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Delta F, Modified Delta Sombor and Delta Sigma Indices of Certain Nanotubes and Networks

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ABSTRACT
In this paper, we introduce the delta F, modified delta Sombor and delta sigma indices and their corresponding polynomials of a graph. Furthermore, we compute these indices and their corresponding polynomials for two families of nanotubes and two families of networks.

KEYWORDS: delta F-index, modified delta Sombor index, delta sigma index, nanotube, network.

I. INTRODUCTION

Let $G = (V(G), E(G))$ be a finite, simple connected graph. Let $d_G(u)$be the degree of a vertex u in G. Let □(G) denote the minimum degree among the vertices of G. We refer [1] for undefined notations and terminologies.

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical Graph Theory is branch of Mathematical Chemistry, which has an important affect on the development of the Chemical Sciences. A topological index is a numerical parameter mathematically derived from the graph structure. Several such topological indices have been considered in Theoretical Chemistry and have found some applications, especially in QSPR/QSAR study see [2, 3].

The δ vertex degree was introduced in [4] and it is defined as

$$\delta_u = d_G(u) - \delta(G) + 1.$$

In [5], Kulli introduced the first and second δ-Banhatti indices of a graph and they are defined as

$$\delta B_1(G) = \sum_{uv \in E(G)} (\delta_u + \delta_v),$$

$$\delta B_2(G) = \sum_{uv \in E(G)} \delta_u \delta_v.$$

Recently some delta indices were studied, for example, in [6, 7, 8, 9].

We introduce the delta F-index of a graph G and it is defined as

$$\delta F(G) = \sum_{uv \in E(G)} \left(\delta_u^2 + \delta_v^2\right).$$

Considering the delta F-index, we define the delta F-polynomial of a graph G as

$$\delta F(G, x) = \sum_{uv \in E(G)} x^{\left(\delta_u^2 + \delta_v^2\right)}.$$

We introduce the delta F_1-index of a graph G and it is defined as

$$\delta F_1(G) = \sum_{u \in V(G)} \delta_u^3.$$

Considering the delta F_1-index, we define the delta F_1-polynomial of a graph G as

$$\delta F_1(G, x) = \sum_{uv \in E(G)} x^{\delta_u^3}.$$

In [4], Kulli introduced the delta Sombor index of a graph and it is defined as

$$\delta S(G) = \sum_{uv \in E(G)} \sqrt{\delta_u^2 + \delta_v^2}.$$

We propose the modified delta Sombor index or sum connectivity delta F-index of a graph G and it is defined as

$$^m\delta S(G) = S\delta F(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\delta_u^2 + \delta_v^2}}.$$

Considering the modified delta Sombor index, we define the modified delta Sombor exponential of a graph G as

$$^{m}\delta S(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{\delta_u^2+\delta_v^2}}}.$$

Recently some Sombor indices were studied, for example, in [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].

We put forward the delta sigma index of a graph G and it is defined as

$$\delta\sigma(G) = \sum_{uv\in E(G)} (\delta_u - \delta_v)^2.$$

Considering the delta sigma index, we define the delta sigma polynomial of a graph G as

$$\delta\sigma(G,x) = \sum_{uv\in E(G)} x^{(\delta_u-\delta_v)^2}.$$

Recently, the sigma index was studied in [28, 29].

In this paper, we determine the delta F, modified delta Sombor and delta sigma indices and their corresponding polynomials of certain nanotubes and networks.

II. RESULTS FOR $KTUC_4C_8(S)$ NANOTUBES

In this section, we focus on the graph structure of a family of $TUC_4C_8(S)$ nanotubes. The 2-*D* lattice of $TUC_4C_8(S)$ is denoted by $KTUC_4C_8[p,q]$, where $p$is the number of columns and $q$is the number of rows. The graph of $KTUC_4C_8[p,q]$ is shown in Figure 1.

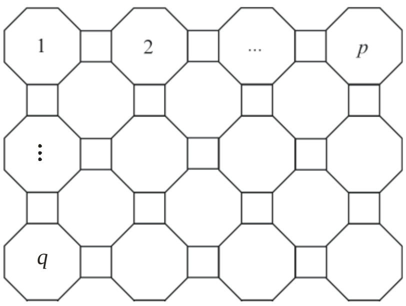

Figure 1: The graph of $KTUC_4C_8[p,q]$ nanotube

Let G be the graph of a nanotube $KTUC_4C_8[p,q]$. By calculation, we obtain that G has $12pq - 2p - 2q$ edges.The graph G has three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 2p + 2q+ 4$.

$E_2 = \{uv\in E(G) \mid d_G(u) =2, d_G(v) = 3\}$, $|E_2| = 4p + 4q- 8$.

$E_3 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 12pq - 8p - 8q + 4$.

Table 1: δ-edge partition of $KTUC_4C_8[p, q]$

$\delta_u,\delta_v\backslash uv \in E(G)$	Number of edges
(1, 1)	$2p+2q+4$
(1, 2)	$4p+4q-8$
(2, 2)	$12pq-8p-8q+4$

Theorem 1. Let G be the graph of a nanotube $KTUC_4C_8[p,q]$. Then

(i) $$\delta F(KTUC_4C_8[p,q]) = 96pq - 40p - 40q - 4.$$

(ii) $$\delta F(KTUC_4C_8[p,q],x) = (2p+2q+4)x^2 + (4p+4q-8)x^5 + (12pq-8p-8q+4)x^8.$$

Proof: From definitions and by using Table 1, we deduce

(i) $$\delta F(KTUC_4C_8[p,q]) = \sum_{uv\in E(G)} (\delta_u^2+\delta_v^2)$$
$$= (1^2+1^2)(2p+2q+4) + (1^2+2^2)(4p+4q-8) + (2^2+2^2)(12pq-8p-8q+4)$$
$$= 96pq - 40p - 40q - 4.$$

(ii) $$\delta F(KTUC_4C_8[p,q],x) = \sum_{uv\in E(G)} x^{(\delta_u^2+\delta_v^2)}$$
$$= (2p+2q+4)x^{(1^2+1^2)} + (4p+4q-8)x^{(1^2+2^2)} + (12pq-8p-8q+4)x^{(2^2+2^2)}$$
$$= (2p+2q+4)x^2 + (4p+4q-8)x^5 + (12pq-8p-8q+4)x^8.$$

Theorem 2. Let G be the graph of a nanotube $KTUC_4C_8[p,q]$. Then

(i) $$^{m}\delta S(KTUC_4C_8[p,q]) = \frac{6}{\sqrt{2}}pq + \left(\frac{4}{\sqrt{5}} - \frac{2}{\sqrt{2}}\right)(p+q) + \left(\frac{6}{\sqrt{2}} - \frac{8}{\sqrt{5}}\right).$$

(ii) $$^{m}\delta S(KTUC_4C_8[p,q],x) = (2p+2q+4)x^{\frac{1}{\sqrt{2}}} + (4p+4q-8)x^{\frac{1}{\sqrt{5}}} + (12pq-8p-8q+4)x^{\frac{1}{2\sqrt{2}}}.$$

Proof: From definitions and by using Table 1, we deduce

(i) $$ {}^{m}\delta S\left(KTUC_4C_8\left[p,q\right]\right)=\sum_{uv\in E(G)}\frac{1}{\sqrt{\delta_u^2+\delta_v^2}}$$

$$=\frac{1}{\sqrt{1^2+1^2}}(2p+2q+4)+\frac{1}{\sqrt{1^2+2^2}}(4p+4q-8)$$

$$+\frac{1}{\sqrt{2^2+2^2}}(12pq-8p-8q+4)$$

$$=\frac{6}{\sqrt{2}}pq+\left(\frac{4}{\sqrt{5}}-\frac{2}{\sqrt{2}}\right)(p+q)+\left(\frac{6}{\sqrt{2}}-\frac{8}{\sqrt{5}}\right).$$

(ii) $$P\delta B\left(KTUC_4C_8\left[p,q\right],x\right)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{\delta_u\delta_v}}}$$

$$=(2p+2q+4)\,x^{\frac{1}{\sqrt{1^2+1^2}}}+(4p+4q-8)\,x^{\frac{1}{\sqrt{1^2+2^2}}}$$

$$+(12pq-8p-8q+4)\,x^{\frac{1}{\sqrt{2^2+2^2}}}$$

$$=(2p+2q+4)\,x^{\frac{1}{\sqrt{2}}}+(4p+4q-8)\,x^{\frac{1}{\sqrt{5}}}$$

$$+(12pq-8p-8q+4)\,x^{\frac{1}{2\sqrt{2}}}.$$

Theorem 3. Let G be the graph of a nanotube

$KTUC_4C_8[p,q]$. Then

(i) $$\delta\sigma\left(KTUC_4C_8\left[p,q\right]\right)=4p+4q-8.$$

(ii) $$\delta\sigma\left(KTUC_4C_8\left[p,q\right],x\right)$$

$$=(12pq-6p-6q+8)\,x^0+(4p+4q-8)\,x^1.$$

Proof: From definitions and by using Table 1, we deduce

(i) $$\delta\sigma\left(KTUC_4C_8\left[p,q\right]\right)=\sum_{uv\in E(G)}\left(\delta_u-\delta_v\right)^2$$

$$=(1-1)^2(2p+2q+4)+(1-2)^2(4p+4q-8)$$

$$+(2-2)^2(12pq-8p-8q+4)$$

$$=4p+4q-8.$$

(ii) $$\delta\sigma\left(KTUC_4C_8\left[p,q\right],x\right)=\sum_{uv\in E(G)}x^{(\delta_u-\delta_v)^2}$$

$$=(2p+2q+4)\,x^{(1-1)^2}+(4p+4q-8)\,x^{(1-2)^2}$$

$$+(12pq-8p-8q+4)\,x^{(2-2)^2}$$

$$=(12pq-6p-6q+8)\,x^0+(4p+4q-8)\,x^1.$$

V. RESULTS FOR $GTUC_4C_8(S)$ NANOTUBES

In this section, we focus on the graph structure of family of $TUC_4C_8(S)$ nanotubes. The 2-dimensional lattice of $TUC_4C_8(S)$ is denoted by $G=GTUC_4C_8[p,q]$ where $p$is the number of columns and $q$is the number of rows. The graph of$GTUC_4C_8[p,q]$ is depicted in Figure 2.

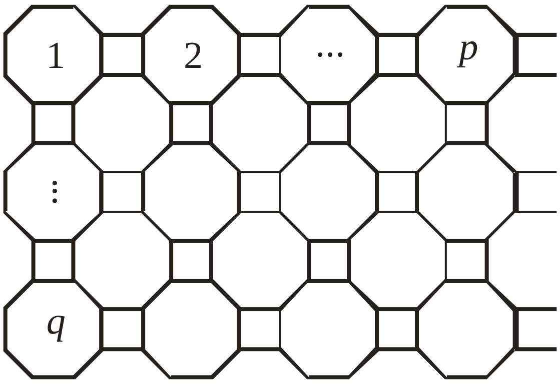

Figure 2: The graph of $GTUC_4C_8[p,q]$ nanotube

Let G be the molecular graph of $GTUC_4C_8[p,q]$ nanotube. By calculation, we obtain that G has $12pq - 2p$ edges. Also by calculation, we
obtain that G has three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 2p$.

$E_2 = \{uv\in E(G) \mid d_G(u) =2, d_G(v) = 3\}$, $|E_2| = 4p$.

$E_3 = \{uv\in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 12pq - 8p$.

Clearly we have $\delta(G)$=2. Hence $\delta_u= d_G(u) - \delta(G)+1=d_G(u) - 1$. Thus there are three types of δ-edges as given in Table 2.

Table 2:δ-edge partition of G

$\delta_u,\delta_v \backslash uv \in E(G)$	Number of edges
(1, 1)	$2p$
(1, 2)	$4p$
(2, 2)	$12pq - 8p$

Theorem 4. Let G be the graph of a nanotube

$GTUC_4C_8[p,q]$. Then

(i) $$\delta F\left(GTUC_4C_8\left[p,q\right]\right)=96pq-40p.$$

(ii) $$\delta F\left(GTUC_4C_8\left[p,q\right],x\right)$$

$$=2px^2+4px^5+(12pq-8p)\,x^8.$$

Proof: From definitions and by using Table 2, we deduce

(i) $\delta F\left(GTUC_4C_8[p,q]\right)=\sum_{uv\in E(G)}\left(\delta_u^2+\delta_v^2\right)$

$$=\left(1^2+1^2\right)2p+\left(1^2+2^2\right)4p$$
$$+\left(2^2+2^2\right)(12pq-8p)$$
$$=96pq-40p.$$

(ii) $\delta F\left(GTUC_4C_8[p,q],x\right)=\sum_{uv\in E(G)}x^{\left(\delta_u^2+\delta_v^2\right)}$

$$=2px^{\left(1^2+1^2\right)}+4px^{\left(1^2+2^2\right)}+(12pq-8p)x^{\left(2^2+2^2\right)}$$
$$=2px^2+4px^5+(12pq-8p)x^8.$$

Theorem 5. Let G be the graph of a nanotube $GTUC_4C_8[p,q]$. Then

(i) ${}^m\delta S\left(GTUC_4C_8[p,q]\right)$

$$=\frac{6}{\sqrt{2}}pq+\left(\frac{4}{\sqrt{5}}-\frac{2}{\sqrt{2}}\right)p.$$

(ii) ${}^m\delta S\left(GTUC_4C_8[p,q],x\right)$

$$=2px^{\frac{1}{\sqrt{2}}}+4px^{\frac{1}{\sqrt{5}}}+(12pq-8p)x^{\frac{1}{2\sqrt{2}}}.$$

Proof: From definitions and by using Table 2, we deduce

(i) ${}^m\delta S\left(GTUC_4C_8[p,q]\right)=\sum_{uv\in E(G)}\frac{1}{\sqrt{\delta_u^2+\delta_v^2}}$

$$=\frac{1}{\sqrt{1^2+1^2}}2p+\frac{1}{\sqrt{1^2+2^2}}4p$$
$$+\frac{1}{\sqrt{2^2+2^2}}(12pq-8p)$$
$$=\frac{6}{\sqrt{2}}pq+\left(\frac{4}{\sqrt{5}}-\frac{2}{\sqrt{2}}\right)p.$$

(ii) ${}^m\delta S\left(GTUC_4C_8[p,q],x\right)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{\delta_u^2+\delta_v^2}}}$

$$=2px^{\frac{1}{\sqrt{1^2+1^2}}}+4px^{\frac{1}{\sqrt{1^2+2^2}}}+(12pq-8p)x^{\frac{1}{\sqrt{2^2+2^2}}}$$
$$=2px^{\frac{1}{\sqrt{2}}}+4px^{\frac{1}{\sqrt{5}}}+(12pq-8p)x^{\frac{1}{2\sqrt{2}}}.$$

Theorem 6. Let G be the graph of a nanotube $GTUC_4C_8[p,q]$. Then

(i) $\delta\sigma\left(GTUC_4C_8[p,q]\right)=4p.$

(ii) $\delta\sigma\left(GTUC_4C_8[p,q],x\right)=(12pq-6p)x^0+4px^1.$

Proof: From definitions and by using Table 2, we deduce

(i) $\delta\sigma\left(GTUC_4C_8[p,q]\right)=\sum_{uv\in E(G)}\left(\delta_u-\delta_v\right)^2$

$$=(1-1)^2\,2p+(1-2)^2\,4p+(2-2)^2\,(12pq-8p)$$
$$=4p.$$

(ii) $\delta\sigma\left(GTUC_4C_8[p,q],x\right)=\sum_{uv\in E(G)}x^{\left(\delta_u-\delta_v\right)^2}$

$$=2px^{(1-1)^2}+4px^{(1-2)^2}+(12pq-8p)x^{(2-2)^2}$$
$$=(12pq-6p)x^0+4px^1.$$

2. Results for Silicate Networks

Silicate networks are obtained by fusing metal oxide or metal carbonates with sand. A silicate network is symbolized by SL_n, where n is the number of hexagons between the center and boundary of SL_n. A 2-*D* silicate network is presented in Figure 3.

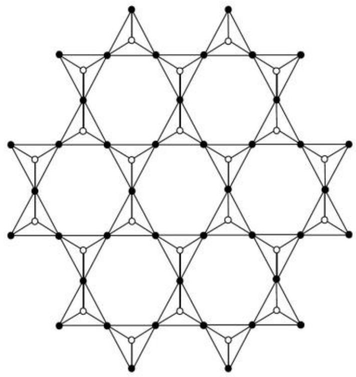

Figure 3. A 2-*D* silicate network

Let G be the graph of a silicate network SL_n. By calculation, we obtain that G has $15n^2+3n$ vertices and $36n^2$ edges. In G, there are two types of vertices as follows:

$V_1 = \{u \in V(G) \mid d_G(u) = 3\}$, $|V_1| = 6n^2 + 6n$.
$V_2 = \{u \in V(G) \mid d_G(u) = 6\}$, $|V_2| = 9n^2 - 3n$.

Therefore, we have $\delta(G)=3$ and hence $\delta_u = d_G(u) - \delta(u) + 1 = d_G(u) - 2$. Thus there are two types of δ-vertices as given in Table 3

Table 3: δ-vertex partition of SL_n

$\delta_u \setminus u \in V(G)$	Number of vertices
1	$6n^2 + 6n$
4	$9n^2 - 3n$

By calculation, in SL_n there are 3 types of edges based on degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_1| = 6n$.
$E_2 = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 6\}$, $|E_2| = 18n^2 + 6n$.
$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 6\}$, $|E_3| = 18n^2 - 12n$.

Hence there are 3 types of □-edges as given in Table 4.

Table 4: δ-edge partition of SL_n

$\delta_u,\delta_v \backslash uv \in E(G)$	Number of edges
(1, 1)	$6n$
(1, 4)	$18n^2 + 6n$
(4, 4)	$18n^2 - 12$

Theorem 7. Let G be the graph of a silicate network SL_n. Then

(i) $\delta F(SL_n) = 882n^2 - 276n.$

(ii) $\delta F(SL_n, x) = (18n^2 - 6n)x^0 + (18n^2 + 6n)x^9.$

Proof: From definitions and by using Table 4, we deduce

(i) $$\delta F(SL_n) = \sum_{uv\in E(G)} (\delta_u^2 + \delta_v^2)$$
$$= (1^2 + 1^2)6n + (1^2 + 4^2)(18n^2 + 6n)$$
$$+ (4^2 + 4^2)(18n^2 - 12n)$$
$$= 882n^2 - 276n.$$

(ii) $$\delta F(SL_n, x) = \sum_{uv\in E(G)} x^{(\delta_u^2 + \delta_v^2)}$$
$$= 6nx^{(1^2+1^2)} + (18n^2 + 6n)x^{(1^2+4^2)}$$
$$+ (18n^2 - 12n)x^{(4^2+4^2)}$$
$$= 6nx^2 + (18n^2 + 6n)x^{17} + (18n^2 - 12n)x^{32}.$$

Theorem 8. Let G be the graph of a silicate network SL_n. Then

(i) $${}^m\delta S(SL_n) = \left(\frac{18}{\sqrt{17}} + \frac{9}{2\sqrt{2}}\right)n^2 + \left(\frac{3}{\sqrt{2}} + \frac{6}{\sqrt{17}}\right)n.$$

(ii) $${}^m\delta S(SL_n, x)$$
$$= 6nx^{\frac{1}{\sqrt{2}}} + (18n^2 + 6n)x^{\frac{1}{\sqrt{17}}} + (18n^2 - 12n)x^{\frac{1}{4\sqrt{2}}}.$$

Proof: From definitions and by using Table 4, we deduce

(i) $${}^m\delta S(SL_n) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\delta_u^2 + \delta_v^2}}$$
$$= \frac{1}{\sqrt{1^2 + 1^2}}6n + \frac{1}{\sqrt{1^2 + 4^2}}(18n^2 + 6n)$$
$$+ \frac{1}{\sqrt{4^2 + 4^2}}(18n^2 - 12n)$$
$$= \left(\frac{18}{\sqrt{17}} + \frac{9}{2\sqrt{2}}\right)n^2 + \left(\frac{3}{\sqrt{2}} + \frac{6}{\sqrt{17}}\right)n.$$

(ii) $${}^m\delta S(SL_n, x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{\delta_u^2 + \delta_v^2}}}$$
$$= 6nx^{\frac{1}{\sqrt{1^2+1^2}}} + (18n^2 + 6n)x^{\frac{1}{\sqrt{1^2+4^2}}}$$
$$+ (18n^2 - 12n)x^{\frac{1}{\sqrt{4^2+4^2}}}$$
$$= 6nx^{\frac{1}{\sqrt{2}}} + (18n^2 + 6n)x^{\frac{1}{\sqrt{17}}} + (18n^2 - 12n)x^{\frac{1}{4\sqrt{2}}}.$$

Theorem 9. Let G be the graph of a silicate network SL_n. Then

(i) $\delta\sigma(SL_n) = 162n^2 + 54n.$

(ii) $\delta\sigma(SL_n, x) = (12pq - 6p)x^0 + 4px^1.$

Proof: From definitions and by using Table 4, we deduce

(i) $$\delta\sigma(SL_n) = \sum_{uv\in E(G)} (\delta_u - \delta_v)^2$$
$$= (1-1)^2 6n + (1-4)^2 (18n^2 + 6n)$$
$$+ (4-4)^2 (18n^2 - 12n)$$
$$= 162n^2 + 54n.$$

(ii) $$\delta\sigma(SL_n, x) = \sum_{uv\in E(G)} x^{(\delta_u - \delta_v)^2}$$
$$= 6nx^{(1-1)^2} + (18n^2 + 6n)x^{(1-4)^2} + (18n^2 - 12n)x^{(4-4)^2}$$
$$= (18n^2 - 6n)x^0 + (18n^2 + 6n)x^9.$$

Theorem 10. Let G be the graph of a silicate network SL_n. Then

(i) $\delta F_1(SL_n) = 582n^2 - 186n.$

(ii) $\delta F_1(SL_n, x) = (6n^2 + 6n)x^1 + (9n^2 - 3n)x^{64}.$

Proof: From definitions and by using Table 3, we deduce

(i) $$\delta F_1(G) = \sum_{u\in V(G)} \delta_u^3$$
$$= (1^3)(6n^2 + 6n) + (4^3)(9n^2 - 3n)$$
$$= 582n^2 - 186n..$$

(ii) $$\delta F_1(SL_n, x) = \sum_{u\in V(G)} x^{\delta_u^3}$$
$$= (6n^2 + 6n)x^{1^3} + (9n^2 - 3n)x^{4^3}$$
$$= (6n^2 + 6n)x^1 + (9n^2 - 3n)x^{64}.$$

6. RESULTS FOR HONEYCOMB NETWORKS

If we recursively use hexagonal tiling in a particular pattern, honeycomb networks are formed. These networks are very useful in chemistry and also in computer graphics. A honeycomb network of dimension n is denoted by HC_n, where n is the number of hexagons between central and boundary hexagon. A 4-dimensional honeycomb network is shown in Figure 4.

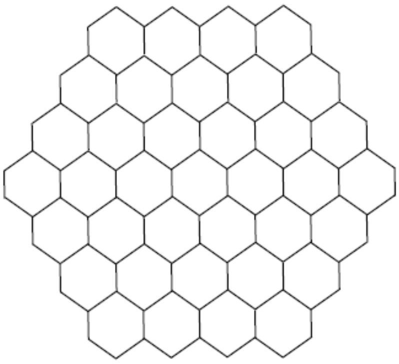

Figure 4. Honeycomb network of dimension four

Let G be the graph of honeycomb network HC_n with $|V(HC_n)|=6n^2$ and $|E(HC_n)| = 9n^2 - 3n$. From Figure 4, it is easy to see that there are two partitions of the vertex set of HC_n as follows:

$V_2 = \{u \in V(G) \mid d_G(u) = 2\}$, $|V_2| = 6n$.

$V_3 = \{u \in V(G) \mid d_G(u) = 3\}$, $|V_3| = 6n^2 - 3n$.

Therefore, we have $\delta(G)=2$ and hence $\delta_u = d_G(u) - \delta(u) + 1 = d_G(u) - 1$. Thus there are two types of δ-vertices as given in Table 5.

Table 5: δ-vertex partition of HC_n

$\delta_u \backslash u \in V(G)$	Number of vertices
1	$6n$
2	$6n^2 - 3n$

In HC_n, by algebraic method, there are three types of edges based on the degree of the vertices of each edge as follows:

$E_4 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_4| = 6$.

$E_5 = \{uv \in E(G) \mid d_G(u) = 2,\ d_G(v) = 3\}$, $|E_5| = 12n - 12$.

$E_6 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_6| = 9n^2 - 15n + 6$.

Hence there are 3 types of δ-edges as given in Table 6.

Table 6: δ-edge partition of HC_n

$\delta_u, \delta_v \backslash uv \in E(G)$	Number of edges
(1, 1)	6
(1, 2)	$12n - 12$
(2, 2)	$9n^2 - 15n + 6$

Theorem 11. Let G be the graph of a honeycomb network HC_n. Then

(i) $\delta F(HC_n) = 72n^2 - 60n.$

(ii) $\delta F(HC_n, x)$

$$= 6x^2 + (12n-12)x^5 + (9n^2 - 15n + 6)x^8.$$

Proof: From definitions and by using Table 6, we deduce

(i) $\delta F(HC_n) = \sum_{uv \in E(G)} \left(\delta_u^2 + \delta_v^2\right)$

$$= (1^2 + 1^2)6 + (1^2 + 2^2)(12n - 12) + (2^2 + 2^2)(9n^2 - 15n + 6) = 72n^2 - 60n.$$

(ii) $\delta F(HC_n, x) = \sum_{uv \in E(G)} x^{\left(\delta_u^2 + \delta_v^2\right)}$

$$= 6x^{(1^2+1^2)} + (12n-12)x^{(1^2+2^2)} + (9n^2 - 15n + 6)x^{(2^2+2^2)} = 6x^2 + (12n-12)x^5 + (9n^2 - 15n + 6)x^8.$$

Theorem 12. Let G be the graph of a honeycomb network HC_n. Then

(i) ${}^m\delta S(HC_n)$

$$= \frac{9}{2\sqrt{2}}n^2 + \left(\frac{12}{\sqrt{5}} - \frac{15}{2\sqrt{2}}\right)n + \frac{9}{\sqrt{2}} - \frac{12}{\sqrt{5}}.$$

(ii) ${}^m\delta S(HC_n, x)$

$$= 6x^{\frac{1}{\sqrt{2}}} + (12n - 12)x^{\frac{1}{\sqrt{5}}} + (9n^2 - 15n + 6)x^{\frac{1}{2\sqrt{2}}}.$$

Proof: From definitions and by using Table 6, we deduce

(i) ${}^m\delta S(HC_n) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\delta_u^2 + \delta_v^2}}$

$$= \frac{1}{\sqrt{1^2 + 1^2}}6 + \frac{1}{\sqrt{1^2 + 2^2}}(12n - 12) + \frac{1}{\sqrt{2^2 + 2^2}}(9n^2 - 15n + 6)$$

$$= \frac{9}{2\sqrt{2}}n^2 + \left(\frac{12}{\sqrt{5}} - \frac{15}{2\sqrt{2}}\right)n + \frac{9}{\sqrt{2}} - \frac{12}{\sqrt{5}}.$$

(ii) ${}^m\delta S(HC_n, x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{\delta_u^2 + \delta_v^2}}}$

$$= 6x^{\frac{1}{\sqrt{1^2+1^2}}} + (12n - 12)x^{\frac{1}{\sqrt{1^2+2^2}}} + (9n^2 - 15n + 6)x^{\frac{1}{\sqrt{2^2+2^2}}}$$

$$= 6x^{\frac{1}{\sqrt{2}}} + (12n - 12)x^{\frac{1}{\sqrt{5}}} + (9n^2 - 15n + 6)x^{\frac{1}{2\sqrt{2}}}.$$

Theorem 13. Let G be the graph of a honeycomb network HC_n. Then

(i) $\delta\sigma(HC_n) = 12n - 12.$

(ii) $\delta\sigma(HC_n, x) = (9n^2 - 15n + 12)x^0 + (12n - 12)x^1.$

Proof: From definitions and by using Table 6, we deduce

(i) $$\delta\sigma(HC_n) = \sum_{uv\in E(G)} (\delta_u - \delta_v)^2$$
$$= (1-1)^2 6 + (1-2)^2 (12n - 12) + (2-2)^2 (9n^2 - 15n + 6)$$
$$= 12n - 12.$$

(ii) $$\delta\sigma(HC_n, x) = \sum_{uv\in E(G)} x^{(\delta_u - \delta_v)^2}$$
$$= 6x^{(1-1)^2} + (12n-12)x^{(1-2)^2} + (9n^2 - 15n + 6)x^{(2-2)^2}$$
$$= (9n^2 - 15n + 12)x^0 + (12n-12)x^1.$$

Theorem 14. Let G be the graph of a silicate network HC_n. Then

(i) $\delta F_1(HC_n) = 48n^2 - 18n.$

(ii) $\delta F_1(HC_n, x) = 6nx^1 + (6n^2 - 3n)x^8.$

Proof: From definitions and by using Table 5, we deduce

(i) $$\delta F_1(G) = \sum_{u\in V(G)} \delta_u^3$$
$$= (1^3)6n + (2^3)(6n^2 - 3n)$$
$$= 48n^2 - 18n..$$

(ii) $$\delta F_1(SL_n, x) = \sum_{u\in V(G)} x^{\delta_u^3}$$
$$= 6nx^{1^3} + (6n^2 - 3n)x^{2^3}$$
$$= 6nx^1 + (6n^2 - 3n)x^8.$$

VI. CONCLUSION

In this study, we have defined the delta F, modified delta Sombor, delta sigma indices and their corresponding polynomials of a graph. Also these delta indices and their corresponding polynomials of certain nanotubes and networks are determined.

REFERENCES

1. V.R. Kulli, College Graph Theory, Vishwa International Publications, Gulbarga, India (2012).
2. S.Wagner and H.Wang, Introduction Chemical Graph Theory, Boca Raton, CRC Press, (2018).
3. M.V.Diudea (ed.) QSPR/QSAR Studies by Molecular Descriptors, NOVA New York, (2001).
4. V.R.Kulli, δ-Sombor index and its exponential of certain nanotubes, Annals of Pure and Applied Mathematics, 23(1) (2021) 37-42.
5. V.R.Kulli, δ-Banhatti, hyper δ-Banhatti indices and their polynomials of certain networks, International Journal of Engineering Sciences and Research Technology, 10(3) (2021) 1-8.
6. S.B.B.Altindag, I.Milovannovic and E.Milovanocic M.Matejic, S.Stankov , Remark on delta and reverse degree indices, Sci. Pub. State Univ. Novi Pazar, Ser. A: Appl. Math. Inform. Mech. 15(1) (2022) 37-47.
7. B.K.Majhi and V.R.Kulli, Sum connectivity delta Banhatti and product connectivity delta Banhatti indices of certain nanotubes, submitted.
8. V.R.Kulli, Delta Nirmala index, submitted.
9. E.Milovannovic and I.Milovanocic, M.Matejic, S.Stankov and S.B.B.Altindag , Bounds for the reverse (delta) first Zagreb indices, Sci. Pub. State Univ. Novi Pazar, Ser. A: Appl. Math. Inform. Mech. 15(1) (2023) 49-59.
10. S.Alikhani and N.Ghanbari, Sombor index of polymers, MATCH Commun. Math. Comput. Chem. 86 (2021).
11. R.Cruz, I.Gutman and J.Rada, Sombor index of chemical graphs, Appl. Math. Comput. 399 (2021) 126018.
12. H.Deng, Z.Tang and R.Wu, Molecular trees with extremal values of Sombor indices, Int. J. Quantum Chem.DOI: 10.1002/qua.26622.
13. I.Gutman, I.Redzepovic and V.R.Kulli, KG-Sombor index of Kragujevac trees, Open Journal of Discrete Applied Matematics,5(2) (2022) 19-25.
14. B.Horoldagva and C.Xu, On Sombor index of graphs, MATCH Commun. Math. Comput. Chem. 86 (2021).
15. V.R.Kulli, Neighborhood Sombor indices, International Journal of Mathematics Trends and Technology, 68(6) (2022) 195-204.
16. V.R.Kulli, Computation of reduced Kulli-Gutman Sombor index of certain networks, Journal of Mathematics and Informatics, 23 (2022) 1-5.
17. V.R.Kulli, Multiplicative KG Sombor indices of some networks, International Journal of Mathematics Trends and Technology, 68(10) (2022) 1-7.
18. V.R.Kulli and I.Gutman, Sombor and KG Sombor indices of benzenoid systems and phenylenes, Annals of Pure and Applied Mathematics, 26(2) (2022) 49-53.
19. V.R.Kulli, N.Harish, and B.Chaluvaraju, Sombor leap indices of some chemical drugs, RESEARCH REVIEW International Journal of Multidisciplinary, 7(10) (2022) 158-166.
20. V.R.Kulli, J.A.Mendez-Bermudez, J.M.Rodriguez and J.M.Sigarreta, Revan Sombor indices: Analytical and statistical study, on random graphs,

Mathematical Boisciences and Engineering, 20(2) (2022) 1801-1819.
21. H.R.Manjunatha, V.R.Kulli and N.D.Soner, The HDR Sombor index, International Journal of Mathematics Trends and Technology, 68(4) (2022) 1-6.
22. V.R.Kulli, F-Sombor and modified Sombor indices of certain nanotubes, Annals of Pure and Applied Mathematics, 27(1) (2023) 13-17.
23. V.R.Kulli, Gourava Sombor indices, International Journal of Engineering sciences and Research Technology, 11(11) (2023) 29-38.
24. V.R.Kulli, Edge version of Sombor and Nirmala indices of some nanotubes and nanotori, International Journal of Mathematics and Computer Research, 11(3) (2023) 3305-3310.
25. V.R.Kulli, Irregularity domination Nirmala and domination Sombor indices of certain drugs, International Journal of Mathematical Archive, 14(8) (2023) 1-7.
26. V.R.Kulli, Modified domination Sombor index and its exponential of a graph, International Journal of Mathematics and Computer Research, 11(8) (2023) 3639-3644.
27. A.A.Shashidhar, H.Ahmed, N.D.Soner and M.Cancan, Domination version: Sombor index of graphs and its significance in predicting physicochemical properties of butane derivatives, Eurasian Chemical Communications, 5 (2023) 91-102.
28. I.Gutman, M.Togan, A.Yurttas, A.S.Cevik and I.N.Cangul, Inverse problem for sigma index, MATCH Commun. Math. Comput. Chem. 79 (2018) 491-508.
29. A.Jahabani and S.Ediz, The sigma index of graph operations, Sigma J. Eng. & Nat. Sci. 37 (1) (2019) 155-162.