2)

Replacing x by xr, $r \in R$ in equation (3.2), we get

$\tau(y)\tau(r)\tau(x)d(z) = 0$, for all $x, y, z \in I$ and $r \in R$. (3.3)

Replacing r by $\tau^{-1}(rd(z))$, $r \in R$ in equation (3.3), we get

$\tau(y)d(z)r\tau(x)d(z) = 0$, for all $x, y, z \in I$ and $r \in R$.

$\tau(y)d(z)R\tau(x)d(z) = 0$, for all $x, y, z \in I$.

Thus, by primeness of R, we get either $\tau(I)d(I) = (0)$. (3.4)

Thus, equation (3.1) becomes $\big(F(y) + \sigma(y)\big)\sigma(x) + \tau(y)d(x) = 0$, for all $x, y \in I$.

Using equation (3.4), it reduces to $\big(F(y) + \sigma(y)\big)\sigma(x) = 0$, for all $x, y \in I$. (3.5)

Replacing x by xr, $r \in R$ in equation (3.5), we get

$\big(F(y) + \sigma(y)\big)\sigma(r)\sigma(x) = 0$, for all $x, y \in I$ and $r \in R$.

Replacing r by $\sigma^{-1}(r)$ in the above equation, we get

$\big(F(y) + \sigma(y)\big)R\sigma(x) = 0$, for all $x, y \in I$.

Using primeness of R, we get $F(x) = -\sigma(x)$ for all $x \in I$. (3.6)

Theorem 3.2: Let R be a prime ring and I be a non-zero left ideal of R. Suppose F is a multiplicative (generalized)-(σ,τ)-reverse derivation on R associated with the map d on R. If $F(xy) + \sigma(yx) = 0$, for all $x, y \in I$, then R is commutative, $F(x) = -\sigma(x)$ for all $x \in I$ and $\tau(I)d(I) = (0)$.

Proof: We have $F(xy) + \sigma(yx) = 0$, for all $x, y \in I$. (3.7)

Replacing x by x^2 in equation (3.7), we obtain

$F(xy)\sigma(x) + \tau(xy)d(x) + \sigma(x)\sigma(yx) = 0$, for all $x, y \in I$. (3.8)

Replacing y by xy in equation (3.7), we get

$F(xy)\sigma(x) + \tau(xy)d(x) + \sigma(x)\sigma(xy) = 0$, for all $x, y \in I$. (3.9)

Subtracting equation (3.8) from equation (3.9), we get

$\sigma(x)\sigma[x,y] = 0$, for all $x, y \in I$. (3.10)

Replacing y by ry, $r \in R$ in equation (3.10), we get

$\sigma(x)\sigma[x,y]\sigma(r) + \sigma(x)\sigma(y)\sigma[x,r] = 0$, for all $x, y \in I$ and $r \in R$.

Using equation (3.10), it reduces to $\sigma(yx)\sigma[x,r] = 0$, for all $x, y \in I$ and $r \in R$.

Since I is nonzero ideal, so by primeness of R, we get $\sigma[x,r] = 0$, for all $x \in I$ and $r \in R$. $[x,r] = 0$, for all $x \in I$ and $r \in R$. (3.11)

Substituting x by tx, $t \in R$ in equation (3.11), we get $t[x,r] + [t,r]x = 0$, for all $x \in I$ and $r, t \in R$.

Using equation (3.11), it reduces to $[t,r]x = 0$, for all $x \in I$ and $r, t \in R$.

Since I is nonzero ideal, so by primeness of R, we get $[t,r] = 0$, for all $r, t \in R$.

Thus R is commutative. (3.12)

9uTherefore $F(xy) + \sigma(yx) = 0$ becomes $F(xy) + \sigma(xy) = 0$, for all $x, y \in I$.

Thus, in view of theorem 3.1, we get $F(x) = -\sigma(x)$ for all $x \in I$ and $\tau(I)d(I) = (0)$.

Theorem 3.3: Let R be a prime ring and I be a non-zero left ideal of R. Suppose F is a multiplicative (generalized)-(σ,τ)-reverse derivation on R associated with the map d on R. If $F(xy) + F(y)F(x) = 0$, for all $x, y \in I$, then either $\sigma(I)[F(x),\sigma(x)] = (0)$ or $\tau(I)[F(x),\tau(x)] = (0)$, for all $x \in I$.

Proof: We have $F(xy) + F(y)F(x) = 0$, for all $x, y \in I$. (3.13)

Replacing x by zx in equation (3.13), we obtain

$(F(xy) + F(y)F(x))\sigma(x) + \tau(xy)d(z) + F(y)\tau(x)d(z) = 0$, for all $x, y, z \in I$.

Using equation (3.13), it reduces to

$\tau(xy)d(z) + F(y)\tau(x)d(z) = 0$, for all $x, y, z \in I$. (3.14)

Replacing y by wy in equation (3.14), we get

$\tau(xwy)d(z) + F(y)\sigma(w)\tau(x)d(z) + \tau(y)d(w)\tau(x)d(z) = 0, \forall x, y, z, w \in I$. (3.15)

Replacing x by xw in equation (3.14), we get

$\tau(xwy)d(z) + F(y)\tau(w)\tau(x)d(z) = 0$, for all $x, y, z, w \in I$. (3.16)

Subtracting equation (3.16) from equation (3.15), we get

$(F(wy) - F(y)\tau(w))\tau(x)d(z) = 0$, for all $x, y, z, w \in I$.

Replacing x by xr, $r \in R$ in the above equation, we get

$\big(F(wy) - F(y)\tau(w)\big)\tau(r)\tau(x)d(z) = 0$, for all $x, y, z, w \in I$.

$\big(F(wy) - F(y)\tau(w)\big)R\tau(x)d(z) = 0$, for all $x, y, z, w \in I$.

Using primeness of R, we get either
$F(wy)-F(y)\tau(w)=0$ or $\tau(x)d(z)=0$
First case, we have $F(wy)-F(y)\tau(w)=0$, for all $y,w\in I$. (3.17)
Using equation (3.13), we have $F(xyz)=-F(z)F(xy)=-F(yz)F(x)$
$F(z)F(xy)-F(yz)F(x)=0$, for all $x,y,z\in I$. (3.18)
Using equation (3.17) in equation (3.18), we get
$F(z)(F(y)\tau(x)-\tau(y)F(x))=0$, for all $x,y,z\in I$.
Replacing z by wrz, $r\in R$ and using equation (3.17) in the above equation, we get
$F(z)\tau(r)\tau(w)(F(y)\tau(x)-\tau(y)F(x))=0$, for all $x,y,z,w\in I$.
$F(z)R\tau(w)(F(y)\tau(x)-\tau(y)F(x))=0$, for all $x,y,z,w\in I$.
Since R is a prime ring, we get $\tau(I)[F(x),\tau(x)]=0$, for all $x\in I$.
Second case, we have $\tau(x)d(z)=0$, for all $x,z\in I$. (3.19)
Equation (3.18), we have $F(z)F(xy)-F(yz)F(x)=0$, for all $x,y,z\in I$.
$F(z)\big(F(y)\sigma(x)+\tau(y)d(x)\big)-(F(z)\sigma(y)+\tau(z)d(y))F(x)=0$, for all $x,y,z\in I$.
Using equation (3.19), it reduces to
$F(z)(F(y)\sigma(x)-\sigma(y)F(x))=0$, for all $x,y,z\in I$.
Replacing z by wrz, $r\in R$ and in the above equation, we get
$(F(z)\sigma(wr)+\tau(wr)d(z))(F(y)\sigma(x)-\sigma(y)F(x))=0$, for all $x,y,z,w\in I$ and $r\in R$.
Using equation (3.19), it reduces to $F(z)\sigma(r)\sigma(w)(F(y)\sigma(x)-\sigma(y)F(x))=0$.
$F(z)R\sigma(w)(F(y)\sigma(x)-\sigma(y)F(x))=0$, for all $x,y,z,w\in I$.
Since R is a prime ring, we get $\sigma(I)[F(x),\sigma(x)]=0$, for all $x\in I$.
Hence, we get the required result.

Theorem 3.4: Let R be a prime ring and I be a non-zero left ideal of R. Suppose F and H are two multiplicative (generalized)-(σ,τ)-reverse derivation on R associated with the maps d and h on R, respectively. If $F(xy)=\sigma(x)oH(y)$, for all $x,y\in I$, then either R is commutative or $\sigma(I)[\sigma(I),H(I)]=(0)$.

Proof: We have $F(xy)=\sigma(x)oH(y)$, for all $x,y\in I$. (3.20)
Replacing x by zx in equation (20), we get
$F(xy)\sigma(z)+\tau(xy)d(z)=\big(\sigma(x)oH(y)\big)\sigma(z)+\sigma(x)[\sigma(z),H(y)]$ for all $x,y,z\in I$.
Using equation (3.20), it reduces to
$\tau(xy)d(z)=\sigma(x)[\sigma(z),H(y)]$, for all $x,y,z\in I$. (3.21)
Replacing x by xw in equation (3.21), we get
$\tau(y)\tau(w)\tau(x)d(z)=\sigma(w)\sigma(x)[\sigma(z),H(y)]$, for all $x,y,z,w\in I$. (3.22)
Left multiplying equation (3.21) by $\sigma(w)$, we get
$\sigma(w)\tau(y)\tau(x)d(z)=\sigma(w)\sigma(x)[\sigma(z),H(y)]$, for all $x,y,z,w\in I$. (3.23)
Subtracting equation (3.23) from equation (3.22), we get
$\big(\tau(y)\tau(w)-\sigma(w)\tau(y)\big)\tau(x)d(z)=0$, for all $x,y,z,w\in I$.
Replacing x by xr, $r\in R$ in the above equation, we get
$\big(\tau(y)\tau(w)-\sigma(w)\tau(y)\big)R\tau(x)d(z)=0$, for all $x,y,z,w\in I$.
Since R is a prime, we get either $\tau(y)\tau(w)-\sigma(w)\tau(y)=0$ or $\tau(x)d(z)=0$
First case, we have $\tau(y)\tau(x)-\sigma(x)\tau(y)=0$, for all $x,y\in I$. (3.24)
Substituting x by xr, $r\in R$ in equation (3.24), we get
$\tau(y)\tau(r)\tau(x)-\sigma(r)\sigma(x)\tau(y)=0$, for all $x,y\in I$. (3.25)
Left multiplying equation (3.21) by $\sigma(r)$, we get
$\sigma(r)\tau(y)\tau(x)-\sigma(r)\sigma(x)\tau(y)=0$, for all $x,y\in I$. (3.26)
Subtracting equation (3.26) from equation (3.25), we get
$(\tau(y)\tau(r)-\sigma(r)\tau(y))\tau(x)=0$, for all $x,y\in I$ and $r\in R$.
Replacing x by xs, $s\in R$ in the above equation, we get
$(\tau(y)\tau(r)-\sigma(r)\tau(y))R\tau(x)=0$, for all $x,y\in I$ and $r\in R$.
Since I is nonzero ideal, so by primeness of R, we get
$[\tau(y),r]_{\tau,\sigma}=0$, for all $y\in I$ and $r\in R$. (3.27)
Replacing y by yt, $t\in R$ in equation (3.27), we get
$\tau(r)[\tau(y),r]_{\tau,\sigma}+[\tau(t),\tau(r)]\tau(y)=0$, for all $y\in I$ and $r,t\in R$.
Using equation (3.27), it reduces to $[\tau(t),\tau(r)]\tau(y)=0$, for all $y\in I$ and $r,t\in R$.
Substituting y by ys, $s\in R$ in the above equation, we get $[\tau(t),\tau(r)]R\tau(y)=0$, for all $y\in I$ and $r,t\in R$. Since I is nonzero ideal, so by primeness of R, we get
$[\tau(t),\tau(r)]=0$, for all $r,t\in R$.
$[t,r]=0$, for all $r,t\in R$. Therefore R is a commutative.
Second case, we have $\tau(x)d(z)=0$, for all $x,z\in I$. (3.28)
Substituting equation (3.28) in equation (3.21), we get $\sigma(I)[\sigma(I),H(I)]=(0)$.

Theorem 3.5: Let R be a prime ring and I be a non-zero left ideal of R. Suppose F and H are two multiplicative (generalized)-(σ,τ)-reverse derivation on R associated with the maps d and h on R, respectively. If $F(xy)=[\sigma(x),H(y)]$, for all $x,y\in I$, then either R is commutative or $\sigma(I)[\sigma(I),H(I)]=(0)$.

Proof: We have $F(xy)=[\sigma(x),H(y)]$, for all $x,y\in I$. (3.29)
Replacing x by zx in equation (3.29), we get
$F(xy)\sigma(z)+\tau(xy)d(z)=[\sigma(x),H(y)]\sigma(z)+\sigma(x)[\sigma(z),H(y)]$, for all $x,y,z\in I$.
Using equation (3.29), it reduces to
$\tau(xy)d(z)=\sigma(x)[\sigma(z),H(y)]$, for all $x,y,z\in I$. (3.30)

The equation (3.30) is same as the equation (3.21) in theorem 3.4, then proceeding in the same way as in theorem 3.4, we get the required result.

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