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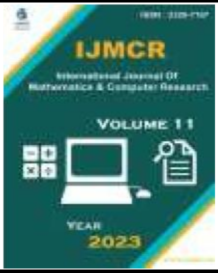

On Periodic Solutions of System of Generalized Rational Difference Equations

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ABSTRACT

In this article, the authors establish periodic solutions for system of generalized rational difference equations (GRDE) with nonzero initial conditions. Additionally, we discussed the sensitivity of the GRDE's generic $k\ell$ variables.

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I. INTRODUCTION

Our aim in this paper is to investigate the periodic nature of solutions of the following systems of generalized rational difference equations

$$u(k+\ell)=\frac{1}{v(k)},\quad v(k+\ell)=\frac{v(k)}{u(k-\ell)v(k-\ell)},$$

$$u(k+\ell)=\frac{1}{v(k)},\quad v(k+\ell)=\frac{v(k)}{u(k-\ell)v(k-\ell)},\quad w(k+\ell)=\frac{u(k-\ell)v(k-\ell)}{u(k-2\ell)v(k-2\ell)\,w(k-2\ell)},$$

and

$$u(k+\ell)=\frac{1}{v(k)},$$

$$v(k+\ell)=\frac{v(k)}{u(k-\ell)v(k-\ell)},$$

$$w(k+\ell)=\frac{u(k-\ell)v(k-\ell)}{u(k-2\ell)v(k-2\ell)\,w(k-2\ell)},$$

$$z(k+\ell)=\frac{u(k-2\ell)v(k-2\ell)\,w(k-2\ell)}{u(k-3\ell)v(k-3\ell)\,w(k-3\ell)\,z(k-3\ell)},$$

with a nonzero real numbers initial conditions. Also, the periodicity of the general k variable will be considered.

The periodicity of the positive solutions of the system of rational difference equations

$$x_{n+1}=\frac{m}{y_n},\quad y_{n+1}=\frac{p\,y_n}{x_{n-1}y_{n-1}},$$

was studied by Cinar in [3].

Also, Cinar [4] has obtained the positive solution of the difference equation system

$$x_{n+1}=\frac{1}{z_n},\quad y_{n+1}=\frac{y_n}{x_{n-1}y_{n-1}},\ z_{n+1}=\frac{1}{x_{n-1}}.$$

Elabbasy et al. [6] has obtained the solution of particular cases of the following general system of difference equations

$$x_{n+1}=\frac{a_1+a_2\,y_n}{a_3\,z_n+a_4\,x_{n-1}\,z_n},$$

$$y_{n+1}=\frac{b_1\,z_{n-1}+b_2\,z_n}{b_3\,x_n\,y_n+b_4\,x_n\,y_{n-1}},$$

$$z_{n+1}=\frac{c_1\,z_{n-1}+c_2\,z_n}{c_3\,x_{n-1}y_{n-1}+c_4\,x_{n-1}y_n+c_5\,x_n\,y_n}.$$

Ozban [8] has investigated the positive solutions of the system of rational difference equations

$$x_{n+1}=\frac{1}{y_{n-k}},\ y_{n+1}=\frac{y_n}{x_{n-m}\,y_{n-m-k}}.$$

Ozban [9] has investigated the solutions of the following system

$$x_{n+1}=\frac{a}{y_{n-3}},\ y_{n+1}=\frac{by_{n-3}}{x_{n-q}y_{n-q}}.$$

Yang [11] has investigated the positive solutions of the systems investigated see[1]-[11].

Definition (Periodicity): A Sequence $\{u(k)\}$ is said to be periodic with period T if $u(k+T)=u(k)$ for all T.

II. MAIN RESULTS

2.1 First System

In this section, we study the periodicity of the solutions of the system of two generalized rational difference equations

$$u(k+\ell)=\frac{1}{v(k)},\ v(k+\ell)=\frac{v(k)}{u(k-\ell)v(k-\ell)},\qquad (1)$$

with a nonzero real numbers initial conditions.

Theorem 1. Suppose that $\{u(k),v(k)\}$ are solutions of system (1). Also, assume that $u(\ell), u(0), v(\ell),$ and $v(0)$ are arbitrary nonzero real numbers. Then all the solutions of generalized rational difference equation system (1) are periodic with period 4ℓ.

Proof: From Eq.(1) we have

$$u(k+\ell)=\frac{1}{v(k)},\ v(k+\ell)=\frac{v(k)}{u(k-\ell)v(k-\ell)},$$

$$u(k+2\ell)=\frac{1}{v(k+\ell)}=\frac{u(k-\ell)v(k-\ell)}{v(k)},\ v(k+2\ell)=\frac{v(k+\ell)}{u(k)v(k)}=\frac{1}{u(k-\ell)v(k-\ell)u(k)},$$

$$u(k+3\ell)=\frac{1}{v(k+2\ell)}=u(k-\ell)v(k-\ell)u(k),\ v(k+3\ell)=\frac{v(k+2\ell)}{u(k+\ell)\,v(k+\ell)}=\frac{1}{u(k)},$$

$$u(k+4\ell)=\frac{1}{v(k+3\ell)}=u(k), v(k+4\ell)=\frac{v(k+3\ell)}{u(k+2\ell)v(k+2\ell)}=v(k).$$

The proof is complete.

2.2 Second System

In this section, we deal with the solutions of the system of generalized difference equations

$$u(k+\ell)=\frac{1}{v(k)},\ v(k+\ell)=\frac{v(k)}{u(k-\ell)v(k-\ell)},\ w(k+\ell)=\frac{u(k-\ell)v(k-\ell)}{u(k-2\ell)v(k-2\ell)\ w(k-2\ell)},\ (2)$$

With a nonzero real numbers initial conditions.

Theorem 2. Suppose that $\{u(k),v(k),w(k)\}$ are solutions of system (2). Also, assume that $u(-2\ell),u(-\ell), u(0),v(-2\ell),\ v(-\ell),$ $v(0),w(-2\ell),w(-\ell)$ and $w(0)$ are arbitrary nonzero real numbers. Then $\{u(k),v(k)\}$ are also periodic with period four and $\{w(k)\}$ is periodic with period 12ℓ.

Proof: It is easy to see that $\{u(k),v(k)\}$ are periodic with period four. So, to prove theorem we prove that $\{w(k)\}$ is periodic with period twelve. From the given equations we see that

$$w(k+\ell)=\frac{u(k-\ell)v(k-\ell)}{u(k-2\ell)v(k-2\ell)w(k-2\ell)},$$

$$w(k+3\ell)=\frac{u(k+\ell)v(k+\ell)}{u(k)v(k)w(k)}=\frac{\frac{1}{v(k)}\frac{v(k)}{u(k-\ell)v(k-\ell)}}{u(k)v(k)w(k)}=\frac{1}{u(k-\ell)v(k-\ell)u(k)v(k)w(k)},$$

$$w(k+4\ell)=\frac{u(k+2\ell)v(k+2\ell)}{u(k+\ell)v(k+\ell)w(k+\ell)}=\frac{u(k-\ell)v(k-\ell)}{v(k)u(k-\ell)v(k-\ell)}\frac{1}{v(k)\frac{u(k-\ell)v(k-\ell)}{u(k-2\ell)v(k-2\ell)u(k-\ell)w(k-2\ell)}}$$

$$=\frac{u(k-2\ell)v(k-2\ell)w(k-2\ell)}{u(k)v(k)},$$

$$w(k+5\ell)=\frac{u(k+3\ell)v(k+3\ell)}{u(k+2\ell)v(k+2\ell)w(k+2\ell)}=\frac{u(k-\ell)v(k-\ell)u(k)}{u(k)\frac{u(k-\ell)v(k-\ell)}{v(k)}\frac{1}{u(k-\ell)v(k-\ell)u(k)}\frac{u(k)v(k)}{u(k-\ell)v(k-\ell)w(k-\ell)}}$$

$$=\left[u(k-\ell)v(k-\ell)\right]^2\ w(k-\ell),$$

$$w(k+6\ell)=\frac{u(k+4\ell)v(k+4\ell)}{u(k+3\ell)v(k+3\ell)w(k+3\ell)}$$

$$=\frac{u(k)v(k)}{u(k-\ell)v(k-\ell)u(k)\frac{1}{u(k)}\frac{1}{u(k-\ell)v(k-\ell)u(k)v(k)w(k)}}=u_2(k)v_2(k)w(k),$$

$$w(k+7\ell)=\frac{u(k+5\ell)v(k+5\ell)}{u(k+4\ell)v(k+4\ell)w(k+4\ell)}$$

$$=\frac{v(k)}{v(k)u(k-\ell)v(k-\ell)u(k)v(k)\frac{u(k-2\ell)v(k-2\ell)w(k-2\ell)}{v(k)u(k)}}$$

$$=\frac{1}{u(k-\ell)v(k-\ell)\,u(k-2\ell)v(k-2\ell)w(k-2\ell)},$$

$$w(k+8\ell)=\frac{u(k+6\ell)v(k+6\ell)}{u(k+5\ell)v(k+5\ell)w(k+5\ell)}$$

$$=\frac{u(k-\ell)v(k-\ell)}{v(k)u(k-\ell)v(k-\ell)\,u(k)\frac{1}{v(k)}\frac{y_n}{u(k-\ell)v(k-\ell)}u_2(k-\ell)v_2(k-\ell)\,w(k-\ell)}$$

$$=\frac{1}{u(k)\,v(k)\,u(k-\ell)v(k-\ell)\,w(k-\ell)},$$

$$w(k+9\ell)=\frac{u(k+7\ell)v(k+7\ell)}{u(k+6\ell)v(k+6\ell)w(k+6\ell)}$$

$$=\frac{u(k-\ell)v(k-\ell)u(k)}{u(k)\frac{u(k-\ell)v(k-\ell)}{v(k)}\frac{1}{u(k-\ell)v(k-\ell)u(k)}u_2(k)v_2(k)w(k)}=\frac{u(k-\ell)v(k-\ell)}{u(k)v(k)z(k)},$$

$$w(k+10\ell)=\frac{u(k+8\ell)v(k+8\ell)}{u(k+7\ell)v(k+7\ell)w(k+7\ell)}$$

$$=\frac{u(k)v(k)}{u(k-\ell)v(k-\ell)u(k)\frac{1}{u(k)}\frac{1}{u(k-\ell)v(k-\ell)u(k-2\ell)v(k-2\ell)w(k-2\ell)}}$$

$$=u(k)v(k)u(k-2\ell)v(k-2\ell)w(k-2\ell),$$

$$w(k+11\ell)=\frac{u(k+9\ell)v(k+9\ell)}{u(k+8\ell)v(k+8\ell)w(k+8\ell)}$$

$$=\frac{v(k)}{v(k)u(k-\ell)v(k-\ell)u(k)v(k)\frac{1}{u(k)v(k)u(k-\ell)v(k-\ell)w(k-\ell)}}=w(k-\ell),$$

$$w(k+12\ell)=\frac{u(k+10\ell)v(k+10\ell)}{u(k+9\ell)v(k+9\ell)w(k+9\ell)}$$

$$=\frac{u(k-\ell)v(k-\ell)}{v(k)u(k-\ell)v(k-\ell)u(k)\frac{1}{v(k)}\frac{v(k)}{u(k-\ell)v(k-\ell)}\frac{u(k-\ell)v(k-\ell)}{u(k)v(k)w(k)}}=w(k).$$

The proof is complete.

2.3 Generalized Third System

In this section, we obtain the solutions of the system of the generalized difference equations

$$u(k+\ell)=\frac{1}{v(k)},\quad v(k+\ell)=\frac{v(k)}{u(k-\ell)v(k-\ell)},\qquad w(k+\ell)=\frac{u(k-\ell)v(k-\ell)}{u(k-2\ell)v(k-2\ell)w(k-2\ell)},$$

$$z(k+\ell)=\frac{u(k-2\ell)v(k-2\ell)\,w(k-2\ell)}{u(k-3\ell)\,v(k-3\ell)w(k-3\ell)},\tag{3}$$

With a nonzero real numbers initial conditions.

Theorem 3. Suppose that $\{u(k),v(k),w(k)\}$ are solutions of system (3). Also assume that $u(-3\ell),u(-2\ell),u(-\ell),u(0),u(-3\ell),$ $v(-2\ell),v(-\ell),v(0),w(-3\ell),w(-2\ell),z(-\ell),$ $z(-3\ell),z(-2\ell),z(-\ell)$ and $z(0)$ are arbitrary nonzero. Then $\{u(k),v(k)\}$ are also periodic with period four, $\{w(k)\}$ is periodic with period twelve, and $\{z(k)\}$ is periodic with period 24ℓ. Proof: As the proof of Theorem 2.

2.1. GENERAL SYSTEM

In this section, we investigate the solutions of the system of the generalized difference equations

$$u_4(k+\ell)=\frac{u_1(k-2\ell)\,u_2(k-2\ell)u_3(k-2\ell)}{u_1(k-3\ell)u_2(k-3\ell)u_3(k-3\ell)u_4(k-3\ell)},\ldots,$$

$$u_j(k+\ell)=\frac{\prod_{i=1}^{j-1}u_i(k-(j-2\ell))}{\prod_{i=1}^{j}u_i(k-(j-\ell))},\ldots,u_n(k+\ell)=\frac{\prod_{i=1}^{k-1}u_i(k-(n-2\ell))}{\prod_{i=1}^{k}u_i(k-(n-\ell))}.\tag{4}$$

Theorem 4. Suppose that $\{u_1(k),u_2(k),u_3(k),\ldots\ldots u_n(k)\}$ are solutions of system (4). Then $\{u_1(k),u_2(k)\}$ are periodic with period four and $u_j(k)$ is periodic with period $(P(u_{j-1}(k))+4(j-1))$, where $P(u_j(k))$ the period of $(u_{j-1}(k))$, j = 3, 4, 5……..

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