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soft g-closed sets in Soft minimal Spaces

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Abstract

In this present paper, we introduce soft $g\tilde{m}$-closed sets and soft $g\tilde{m}$-open sets in soft minimal spaces and to investigate its properties. Also we introduce some new separation axiom called $T_{1/2}$ -soft minimal space and its basic properties are discussed.
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1 Introduction

V. Popa and T.Noiri [14] introduced the concept of minimal structure (briefly m-structure). They also introduced the notion of m_X-open set and m_X-closed set and characterize those sets using m_X-closure and m_X-interior operators respectively. C. Boonpok [1] introduced the concept of biminimal structure space and studied $m^1_X m^2_X$-open sets and $m^1_X m^2_X$-closed sets in biminimal structure spaces. R. Gowri and S. Vembu [7] introduced the concept of Soft minimal and soft biminimal spaces. Also they introduced the notion of $\tilde{m}$-soft closed, $\tilde{m}$-soft open, $\tilde{m}_1\tilde{m}_2$-soft closed, $\tilde{m}_1\tilde{m}_2$-soft open set and characterize those sets using m_X-closure and m_X-interior operators respectively. C. Viriyapong et.al [16] introduced the concept of generalized m-closed sets in biminimal structure spaces and we obtain some properties of generalized m-closed sets. Russian researcher Molodtsov [12], initaited the concept of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences. In this paper, we introduce soft $g\tilde{m}$-closed sets in soft minimal spaces which are defined over an initial universe with a fixed set of parameters. Also we introduce $T_{1/2}$- soft minimal spaces and detailed study of some of its properties.

2 Preliminaries

Definition 2.1 [6] *Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a nonempty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.*

In other words, a soft set over U is a parametrized family of subsets of the universe U. For $\epsilon \in A$. $F(\epsilon)$ may be considered as the set of ϵ - approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

Definition 2.2 [14] *Let X be an initial universe set, E be the set of parameters and $A \subseteq E$. Let F_A be a nonempty soft set over X and $\tilde{P}(F_A)$ is the soft power set of F_A. A subfamily $\tilde{m}$ of $\tilde{P}(F_A)$ is called a soft minimal set over X if $F_\emptyset \in \tilde{m}$ and $F_A \in \tilde{m}$.*

$(F_A, \tilde{m})$ or $(X, \tilde{m}, E)$ is called a soft minimal space over X. Each member of $\tilde{m}$ is said to be $\tilde{m}$ -soft open set and the complement of an $\tilde{m}$-soft open set is said to be $\tilde{m}$-soft closed set over X.

Example 2.3 [14] *Let $U = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. Then*

$$\begin{aligned}
&F_{A_1} = \{(x_1, \{u_1\})\},\\
&F_{A_2} = \{(x_1, \{u_2\})\},\\
&F_{A_3} = \{(x_1, \{u_1, u_2\})\},\\
&F_{A_4} = \{(x_2, \{u_1\})\},\\
&F_{A_5} = \{(x_2, \{u_2\})\},\\
&F_{A_6} = \{(x_2, \{u_1, u_2\})\},\\
&F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\},\\
&F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\},\\
&F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\},\\
&F_{A_{10}} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\},\\
&F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\},\\
&F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\},\\
&F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\},\\
&F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\},\\
&F_{A_{15}} = F_A,\\
&F_{A_{16}} = F_\emptyset \text{ are all soft subsets of } F_A
\end{aligned}$$

soft minimal $\tilde{m} = \{F_A, F_\emptyset, F_{A_2}, F_{A_5}, F_{A_7}, F_{A_{11}}\}$

Definition 2.4 [7] *Let (X, m_X) be an m-space. A subset A of X is said to be generalized m-closed (briefly gm-closed) if $m_X - Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $m_X -$ open.*

Definition 2.5 [7] *An m-space (X, m_X) is called an $m - T_{1/2}$-space if every gm-closed set of (X, m_X) is m_X-closed.*

Definition 2.6 [14] *Let $(F_A, \tilde{m})$ be a soft minimal space with nonempty set F_A is said to have property B if the union of any family belonging to $\tilde{m}$ belongs to $\tilde{m}$.*

Definition 2.7 [14] *Let $(F_A, \tilde{m})$ be a soft minimal space over X. For a soft subset F_B of F_A, the $\tilde{m}$-soft closure of F_B and $\tilde{m}$-soft interior of F_B are defined as follows:*

(1) $\tilde{m}Cl(F_B) = \cap \{F_\alpha : F_B \tilde{\subseteq} F_\alpha, F_A - F_\alpha \in \tilde{m}\}$,

(2) $\tilde{m}Int(F_B) = \cup \{F_\beta : F_\beta \tilde{\subseteq} F_B, F_\beta \in \tilde{m}\}$.

Lemma 2.8 [14] *Let $(F_A, \tilde{m})$ be a soft minimal space over X. For a soft subset F_B and F_C of F_A,the following properties hold:*

(1) $\tilde{m}cl(F_A - F_B) = F_A - \tilde{m}Int(F_B)$ *and* $\tilde{m}Int(F_A - F_B) = F_A - \tilde{m}cl(F_B)$*,*

(2) If $(F_A - F_B) \in \tilde{m}$*, then* $\tilde{m}cl(F_B) = F_B$ *and if* $F_B \in \tilde{m}$*,*
then $\tilde{m}Int(F_B) = F_B$*,*

(3) $\tilde{m}cl(F_\emptyset) = F_\emptyset$*,* $\tilde{m}cl(F_A) = F_A$*,* $\tilde{m}Int(F_\emptyset) = F_\emptyset$ *and* $\tilde{m}Int(F_A) = F_A$*,*

(4) If $F_B \tilde{\subseteq} F_C$*, then* $\tilde{m}cl(F_B) \tilde{\subseteq} \tilde{m}cl(F_C)$ *and* $\tilde{m}Int(F_B) \tilde{\subseteq} \tilde{m}Int(F_C)$*,*

(5) $F_B \tilde{\subseteq} \tilde{m}cl(F_B)$ *and* $\tilde{m}Int(F_B) \tilde{\subseteq} F_B$*,*

(6) $\tilde{m}cl(\tilde{m}cl(F_B)) = \tilde{m}cl(F_B)$ *and* $\tilde{m}Int(\tilde{m}Int(F_B)) = \tilde{m}Int(F_B)$*.*

Lemma 2.9 [14] *Let* F_A *be a nonempty set and* $\tilde{m}$ *on* X *satisfying property B. For a soft subset* F_B *of* F_A*, the following properties hold:*

(1) $F_B \in \tilde{m}$ *if and only if* $\tilde{m}Int(F_B) = F_B$*,*

(2) If F_B *is* $\tilde{m}$*-closed if and only if* $\tilde{m}Cl(F_B) = F_B$*,*

(3) $\tilde{m}Int(F_B) \in \tilde{m}$ *and* $\tilde{m}Cl(F_B) \in \tilde{m}$*-closed.*

3 Soft generalized closed sets in soft minimal spaces

In this section, we introduce the concept of soft g-closed sets in soft minimal spaces and study some of their properties.

Definition 3.1 *A soft subset* F_B *of a soft minimal space* $(F_A, \tilde{m})$ *is said to be soft generalized* $\tilde{m}$*-closed sets (briefly* $sg\tilde{m} - closed$*) if* $\tilde{m}Cl(F_B) \tilde{\subseteq} U_B$ *whenever* $F_B \tilde{\subseteq} U_B$ *and* U_B *is soft* $\tilde{m} - open$*. The complement of soft generalized* $\tilde{m}$*-closed set is said to be soft generalized* $\tilde{m}$*-open sets (briefly* $sg\tilde{m}$*-open).*

The family of all $sg\tilde{m}$ *closed (resp.* $sg\tilde{m}$*-open) sets of* $(F_A, \tilde{m})$ *is denoted by* $sg\tilde{m}Cl(F_A)$ *(resp.* $sg\tilde{m}O(F_A)$*)*

Example 3.2 *Let* $X = \{u_1, u_2\}$*,* $E = \{x_1, x_2, x_3\}$*,* $A = \{x_1, x_2\} \subseteq E$ *and*
$F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$*. Then*

$F_{A_1} = \{(x_1, \{u_1\})\}$, $F_{A_2} = \{(x_1, \{u_2\})\}$,
$F_{A_3} = \{(x_1, \{u_1, u_2\})\}$, $F_{A_4} = \{(x_2, \{u_1\})\}$,
$F_{A_5} = \{(x_2, \{u_2\})\}$, $F_{A_6} = \{(x_2, \{u_1, u_2\})\}$,
$F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}$, $F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$,
$F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}$, $F_{A_{10}} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\}$,
$F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$, $F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}$
$F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}$, $F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$,
$F_{A_{15}} = F_A$, $F_{A_{16}} = F_\emptyset$

soft minimal $(\tilde{m}) = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_5}, F_{A_7}, F_{A_9}, F_{A_{11}}, F_{A_{14}}, F_A\}$

Then the soft generalized $\tilde{m}$*-closed sets* $\{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_4}, F_{A_7}, F_{A_{11}}, F_{A_{13}}, F_A\}$

Proposition 3.3 *Every soft $\tilde{m}$-closed set is soft $g\tilde{m}$-closed.*

Proof: Let F_B be a soft $\tilde{m}$-closed and U_B be a soft $\tilde{m}$-open such that $F_B \tilde{\subseteq} U_B$. Then $\tilde{m}Cl(F_B) \tilde{\subseteq} U_B$, since F_B is a soft $g\tilde{m}$-closed sets $\tilde{m}Cl(F_B) = F_B$ and $\tilde{m}Cl(F_B)$ $\tilde{\subseteq} U_B$. Therefore F_B is a soft $g\tilde{m}$ closed sets. □

Remark 3.4 *The converse of the Proposition 3.3 is not true as seen from the following example.*

Example 3.5 *Let $(F_A, \tilde{m})$ be a soft minimal space where $X = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. $\tilde{m} = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_5}, F_{A_7}, F_{A_9}, F_{A_{11}}, F_{A_{14}}, F_A\}$. Then the soft $g\tilde{m}$-closed sets $\{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_4}, F_{A_5}, F_{A_7}, F_{A_{11}}, F_{A_{13}}, F_A\}$. Here, F_{A_5} is soft $g\tilde{m}$-closed but not soft $\tilde{m}$-closed.*

Theorem 3.6 *Union of two soft $g\tilde{m}$-closed sets is soft $g\tilde{m}$-closed.*

Proof: Let F_B and G_B are soft $g\tilde{m}$-closed set. Let U_B be a soft $\tilde{m}$-open set of $(F_A, \tilde{m})$ suct that $F_B \cup G_B \tilde{\subseteq} U_B$. Then $F_B \tilde{\subseteq} U_B$ and $G_B \tilde{\subseteq} U_B$. Since F_B and G_B are soft $g\tilde{m}$-closed, $\tilde{m}Cl(F_B) \tilde{\subseteq} U_B$ and $\tilde{m}Cl(G_B) \tilde{\subseteq} U_B$. Hence $\tilde{m}Cl(F_B \cup G_B) \tilde{\subseteq}$ $\tilde{m}Cl(F_B) \cup \tilde{m}Cl(G_B) \tilde{\subseteq} U_B$. Therefore $F_B \cup G_B$ is soft $g\tilde{m}$-closed set. □

Remark 3.7 *The intersection of two soft $g\tilde{m}$-closed sets need not be soft $g\tilde{m}$-closed as seen from the following example.*

Example 3.8 *Let $(F_A, \tilde{m})$ be a soft minimal space where $X = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. $\tilde{m} = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_5}, F_{A_7}, F_{A_9}, F_{A_{11}}, F_{A_{14}}, F_A\}$. Then the soft $g\tilde{m}$-closed sets $\{F_\emptyset, F_{A_1}, F_{A_3}, F_{A_4}, F_{A_7}, F_{A_{11}}, F_{A_{13}}, F_A\}$. Then F_{A_3} and $F_{A_{11}}$ are soft $g\tilde{m}$-closed but $F_{A_3} \cap F_{A_{11}} = F_{A_5}$ is not soft $g\tilde{m}$-closed set.*

Theorem 3.9 *If F_B is soft $\tilde{m}$-open and soft $g\tilde{m}$-closed then F_B is soft $\tilde{m}$-closed.*

Proof: Let F_B is soft $\tilde{m}$-open and soft $g\tilde{m}$-closed. Let $F_B \tilde{\subset} F_B$ where F_B is soft $\tilde{m}$-open. Since F_B is soft $g\tilde{m}$-closed we have $\tilde{m}Cl(F_B) \tilde{\subseteq} F_B$. Then $F_B = \tilde{m}Cl(F_B)$. Hence F_B is soft $\tilde{m}$-closed. □

Theorem 3.10 *If F_B is soft $g\tilde{m}$-closed and G_B is soft $\tilde{m}$-closed, then $F_B \cap G_B$ is soft $g\tilde{m}$-closed.*

Proof: Let U_B be a soft $\tilde{m}$-open such that $F_B \cap G_B \tilde{\subseteq} U_B$. Thus $F_B \tilde{\subseteq} U_B \cup {G_B}^c$. Then $\tilde{m}Cl(F_B) \tilde{\subseteq} U_B \cup {G_B}^c$. Then $\tilde{m}Cl(F_B) \cap G_B \tilde{\subseteq} U_B$. Since G_B is soft $\tilde{m}$-closed. Therefore, $\tilde{m}Cl(F_B) \cap G_B \tilde{\subseteq} U_B$. Hence $F_B \cap G_B$ is soft $g\tilde{m}$-closed. □

Theorem 3.11 *Let soft subset F_B of a soft minimal space $(F_A, \tilde{m})$. If F_B is soft $g\tilde{m}$-closed, then $\tilde{m}Cl(F_B) - F_B$ contains no nonempty soft $\tilde{m}$-closed set.*

Proof: Let G_B be a nonempty soft $\tilde{m}$-closed set such that $G_B \tilde{\subseteq} \tilde{m}Cl(F_B) - F_B$. Since F_B is soft $g\tilde{m}$-closed, $F_B \tilde{\subseteq} {G_B}^c$. That is $F_B \tilde{\subseteq} F_A - G_B$, where ${G_B}^c$ is soft $\tilde{m}$-open implies that $\tilde{m}Cl(F_B) \tilde{\subseteq} {G_B}^c$. Hence $G_B \tilde{\subseteq} [\tilde{m}Cl(F_B)]^c$. Now $G_B \tilde{\subseteq}$ $[\tilde{m}Cl(F_B)] \cap [\tilde{m}Cl(F_B)]^c = F_\emptyset$. Therefore $\tilde{m}Cl(F_B) - F_B$ contains no nonempty soft $\tilde{m}$-closed set. □

Remark 3.12 *The converse of the above theorem is not true as seen from the following example*

Example 3.13 *Let* $X = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ *and* $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. *Then*

$F_{A_1} = \{(x_1, \{u_1\})\}$, $F_{A_2} = \{(x_1, \{u_2\})\}$,
$F_{A_3} = \{(x_1, \{u_1, u_2\})\}$, $F_{A_4} = \{(x_2, \{u_1\})\}$,
$F_{A_5} = \{(x_2, \{u_2\})\}$, $F_{A_6} = \{(x_2, \{u_1, u_2\})\}$,
$F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}$, $F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$,
$F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}$, $F_{A_{10}} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\}$,
$F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$, $F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}$
$F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}$, $F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$,
$F_{A_{15}} = F_A$, $F_{A_{16}} = F_\emptyset$
$\tilde{m} = \{F_\emptyset, F_{A_2}, F_{A_3}, F_{A_7}, F_{A_8}, F_{A_{12}}, F_{A_{14}}, F_A\}$
soft $g\tilde{m}$*-closed sets* $\{F_\emptyset, F_{A_1}, F_{A_3}, F_{A_4}, F_{A_8}, F_{A_{10}}, F_{A_{11}}, F_A\}$.
Take $F_B = F_{A_9}$. *Then* $\tilde{m}Cl(F_B) - F_B = \tilde{m}Cl(F_{A_9}) - F_{A_9} = F_{A_2}$ *which does not contain any nonempty soft* $\tilde{m}$*-closed set. But* $F_B = F_{A_9}$ *is not soft* $g\tilde{m}$*-closed.*

Corollary 3.14 *Let* $(F_A, \tilde{m})$ *be a soft minimal space satisfying property B. Let soft subset* F_B *is soft* $g\tilde{m}$*-closed set in* $(F_A, \tilde{m})$*, then* F_B *is soft* $\tilde{m}$*-closed if and only if* $\tilde{m}Cl(F_B) - F_B$ *is soft* $\tilde{m}$*-closed.*

Proof: Let F_B is soft $\tilde{m}$-closed then $\tilde{m}Cl(F_B) = F_B$. That is $\tilde{m}Cl(F_B) - F_B = F_\emptyset$. Thus $F_\emptyset$ is soft $\tilde{m}$-closed. Hence $\tilde{m}Cl(F_B) - F_B = F_\emptyset$ is soft $\tilde{m}$-closed. conversely, Let $\tilde{m}Cl(F_B) - F_B$ is soft $\tilde{m}$-closed. If F_B is soft $g\tilde{m}$-closed By Theorem 3.11, $\tilde{m}Cl(F_B) - F_B = F_\emptyset$. Hence F_B is soft $g\tilde{m}$-closed. Therefore F_B is soft $\tilde{m}$-closed. □

Theorem 3.15 *If* F_B *is a soft* $g\tilde{m}$*-closed set in* $(F_A, \tilde{m})$ *such that* $F_B \tilde{\subseteq} G_B \tilde{\subseteq} \tilde{m}Cl(F_B)$ *then* G_B *is also a soft* $g\tilde{m}$*-closed set in* $(F_A, \tilde{m})$.

Proof: Let $G_B \tilde{\subseteq} U_B$ where U_B is soft $\tilde{m}$-open. Since F_B is soft $g\tilde{m}$-closed set in $(F_A, \tilde{m})$, we have $\tilde{m}cl(F_B) - F_B$ contains no nonempty soft $\tilde{m}$-closed set. Now $G_B \tilde{\subseteq} \tilde{m}Cl(F_B)$ implies that $\tilde{m}Cl(G_B) \tilde{\subseteq} \tilde{m}Cl(F_B)$. We have $\tilde{m}Cl(G_B) - G_B \tilde{\subseteq} \tilde{m}Cl(F_B) - F_B$. This implies $\tilde{m}Cl(G_B) - G_B$ contains no nonempty soft $\tilde{m}$-closed By Theorem 3.11, G_B is soft $g\tilde{m}$-closed. □

Theorem 3.16 *For each* $x \in F_A$, $\{x\}$ *is soft* $\tilde{m}$*-closed in* F_A *or* $\{x\}^c$ *is soft* $g\tilde{m}$*-closed set.*

Proof: If $\{x\}$ is not soft $\tilde{m}$-closed. Then the only soft $\tilde{m}$-open set containing $\{x\}^c$. This implies $\tilde{m}cl[\{x\}]^c \tilde{\subseteq} F_A$. Hence $\{x\}^c$ is soft $g\tilde{m}$-closed in F_A. □

Theorem 3.17 *Let* $(F_A, \tilde{m})$ *be a soft minimal space satisfying property B. If* F_B *is* $sg\tilde{m}$*-closed of* $(F_A, \tilde{m})$ *then* $\tilde{m}Cl(\{x\}) \cap F_B \neq F_\emptyset$ *holds for each* $x \in \tilde{m}Cl(F_B)$

Proof: Let $x \in \tilde{m}Cl(F_B)$. Assume that $\tilde{m}Cl(\{x\}) \cap F_B = F_\emptyset$. Then $F_B \tilde{\subseteq} [\tilde{m}Cl(\{x\})]^c$. Since F_B is $sg\tilde{m}$-closed and $[\tilde{m}Cl(\{x\})]^c$ is soft $\tilde{m}$-open, thus $\tilde{m}Cl(F_B) \tilde{\subseteq} [\tilde{m}Cl(\{x\})]^c$. Consequently $\tilde{m}Cl(F_B) \cap \tilde{m}Cl(\{x\})$. This is a contradiction. □

4 Soft generalized $\tilde{m}$-open set

Definition 4.1 *A soft subset F_B is called a soft $g\tilde{m}$-open in a soft minimal space $(F_A, \tilde{m})$ if the relative complement of F_B is soft $g\tilde{m}$-closed in F_A*

Theorem 4.2 *A soft subset F_B is soft $g\tilde{m}$-open if and only if $G_B \tilde{\subseteq} \tilde{m}Int(F_B)$ whenever G_B is soft $\tilde{m}$-closed and $G_B \tilde{\subseteq} F_B$.*

Proof: Let F_B be a soft $g\tilde{m}$-open set. Let G_B be a soft $\tilde{m}$-closed set such that $G_B \tilde{\subseteq} F_B$. Then ${F_B}^c \tilde{\subseteq} {G_B}^c$ where ${G_B}^c$ is soft $\tilde{m}$-open. ${F_B}^c$ is soft $g\tilde{m}$-closed implies that $[\tilde{m}Cl(F_B)]^c \tilde{\subseteq} {G_B}^c$. That is $[\tilde{m}Int(F_B)]^c \tilde{\subseteq} {G_B}^c$. Therefore $G_B \tilde{\subseteq} \tilde{m}Int(F_B)$. Conversely suppose G_B is soft $\tilde{m}$-closed and $G_B \tilde{\subseteq} F_B$. Also $G_B \tilde{\subseteq} \tilde{m}Int(F_B)$. Let ${U_B}^c \tilde{\subseteq} F_B$ where ${U_B}^c$ is soft $\tilde{m}$-closed. By hypothesis ${U_B}^c \tilde{\subseteq} \tilde{m}Int(F_B)$. Thus $[\tilde{m}Int(F_B)]^c \tilde{\subseteq} U_B$. (i.e) $[\tilde{m}Cl(F_B)]^c \tilde{\subseteq} U_B$. Therefore ${F_B}^c$ is soft $g\tilde{m}$-closed. Hence F_B is soft $g\tilde{m}$-open set □

Theorem 4.3 *If $\tilde{m}Int(F_B) \tilde{\subseteq} F_B$ and F_B is soft $g\tilde{m}$-open set then G_B is soft $g\tilde{m}$-open.*

Proof: $\tilde{m}Int(F_B) \tilde{\subseteq} G_B \tilde{\subseteq} F_B$ implies ${F_B}^c \tilde{\subseteq} {G_B}^c \tilde{\subseteq} [\tilde{m}Cl(F_B)]^c$ and ${F_B}^c$ is soft $g\tilde{m}$-closed. Hence G_B is soft $g\tilde{m}$-open. □

Theorem 4.4 *A soft subset F_B is soft $g\tilde{m}$-closed if and only if $\tilde{m}Cl(F_B) - F_B$ is soft $g\tilde{m}$-open.*

Proof: Let F_B is soft $g\tilde{m}$-closed. Let $G_B \tilde{\subseteq} \tilde{m}Cl(F_B) - F_B$ where G_B is soft $\tilde{m}$-closed. By theorem 4.3 $G_B = F_\emptyset$. Therefore $G_B \tilde{\subseteq} \tilde{m}Int[\tilde{m}Cl(F_B) - F_B]$. By Theorem 4.2 $\tilde{m}Cl(F_B) - F_B$ is soft $g\tilde{m}$-open. Conversely, Let $F_B \tilde{\subseteq} G_B$ where G_B is soft $\tilde{m}$-open set. Then $\tilde{m}Cl(F_B) \cap {G_B}^C \tilde{\subseteq} \tilde{m}Cl(F_B) \cap {F_B}^c = \tilde{m}Cl(F_B) - F_B$. Since $\tilde{m}Cl(F_B) \cap {G_B}^c$ is soft $\tilde{m}$-closed and $\tilde{m}Cl(F_B) - F_B$ is soft $g\tilde{m}$-open. It follows from the Theorem 4.2 $\tilde{m}Cl(F_B) \cap {U_B}^c \tilde{\subseteq} \tilde{m}Int[\tilde{m}Cl(F_B) \cap {F_B}^c] = \tilde{m}Int[\tilde{m}Cl(F_B) - F_B] = F_\emptyset$. Hence F_B is soft $g\tilde{m}$-closed. □

Remark 4.5 *The converse of the above Theorem 4.4 is not true as shown in the following example*

Example 4.6 *Let $(F_A, \tilde{m})$ be a soft minimal space where $X = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$.*
$\tilde{m} = \{F_\emptyset, F_{A_1}, F_{A_4}, F_{A_5}, F_{A_{10}}, F_{A_{11}}, F_{A_{13}}, F_A\}$. Then the soft $g\tilde{m}$-closed sets are $\{F_\emptyset, F_{A_2}, F_{A_4}, F_{A_5}, F_{A_7}, F_{A_8}, F_{A_{12}}, F_A\}$. Here, $F_B = F_{A_1}$. Then $\tilde{m}Cl(F_{A_1}) - F_{A_1} = F_{A_4}$ which is soft $g\tilde{m}$-open. But F_{A_1} is not soft $g\tilde{m}$-closed.

Remark 4.7 *For any soft subset F_B of F_A, $\tilde{m}Int(\tilde{m}Cl(F_B) - F_B) = F_\emptyset$.*

Theorem 4.8 *IF F_B is soft $g\tilde{m}$-open set in $(F_A, \tilde{m})$ such that $\tilde{m}Int(F_B) \tilde{\subseteq} G_B \tilde{\subseteq} F_B$ and G_B is soft $g\tilde{m}$-open set in $(F_A, \tilde{m})$.*

Proof: Let F_B is soft $g\tilde{m}$-open set in $(F_A, \tilde{m})$ such that $\tilde{m}Int(F_B) \tilde{\subseteq} G_B \tilde{\subseteq} F_B$. Let U_B be a soft $\tilde{m}$-closed such that $U_B \tilde{\subseteq} G_B$. Then $U_B \tilde{\subseteq} F_B$. Since F_B is soft $g\tilde{m}$-open set $U_B \tilde{\subseteq} \tilde{m}Int(F_B)$. Now $\tilde{m}Int(\tilde{m}Cl(F_B)) \tilde{\subseteq} \tilde{m}Int(G_B) = U_B \tilde{\subseteq} \tilde{m}Int(G_B)$. That is $U_B \tilde{\subseteq} \tilde{m}Int(G_B)$, G_B is soft $\tilde{m}$-open. Hence G_B is soft $g\tilde{m}$-open. □

Theorem 4.9 *Every soft $\tilde{m}$-open set is soft $g\tilde{m}$-open set.*

Proof: Let F_B be a soft $g\tilde{m}$-open set. Let U_B is a soft $g\tilde{m}$-closed such that $U_B \tilde{\subseteq} F_B$. Then $U_B \tilde{\subseteq} \tilde{m}Int(F_B)$. Since F_B is soft $g\tilde{m}$-open set $\tilde{m}Int(F_B) = F_B$, $U_B \tilde{\subseteq} \tilde{m}Int(F_B)$. Therefore F_B is a soft $g\tilde{m}$-open set.

The converse is not true in general . The following Example supports our claim. □

Example 4.10 *Let $X = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. Then $\tilde{m} = \{F_\emptyset, F_{A_1}, F_{A_8}, F_{A_{10}}, F_A\}$. Then the soft $g\tilde{m}$-closed sets are $\{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, F_{A_5}, F_{A_6}, F_{A_8}, F_{A_{10}}, F_A\}$. Here, F_{A_5} is soft $g\tilde{m}$-open but not soft $\tilde{m}$-open.*

Theorem 4.11 *If F_B and G_B are two soft $g\tilde{m}$-open subset of a soft minimal space $(F_A, \tilde{m})$ then $F_B \cap G_B$ is soft $g\tilde{m}$-open.*

Proof: Assume that U_B is soft $\tilde{m}$-closed set containing in $(F_B \cap G_B)$. Since F_B and G_B are soft $g\tilde{m}$-open set then by theorem 4.2 $U_B \tilde{\subseteq} \tilde{m}Int(F_B)$ and $U_B \tilde{\subseteq} \tilde{m}Int(G_B)$. Then we have $U_B \tilde{\subseteq} \tilde{m}Int(F_B) \cap \tilde{m}Int(G_B) = \tilde{m}Int(F_B \cap G_B)$. Hence $U_B \tilde{\subseteq} \tilde{m}Int(F_B \cap G_B)$. Therefore $F_B \cap G_B$ is soft $g\tilde{m}$-open. □

Theorem 4.12 *If F_B and G_B are two soft $g\tilde{m}$-open subset of a soft minimal space $(F_A, \tilde{m})$ then $F_B \cup G_B$ is soft $g\tilde{m}$-open.*

Proof: Assume that U_B be a soft $\tilde{m}$-closed subset of $F_B \cup G_B$. Then $U_B \cap \tilde{m}cl(F_B) \tilde{\subseteq} F_B$ and hence by theorem 4.2 $U_B \cap \tilde{m}cl(F_B) \tilde{\subseteq} \tilde{m}Int(F_B)$. Simillarly $U_B \cap \tilde{m}cl(G_B) \tilde{\subseteq} \tilde{m}Int(G_B)$. $U_B \tilde{\subseteq} \tilde{m}Int(F_B) \cup \tilde{m}Int(G_B) \tilde{\subseteq} \tilde{m}Int(F_B \cup G_B)$. Hence $U_B \tilde{\subseteq} \tilde{m}Int(F_B \cup G_B)$ and by theorem 4.2 $F_B \cup G_B$ is soft $g\tilde{m}$-open. □

5 $T_{1/2}$ - soft minimal space

Definition 5.1 *A soft minimal space $(F_A, \tilde{m})$ is said to be a $T_{1/2}$ - space if every soft $g\tilde{m}$-closed set of $(F_A, \tilde{m})$ is soft $\tilde{m}$-closed*

Theorem 5.2 *Let $(F_A, \tilde{m})$ be a soft minimal space, then*
(1) Every $T_{1/2}$-space is T_0-space.
(2) Every T_1-space is $T_{1/2}$-space.

Proof: It is obivious □

Theorem 5.3 *Let $(F_A, \tilde{m})$ be a soft minimal space, the following properties are equivalent:*

(1) $(F_A, \tilde{m})$ is an $T_{1/2}$ - space.

(2) For each $x \in F_A$, the singleton $\{x\}$ is not soft $\tilde{m}$-open and soft $\tilde{m}$-closed.

Proof: (1) $\Rightarrow$ (2)
Let $(F_A, \tilde{m})$ be a $T_{1/2}$ - space and $x \in F_A$. If x is not soft $\tilde{m}$-closed subset of $(F_A, \tilde{m})$. Then $\{x\}^c$ is not soft $\tilde{m}$-open and then $\{x\}^c$ is soft $g\tilde{m}$-closed. Since $(F_A, \tilde{m})$ is $T_{1/2}$ - space. This implies that $\{x\}^c$ is soft $\tilde{m}$-closed and hence $\{x\}$ is soft $\tilde{m}$-open.
(2) $\Rightarrow$ (1)
Let G_B be a soft $g\tilde{m}$-closed and $x \in \tilde{m}Cl(G_B)$.
We have the following two cases:
case(i)
If the singleton $\{x\}$ is soft $\tilde{m}$-closed. By Theorem 3.11 $\tilde{m}Cl(G_B) - G_B$ does not contain any non empty closed set. This shows that $x \in G_B$.
case(ii)
If the singleton $\{x\}$ is soft $\tilde{m}$-open. If $\notin G_B$. Then $G_B \tilde{\subseteq} \{x\}^c$. Since $\{x\}^c$ is soft $\tilde{m}$-closed. Then $\tilde{m}Cl(G_B) \tilde{\subseteq} \tilde{m}Cl\{x\}^c = \{x\}^c$. Thus $x \notin \tilde{m}Cl(G_B)$. In either case, $\tilde{m}Cl(G_B) = G_B$. That is G_B is soft $\tilde{m}$-closed. Thus $(F_A, \tilde{m})$ is a $T_{1/2}$ -soft minimal space. □

Corollary 5.4 *The property of $T_{1/2}$-space is strictly between T_0 and T_1.*

References

[1] C. Boonpok, Biminimal Structure Spaces, International Mathematical Forum, 15(5)(2010), 703-707.

[2] N. Cagman, S. Karatas, and S. Enginoglu, Soft Topology., Comput. Math. Appl., Vol. 62,(2011), 351-358.

[3] F. Cammaroto and T. Noiri, On Λ- sets and related topological spaces, Acta Math. Hungar., 109 (3)(2005), 261-279.

[4] D. Chen, The Parametrization Reduction of Soft Set and its Applications, Comput.Math.Appl.49 (2005), 757-763.

[5] T. Fukutake, On generalized closed sets in bitopological spaces, Bull. Fukuoka Univ. Ed. Part III, 35 (1986), 19-28.

[6] R. Gowri and G. Jegadeesan, soft generalized closed set in soft cech closure space, Global Journal of Pure and Applied Mathematics, Vol.12, No.1 (2016), 909-916

[7] R.Gowri, S.Vembu, Soft minimal and soft biminimal spaces, Int Jr. of Mathematical Science and Appl., Vol. 5, No.2,(2015), 447-455.

[8] B.M Ittanagi, Soft Bitopological Spaces, International Journal of Computer Applications, Vol. 107, No.7(2014).

[9] J.C Kelly, Bitopological Spaces, Proc. London Math. Soc., 13 (1963), 71-81

[10] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo (2), 19 (1970), 89-96.

[11] H. Maki, K.C Rao and A. Nagoor Gani, On generalized semi-open and preopen sets, Pure Appl. Math. Sci., 49 (1999), 17-29.

[12] D.A Molodtsov, Soft Set Theory First Results. Comp.and Math.with App., Vol.37, (1999), 19-31.

[13] T. Noiri and V. Popa, A generalized of some forms of g-irresolute functions, European J. of Pure and Appl. Math., 2(4)(2009), 473-493.

[14] V. Popa, T. Noiri, On M-continuous functions, Anal. Univ.Dunarea de Jos-Galati, Ser. Mat. Fiz. Mec. Teor., Fasc. II, 18, No.23 (2000), 31-41.

[15] M. Shabir, M. Naz, On soft topological spaces, Comput.Math. Appl.,61, (2011), 1786-1799.

[16] C. Viriyapong, M. Tunapan, W. Rattanametawee and C. Boonpok, Generalized m-Closed Sets in Biminimal Structure Spaces Int. Journal of Math. Analysis, Vol.5, No.7, (2011), 333 - 346