[Volume 4 issue 07 July 2016]
PageNo.1554-1562 ISSN : 2320-7167

INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTER RESEARCH

Solution Procedure to Solve Fractional Transportation Problem with Fuzzy Cost and Profit Coefficients.

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1. Abstract

This paper deals with fractional transportation problem with fuzzy profit and cost coefficients. In this paper fuzziness in the objective function is handled with fuzzy programming techniques in the sense of multi objective approach. Cost and Profit coefficients are trapezoidal fuzzy numbers and for each set of crisp part of the fuzzy number a single fractional objective is considered. Each of the fractional objective function is solved independently using method given in Erik Bajalinov [1] .Expanding each of the fractional objective about respective optimal solution using Taylor series [2] method ,it is converted into linear transportation problem. Then we present a compensatory approach to solve multi objective linear transportation problem with fuzzy coefficients by using Werner's [3] μ_{and} operator. This approach gives compromise solution which is both compensatory and Pareto optimal. A numerical example is given at the end to illustrate the approach

Key Words: Fuzzy Numbers, Fractional transportation problem, Pareto optimal solution.

2. Introduction

The classical transportation problem is special type of linear programming problem. There are m supply centres and n demand centres. Homogenous goods are transported from the place of origin generally called as supply centre and shipped to place of consumption centres generally called as demand centres. The objective is to either minimise total cost, time, damages to the product or maximise the profit due to transportation. If P(x) is total profit of transportation and Q(x) is total cost of transportation then one can think of maximising the ratio $\frac{P(x)}{Q(x)}$. If P(x) and Q(x) are linear functions, which are in general, then the ratio is linear fraction and the problem becomes fractional transportation problem. If profit coefficients, cost coefficients, demand and supply are crisp numbers then the procedure to solve such problem is standard. When cost and profit coefficients are fuzzy numbers then the approach required is different. There are several methods to solve Fuzzy programming problem. A compensatory approach to multi objective Linear Transportation problem with Fuzzy Cost coefficients is give y Hale Gonce Kocken et.al. [4]. Stefan et.al. [5] introduced a concept of the optimal solution of the transportation problem. H .J, Zimmerma gave several operator to solve linear programming with several objective functions [6]. Luhadjula [7] discussed Compensatory operator in Fuzzy programming with multiple objectives .Doke D.M. et.al. gave a Approach to solve Multi Objective linear fractional programme problem [8]. Doke D.M. et.al. [9] gave a approach to solve multi objective fractional transportation .

3. Method to Solve Fractional Transportation Problem with Fuzzy Cost and Profit coefficients:

The fractional transportation problem with fuzzy cost coefficients is formulated as follows,

Maximise Q(x) = $\frac{\sum_{i=1}^{m}\sum_{j=}^{n}\tilde{p}_{ij}x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{d}_{ij}x_{ij}}$ ....(1)

Such that $\sum_{j=1}^{n} x_{ij} = a_i$ Where i= 1,2,...,m ....(2.i)

$\sum_{i=1}^{m} x_{ij} = b_j$ Where j= 1,2,...,n ....(2.ii)

$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ (Balanced Condition) ……….(2.iii)

$x_{ij} \geq 0$ For i=1,2,…,m and j=1,2,…,n ……(2.iv)

x_{ij} is decision variable which refers to the quantity of the product to be transported from point of supply i to place of demand j .

Note that a_i and b_j are crisp numbers (i=1,2,…,m and j=1,2,…,n)

$\tilde{p}_{ij}$ is fuzzy unit transportation profit from supply point i to demand point j.

$\tilde{d}_{ij}$ is fuzzy unit transportation cost from supply point i to demand point j.

For the fuzzy transportation problem the coefficients of the objectives $\tilde{p}_{ij}$ and $\tilde{d}_{ij}$ are considered as trapezoidal fuzzy numbers (TFN). TFN is defined as under,

$\tilde{p}_{ij} = (p_{ij}^1 , p_{ij}^2 , p_{ij}^3 , p_{ij}^4)$

$\tilde{d}_{ij} = (d_{ij}^1 , d_{ij}^2 , d_{ij}^3 , d_{ij}^4)$

The membership function of TFN $\tilde{p}_{ij}$ is given below.

$$\tilde{p}_{ij}(x) = \begin{cases} 0 & if \ x < p_{ij}^1 \\ \frac{x-p_{ij}^1}{p_{ij}^2-p_{ij}^1} & if \ p_{ij}^1 \leq x < p_{ij}^2 \\ \frac{p_{ij}^4-x}{p_{ij}^4-p_{ij}^3} & if \ p_{ij}^3 < x \leq p_{ij}^4 \\ 0 & if \ x \geq p_{ij}^4 \end{cases}$$

Similarly $\tilde{d}_{ij}(x)$ is a TFN.

When fuzzy profit and cost are given as TFN as defined above then the objective function can be written as,

$\widetilde{Q(x)} = (Q^1(x) , Q^2(x) , Q^3(x) , Q^4(x))$

$$= \left(\frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^{1} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{1} x_{ij}} , \quad \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^{2} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{2} x_{ij}} , \right.$$

$$\left. \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^{3} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{3} x_{ij}} , \quad \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^{4} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{4} x_{ij}} \right) \qquad \text{..................(3)}$$

Since $\widetilde{Q(x)}$ is a TFN for given $\bar{x} \in$ S, we need to define maximum fuzzy value objective function.

Thus problem reduces to

Max $\widetilde{Q(x)} = (Max\ Q^1(x) , MaxQ^2(x) , Max\ Q^3(x) , MaxQ^4(x))$

Problem can be formulated as Multi objective fractional transportation problem as,

$$Max\ Q^1(x) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^{1} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{1} x_{ij}} \qquad \text{..................(4.i)}$$

$$MaxQ^2(x) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^{2} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{2} x_{ij}} \qquad \text{..................(4.ii)}$$

$$Max\ Q^3(x) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^{3} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{3} x_{ij}} \qquad \text{..................(4.iii)}$$

$$MaxQ^4(x) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^{4} x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}^{4} x_{ij}} \qquad \text{...................(4.iv)}$$

Subject to 2(.i),(2.ii),2(.iii),(2.iv).

Solve each of the problems as fractional transportation problem using standard method of fractional programming. Expand each of the objective function Q^i(x) (i = 1,2,3,4) about the optimal solution using Taylor series approach and convert them into Linear function.

Denote these linear functions by $Q_i(x)$ (i= 1,2,3,4).

Note that the optimal solution of fractional transportation is same as that of linear transportation problem which is obtained from fractional transportation to linear transportation

Construct membership function as under

$L_i = \min_{x \in S} Q_i$(x) And $U_i = \max_{x \in S} Q_i$(x) for i = 1,2,3,4. ....(5)

Where S is feasible region. Feasible region satisfies demand and supply constraints along with non negativity conditions. Once we have lower and upper bound of objectives then we can construct membership functions. There are several membership functions such as linear, hyperbolic these are discussed i details y Zimmerman.

We will use linear membership function as defined below,

$$\mu_i\ (Q_i\ (x)) = 1 \qquad \text{if } Q_i < L_i$$

$$= \frac{U_i - Q_i}{U_i - L_i} \qquad \text{if } L_i \le Q_i \le U_i$$

$$= 0 \qquad if \quad Q_i \ge U_i$$

For i=1,2,3,4

Note that Membership function is linear and strictly increasing.

4. Werner's Compensatory μ_{and} Operator for MOLTP with Fuzzy Cost Coefficients.

For every objective function (Q_i (x)) after satisfying its most basic satisfaction level in the transportation system to promote its satisfaction degree as high as possible we can make the following arrangement.

$\mu_i\ (Q_i\ (x)) = \lambda + \lambda_i$ for i =1,2,3,4.

Where $\lambda = \min (\mu_i\ (\breve{Q}_i\ (x))$

This arrangement is introduced to the constraint with following expressions.

$\mu_i\ (Q_i\ (x)) \ge \lambda + \lambda_i$ and $\lambda + \lambda_i \le 1$ for i = 1,2,3,4.

So μ_{and} used in this paper can be formulated as

$$\mu_{and} = \gamma \min_i \mu_i\ (\breve{Q}_i\ (x)) + \frac{(1-\gamma)}{4} (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)$$

$$\mu_{and} = \gamma \lambda + \frac{(1-\gamma)}{4} (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)$$

Consider $\mu_i\ (Q_i\ (x)) \ge \lambda + \lambda_i$ for i = 1,2,3,4

$$\frac{U_i - Q_i}{U_i - L_i} \ge \lambda + \lambda_i$$

$$(U_i - Q_i) \ge (\lambda + \lambda_i)\ (U_i - L_i)$$

$$Q_i + (U_i - L_i)(\lambda + \lambda_i) \le U_i \qquad \text{for } i = 1,2,3,4$$

Thus LPP is

Maximise $\mu_{and} = \gamma \lambda + \frac{(1-\gamma)}{4} (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)$ **....................(6)**

Subject to,

$\sum_{j=1}^{n} x_{ij} = a_i$ **for i= 1,2,...,m.**

$\sum_{i=1}^{m} x_{ij} = b_j$ **for j= 1,2,...,n.**

$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ **(Balanced Condition)**

$Q_i + (U_i - L_i)(\lambda + \lambda_i) \leq U_i$ **for i = 1,2,3,4**

$\lambda + \lambda_i \leq 1$ **for i = 1,2,3,4.**

$x_{ij} \geq 0$ **for i= 1,2,...,m and j = 1,2,...n.**

$\lambda, \lambda_i \in \{0,1\}$

Solve this problem using any of the standard linear optimization method. In this paper it solved using Matlab LPSOVE function.

5. **Illustration.**

Consider the problem with following characteristics.

Supplies $a_1 = 24$, $a_2 = 8$, $a_3 = 18$

$b_1 = 11$, $b_2 = 9$, $b_3 = 21$, $b_4 = 9$

Fuzzy Profit Coefficients p_{ij}

	Destination 1	Destination 2	Destination 3	Destination 4
Origin 1	(6,12,15,20)	(7,9,12,16)	(6,9,10,12)	(3,7,8,16)
Origin 2	(15,16,17,20)	(13,16,18,19)	(18,20,21,25)	(10,12,14,16)
Origin 3	(6,9,14,20)	(10,12,15,20)	(6,9,10,12)	(15,20,21,24)

Fuzzy Cost Coefficients d_{ij}

	Destination 1	Destination 2	Destination 3	Destination 4
Origin 1	(2,3,6,10)	(4,8,10,14)	(8,10,11,12)	(5,8,10,12)
Origin 2	(6,11,13,20)	(10,11,12,14)	(16,18,20,25)	(14,16,17,20)
Origin 3	(3,4,5,8)	(8,10,11,14)	(7,10,10,12)	(14,15,15,18)

Solving equations (4) using fractional transportation programming method, we have following optimal solutions.

x_1 = [3,9,3,9,8,0,0,0,0,0,18,0];

x_2 = [11,0,13,0,0,8,0,0,0,1,8,9];

x_3 = [11,0,13,0,0,8,0,0,0,1,8,9];

x_4 = [0,0,21,3,0,8,0,0,11,1,0,6];

Evaluating each of the four objectives at above solutions we have following values of objectives and lower and upper bouds

	Q_1	Q_2	Q_3	Q_4
x_1	**1.2421**	***1.1020***	***1.1087***	***1,1243***
x_2	1.1136	**1.3466**	1.3616	1.1846
x_3	1.1136	1.3466	**1.3616**	1.1846
x_4	***1.0438***	1.2210	1.3119	**1.2721**
L_i	1.0438	1.1020	1.1087	1.1243
U_i	1.2421	1.3466	1.3616	1.2721

Following is matlab programme to covert fractional objective into linear and to solve LPP

```
% Enter Fuzzy Profit Coefficients as TFN P1, P2, P3 ,P4
P1=[6,7,6,3,15,13,18,10,6,10,6,15];
P2=[12,9,9,7,16,16,20,12,9,12,9,20];
P3=[15,12,10,8,17,18,21,14,14,15,10,21];
P4=[20,16,12,16,20,19,25,16,20,20,12,14];
% Enter Fuzzy Coefficients as TFN D1, D2, D3 ,D4
D1=[2,4,8,5,6,10,16,14,3,8,7,14];
D2=[3,8,10,8,11,11,18,16,4,10,10,15];
D3=[6,10,11,10,13,12,20,17,5,11,10,15];
D4=[10,14,12,12,20,14,25,20,8,14,12,18];
% enter Solutions of four Single objective crisp
%fractional transportation problem
X1=[3,9,3,9,8,0,0,0,0,0,18,0];
X2=[11,0,13,0,0,8,0,0,0,1,8,9];
X3=[11,0,13,0,0,8,0,0,0,1,8,9];
X4=[0,0,21,3,0,8,0,0,11,1,0,6];
```

```
% obtain Coefficients of Linear TP using Taylor Series
% Expansion.
C1P1=P1*(D1*X1');
C1D1 =D1*(P1*X1');
Q1=(C1P1-C1D1)/((D1*X1')^2);
C2P2=P2*(D2*X2');
C2D2 =D2*(P2*X2');
Q2=(C2P2-C2D2)/((D2*X2')^2);
C3P3=P3*(D3*X3');
C3D3 =D3*(P3*X3');
Q3=(C3P3-C3D3)/((D3*X3')^2);
C4P4=P4*(D4*X4');
C4D4 =D4*(P4*X4');
Q4=(C4P4-C4D4)/((D4*X4')^2);
% Enter different.
 matrices to solve LPP
Q=[0,0,0,0,0,0,0,0,0,0,0,0];
A=[Q1,.1983,.1983,0,0,0;
   Q2,.2446,0,.2446,0,0;
   Q3,.2529,0,0,.2529,0;
   Q4,.1478,0,0,0,.1478;
   Q,1,1,0,0,0;    Q,1,0,1,0,0;     Q,1,0,0,1,0;     Q,1,0,0,0,1];
B=[1.2421; 1.3466 ;  1.3466;  1.2729  ;1  ;1  ;1  ;1];
AEQ = [1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0;
             0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0;
            0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,0;
```

```
           1,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0;
          0,1,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0;
          0,0,1,0,0,0,1,0,0,0,1,0,0,0,0,0,0;
          0,0,0,1,0,0,0,1,0,0,0,1,0,0,0,0,0;
      1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0];
  BEQ=[24; 8; 18; 11; 9; 21; 9; 50];
  LB=[0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0];
  UB=[50; 50; 50; 50; 50; 50; 50; 50; 50; 50; 50; 50; 1; 1; 1; 1; 1];
  % Enter Value of  γ = 0
  % Matlab LPSOLVE minimises the function thus to maximise multiply by -1
  F=[0;0;0;0;0;0;0;0;0;0;0;0;-1;-0.25;-0.25;-0.25;-0.25];
  X=linprog(F,A,B,AEQ,BEQ,LB,UB);
 % The solution is X'
```

Optimization terminated.

For $\gamma = 0$ we have following solution say $\mathbf{X_1}$.

5.3068 4.7219 9.4052 4.5661 1.6663 0.9209 4.2030 1.2098 4.0269 3.3573 7.3918 3.2241 0.6598 0.3402 0.3402 0.3402 0.3402

After rounding of suitably we have following solution and corresponding values of objective functions..

$x_{11} = 5$, $x_{12} = 5$, $x_{13} = 9$, $x_{14} = 5$, $x_{21} = 2$, $x_{22} = 1$, $x_{23} = 4$, $x_{24} = 1$, $x_{31} = 4$, $x_{32} = 3$, $x_{33} = 8$, $x_{34} = 3$

Corresponding values are Q ($\mathbf{X_1}$) = (1.1247, 1.1845, 1.2159, 1.1968)

For $\gamma = 1$ we have following solution say $\mathbf{X_2}$.

5.2504 4.6293 9.6375 4.4828 1.7081 1.0142 3.9925 1.2852 4.0416 3.3564 7.3700 3.2319 1.0000 0.0000 0.0000 0.0000 0.0000

After rounding of suitably we have following solution and corresponding values.

$x_{11} = 5$, $x_{12} = 4$, $x_{13} = 10$, $x_{14} = 5$, $x_{21} = 2$, $x_{22} = 1$, $x_{23} = 4$, $x_{24} = 1$, $x_{31} = 4$, $x_{32} = 4$, $x_{33} = 7$, $x_{34} = 3$.

Corresponding values are Q ($\mathbf{X_2}$) = (1.1629, 1.2054, 1.2500, 1.2153)

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