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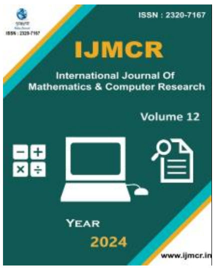

Modified Elliptic Sombor Index and its Exponential of a Graph

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ARTICLE INFO	ABSTRACT
Published Online: 04 January 2024 Corresponding Author: **V.R.Kulli**	The elliptic Sombor index is a recently introduced topological index. In this paper, we put forward the modified elliptic Sombor index and its corresponding exponential of a graph and compute exact formulas for some families of benzenoid systems such as triangular benzenoids, benzenoid rhombus, benzenoid hourglass and jagged rectangle benzenoid systems.

KEYWORDS: modified elliptic Sombor index, modified elliptic Sombor exponential, benzenoid.

I. INTRODUCTION

Let G be a finite, simple, connected graph. Its vertex and edge sets are *V(G) and E(G)* respectively. The degree d_u of a vertex u in G is the number of vertices adjacent to u. We refer the book [1] for undefined notations and terminologies.

Chemical Graph Theory has an important effect on the development of Mathematical Chemistry. Topological indices are useful for establishing correlation between the structure of a molecular compound and its physicochemical properties. Topological indices have found some applications, especially in QSPR/QSAR research [2, 3].

The elliptic Sombor index [4] of a graph G is defined as

$$ESO(G)=\sum_{uv\in E(G)}(d_u+d_v)\sqrt{d_u^2+d_v^2}.$$

Recently, some Sombor indices were studied in [5-25].

We define the elliptic Sombor exponential of a graph G as

$$ESO(G,x)=\sum_{uv\in E(G)}x^{(d_u+d_v)\sqrt{d_u^2+d_v^2}}.$$

We put forward the modified elliptic Sombor index of a graph G and it is defined as

$$^{m}ESO(G)=\sum_{uv\in E(G)}\frac{1}{(d_u+d_v)\sqrt{d_u^2+d_v^2}}.$$

We define the modified elliptic Sombor exponential of a graph G as

$$^{m}ESO(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{(d_u+d_v)\sqrt{d_u^2+d_v^2}}}.$$

In this paper, we determine the elliptic Sombor indices and their corresponding exponentials of some families of benzenoid systems.

II. TRIANGULAR BENZENOIDS

In this section, we consider a family of triangular benzenoids. This family of benzenoids is denoted by T_p, where p is the number of hexagons in the base graph. Clearly T_p has $\frac{1}{2}p(p-1)$ hexagons. The graph of T_4 is shown in Figure 1.

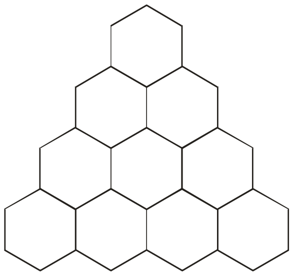

Figure 1. Molecular graph of T_4

Let G be the graph of a triangular benzenoid T_p. The graph G has $p^2 + 4p + 1$vertices and $\frac{3p(p+3)}{2}$ edges. We obtain that G has three types of edges as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 6$.

$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 6p - 6$.

$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}, |E_3| = \frac{3p(p-1)}{2}$.

Theorem 1. The elliptic Sombor index of a triangular benzenoid T_p is given by

$$ESO(G)$$
$$= 27\sqrt{2}p^2 + \left(30\sqrt{13} - 27\sqrt{2}\right)p + 48\sqrt{2} - 30\sqrt{13}.$$

Proof: We have

$$ESO(G) = \sum_{uv\in E(G)} (d_u + d_v)\sqrt{d_u^2 + d_v^2}$$
$$= 6(2+2)\sqrt{2^2 + 2^2} + (6p-6)(2+3)\sqrt{2^2+3^2}$$
$$+ \frac{3p(p-1)}{2}(3+3)\sqrt{3^2+3^2}$$
$$= 27\sqrt{2}p^2 + \left(30\sqrt{13} - 27\sqrt{2}\right)p + 48\sqrt{2} - 30\sqrt{13}.$$

Theorem 2. The elliptic Sombor exponential of a triangular benzenoid T_p is given by

$$ESO(G,x) = 6x^{8\sqrt{2}} + (6p-6)x^{5\sqrt{13}} + \frac{3p(p-1)}{2}x^{18\sqrt{2}}.$$

Proof: We have

$$ESO(G,x) = \sum_{uv\in E(G)} x^{(d_u+d_v)\sqrt{d_u^2+d_v^2}}$$
$$= 6x^{(2+2)\sqrt{2^2+2^2}} + (6p-6)x^{(2+3)\sqrt{2^2+3^2}}$$
$$+ \frac{3p(p-1)}{2}x^{(3+3)\sqrt{3^2+3^2}}$$
$$= 6x^{8\sqrt{2}} + (6p-6)x^{5\sqrt{13}} + \frac{3p(p-1)}{2}x^{18\sqrt{2}}.$$

Theorem 3. The modified elliptic Sombor index of a triangular benzenoid T_p is given by

$$^{m}ESO(G)$$
$$= \frac{1}{12\sqrt{2}}p^2 + \left(\frac{6}{5\sqrt{13}} - \frac{1}{12\sqrt{2}}\right)p + \frac{3}{4\sqrt{2}} - \frac{6}{5\sqrt{13}}.$$

Proof: We have

$$^{m}ESO(G) = \sum_{uv\in E(G)} \frac{1}{(d_u+d_v)\sqrt{d_u^2+d_v^2}}$$
$$= \frac{6}{(2+2)\sqrt{2^2+2^2}} + \frac{(6p-6)}{(2+3)\sqrt{2^2+3^2}}$$
$$+ \frac{3p(p-1)}{2(3+3)\sqrt{3^2+3^2}}$$
$$= \frac{1}{12\sqrt{2}}p^2 + \left(\frac{6}{5\sqrt{13}} - \frac{1}{12\sqrt{2}}\right)p + \frac{3}{4\sqrt{2}} - \frac{6}{5\sqrt{13}}.$$

Theorem 4. The modified elliptic Sombor exponential of a triangular benzenoid T_p is given by

$$^{m}ESO(G,x) = 6x^{\frac{1}{8\sqrt{2}}} + (6p-6)x^{\frac{1}{5\sqrt{13}}} + \frac{3p(p-1)}{2}x^{\frac{1}{18\sqrt{2}}}.$$

Proof: We have

$$^{m}ESO(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{(d_u+d_v)\sqrt{d_u^2+d_v^2}}}$$
$$= 6x^{\frac{1}{(2+2)\sqrt{2^2+2^2}}} + (6p-6)x^{\frac{1}{(2+3)\sqrt{2^2+3^2}}}$$
$$+ \frac{3p(p-1)}{2}x^{\frac{1}{(3+3)\sqrt{3^2+3^2}}}$$
$$= 6x^{\frac{1}{8\sqrt{2}}} + (6p-6)x^{\frac{1}{5\sqrt{13}}} + \frac{3p(p-1)}{2}x^{\frac{1}{18\sqrt{2}}}.$$

III. BENZENOID RHOMBUS

In this section, we consider a family of benzenoid rhombus. This family of benzenoids is denoted by R_p. The benzenoid rhombus R_p is obtained from two copies of a triangular benzenoid T_p by identifying hexagons in one of their base rows. The graph of R_4 is shown in Figure 2.

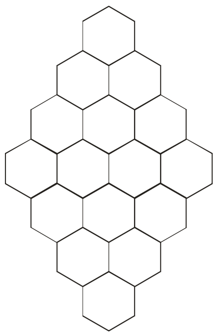

Figure 2. Molecular graph of R_4

Let G be the graph of a benzenoid rhombus R_p. The graph G has $2p^2 + 4p$ vertices and $3p^2 + 4p - 1$ edges. We obtain that G has three types of edges as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$,
$|E_1| = 6$.

$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$,
$|E_2| = 8p - 8$.

$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$,
$|E_3| = 3p^2 - 4p + 1$.

Theorem 5. The elliptic Sombor index of a benzenoid rhombus R_p is given by

$$ESO(G)$$
$$= 54\sqrt{2}p^2 + \left(40\sqrt{13} - 72\sqrt{2}\right)p + 66\sqrt{2} - 40\sqrt{13}.$$

Proof: We have

$$ESO(G) = \sum_{uv\in E(G)} (d_u + d_v)\sqrt{d_u^2 + d_v^2}$$
$$= 6(2+2)\sqrt{2^2+2^2} + (8p-8)(2+3)\sqrt{2^2+3^2}$$

$$+\left(3p^2-4p+1\right)(3+3)\sqrt{3^2+3^2}$$
$$=54\sqrt{2}p^2+\left(40\sqrt{13}-72\sqrt{2}\right)p+66\sqrt{2}-40\sqrt{13}.$$

Theorem 6. The elliptic Sombor exponential of a benzenoid rhombus R_p is given by

$$ESO(G,x)=6x^{8\sqrt{2}}+(8p-8)x^{5\sqrt{13}}$$
$$+\left(3p^2-4p+1\right)x^{18\sqrt{2}}.$$

Proof: We have

$$ESO(G,x)=\sum_{uv\in E(G)} x^{(d_u+d_v)\sqrt{d_u^2+d_v^2}}$$
$$=6x^{(2+2)\sqrt{2^2+2^2}}+(8p-8)x^{(2+3)\sqrt{2^2+3^2}}$$
$$+\left(3p^2-4p+1\right)x^{(3+3)\sqrt{3^2+3^2}}$$
$$=6x^{8\sqrt{2}}+(8p-8)x^{5\sqrt{13}}+\left(3p^2-4p+1\right)x^{18\sqrt{2}}.$$

Theorem 7. The modified elliptic Sombor index of a benzenoid rhombus R_p is given by

$$^{m}ESO(G)$$
$$=\frac{1}{6\sqrt{2}}p^2+\left(\frac{8}{5\sqrt{13}}-\frac{2}{9\sqrt{2}}\right)p+\frac{3}{4\sqrt{2}}-\frac{8}{5\sqrt{13}}+\frac{1}{18\sqrt{2}}.$$

Proof: We have

$$^{m}ESO(G)=\sum_{uv\in E(G)}\frac{1}{(d_u+d_v)\sqrt{d_u^2+d_v^2}}$$
$$=\frac{6}{(2+2)\sqrt{2^2+2^2}}+\frac{(8p-8)}{(2+3)\sqrt{2^2+3^2}}$$
$$+\frac{\left(3p^2-4p+1\right)}{(3+3)\sqrt{3^2+3^2}}$$
$$=\frac{1}{6\sqrt{2}}p^2+\left(\frac{8}{5\sqrt{13}}-\frac{2}{9\sqrt{2}}\right)p+\frac{3}{4\sqrt{2}}-\frac{8}{5\sqrt{13}}+\frac{1}{18\sqrt{2}}.$$

Theorem 8. The modified elliptic Sombor exponential of a benzenoid rhombus R_p is given by

$$^{m}ESO(G,x)$$
$$=6x^{\frac{1}{8\sqrt{2}}}+(8p-8)x^{\frac{1}{5\sqrt{13}}}+\left(3p^2-4p+1\right)x^{\frac{1}{18\sqrt{2}}}$$

Proof: We have

$$^{m}ESO(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{(d_u+d_v)\sqrt{d_u^2+d_v^2}}}$$
$$=6x^{\frac{1}{(2+2)\sqrt{2^2+2^2}}}+(6p-6)x^{\frac{1}{(2+3)\sqrt{2^2+3^2}}}$$
$$+\frac{3p(p-1)}{2}x^{\frac{1}{(3+3)\sqrt{3^2+3^2}}}$$
$$=6x^{\frac{1}{8\sqrt{2}}}+(8p-8)x^{\frac{1}{5\sqrt{13}}}+\left(3p^2-4p+1\right)x^{\frac{1}{18\sqrt{2}}}.$$

IV. BENZENOID HOURGLASS

In this section, we consider a family of benzenoid hourglass, which is denoted by X_p. This family is obtained from two copies of a triangular benzenoid T_p by overlapping hexagons. The graph of benzenoid hourglass is presented in Figure 3.

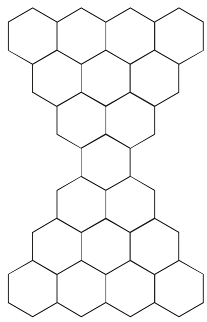

Figure 3. Molecular graph of benzenoid hourglass

Let G be the graph of a benzenoid hourglass X_p. This graph G has $2(p^2 + 4p - 2)$ vertices and $3p^2+9p-4$ edges. We find that G has three types of edges as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 8$.

$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 12p - 16$.

$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 3p^2 - 3p + 4$.

Theorem 9. The elliptic Sombor index of a benzenoid hourglass X_p is given by

$$ESO(G)$$
$$=54\sqrt{2}p^2+\left(60\sqrt{13}-54\sqrt{2}\right)p+136\sqrt{2}-80\sqrt{13}.$$

Proof: We have

$$ESO(G)=\sum_{uv\in E(G)}(d_u+d_v)\sqrt{d_u^2+d_v^2}$$
$$=8(2+2)\sqrt{2^2+2^2}+(12p-16)(2+3)\sqrt{2^2+3^2}$$
$$+\left(3p^2-3p+4\right)(3+3)\sqrt{3^2+3^2}$$
$$=54\sqrt{2}p^2+\left(60\sqrt{13}-54\sqrt{2}\right)p+136\sqrt{2}-80\sqrt{13}.$$

Theorem 10. The elliptic Sombor exponential of a benzenoid hourglass X_p is given by

$$ESO(G,x) = 8x^{8\sqrt{2}} + (12p-16)\,x^{5\sqrt{13}} + \left(3p^2-3p+4\right)x^{18\sqrt{2}}$$

Proof: We have

$$ESO(G,x) = \sum_{uv\in E(G)} x^{(d_u+d_v)\sqrt{d_u^2+d_v^2}}$$

$$= 8x^{(2+2)\sqrt{2^2+2^2}} + (12p-16)x^{(2+3)\sqrt{2^2+3^2}} + \left(3p^2-3p+4\right)x^{(3+3)\sqrt{3^2+3^2}}$$

$$= 8x^{8\sqrt{2}} + (12p-16)x^{5\sqrt{13}} + \left(3p^2-3p+4\right)x^{18\sqrt{2}}.$$

Theorem 11. The modified elliptic Sombor index of a benzenoid hourglass X_p is given by

$$^{m}ESO(G) = \frac{1}{6\sqrt{2}}p^2 + \left(\frac{12}{5\sqrt{13}} - \frac{1}{6\sqrt{2}}\right)p + \frac{1}{\sqrt{2}} - \frac{16}{5\sqrt{13}} + \frac{2}{9\sqrt{2}}.$$

Proof: We have

$$^{m}ESO(G) = \sum_{uv\in E(G)} \frac{1}{(d_u+d_v)\sqrt{d_u^2+d_v^2}}$$

$$= \frac{8}{(2+2)\sqrt{2^2+2^2}} + \frac{(12p-16)}{(2+3)\sqrt{2^2+3^2}} + \frac{\left(3p^2-3p+4\right)}{(3+3)\sqrt{3^2+3^2}}$$

$$= \frac{1}{6\sqrt{2}}p^2 + \left(\frac{12}{5\sqrt{13}} - \frac{1}{6\sqrt{2}}\right)p + \frac{1}{\sqrt{2}} - \frac{16}{5\sqrt{13}} + \frac{2}{9\sqrt{2}}.$$

Theorem 12. The modified elliptic Sombor exponential of a benzenoid hourglass X_p is given by

$$^{m}ESO(G,x) = 6x^{\frac{1}{8\sqrt{2}}} + (8p-8)x^{\frac{1}{5\sqrt{13}}} + \left(3p^2-4p+1\right)x^{\frac{1}{18\sqrt{2}}}$$

Proof: We have

$$^{m}ESO(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{(d_u+d_v)\sqrt{d_u^2+d_v^2}}}$$

$$= 8x^{\frac{1}{(2+2)\sqrt{2^2+2^2}}} + (12p-16)x^{\frac{1}{(2+3)\sqrt{2^2+3^2}}} + \frac{3p(p-1)}{2}x^{\frac{1}{(3+3)\sqrt{3^2+3^2}}}$$

$$= 6x^{\frac{1}{8\sqrt{2}}} + (8p-8)x^{\frac{1}{5\sqrt{13}}} + \left(3p^2-4p+1\right)x^{\frac{1}{18\sqrt{2}}}.$$

V. JAGGED RECTANGLE BENZENOID SYSTEMS

In this section, we focus in the molecular graph structure of a jagged rectangle benzenoid system. This system is denoted by $B_{m,n}$ for all $m, n \in N$. Three chemical graphs of a jagged rectangle benzenoid system are presented in Figure 4.

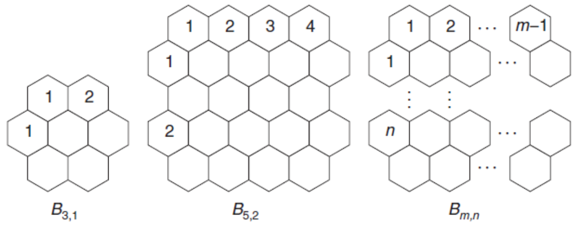

Figure 4

Let G be the graph of a jagged rectangle benzenoid system $B_{m,n}$. By calculation, we obtain that G has $4mn + 4m + 2n - 2$ vertices and $6mn + 5m + n - 4$ edges. We obtain that the edge set of $B_{m,n}$ can be divided into three partitions as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 2n + 4$.

$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 4m + 4n - 4$.

$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 6mn + m - 5n - 4$.

Theorem 13. The elliptic Sombor index of a jagged rectangle benzenoid system $B_{m,n}$ is given by

$$ESO(G) = 108\sqrt{2}mn + \left(20\sqrt{13} + 18\sqrt{2}\right)m + \left(20\sqrt{13} - 74\sqrt{2}\right)n - 20\sqrt{13} - 40\sqrt{2}.$$

Proof: We have

$$ESO(G) = \sum_{uv\in E(G)} (d_u+d_v)\sqrt{d_u^2+d_v^2}$$

$$= (2n+4)(2+2)\sqrt{2^2+2^2} + (4m+4n-4)(2+3)\sqrt{2^2+3^2} + (6mn+m-5n-4)(3+3)\sqrt{3^2+3^2}$$

$$= 108\sqrt{2}mn + \left(20\sqrt{13} + 18\sqrt{2}\right)m + \left(20\sqrt{13} - 74\sqrt{2}\right)n - 20\sqrt{13} - 40\sqrt{2}.$$

Theorem 14. The elliptic Sombor exponential of a jagged rectangle benzenoid system $B_{m,n}$ is given by

$$ESO(G,x) = (2n+4)\,x^{8\sqrt{2}} + (4m+4n-4)\,x^{5\sqrt{13}} + (6mn+m-5n-4)\,x^{18\sqrt{2}}.$$

Proof: We have

$$ESO(G,x) = \sum_{uv\in E(G)} x^{(d_u+d_v)\sqrt{d_u^2+d_v^2}}$$

$$= (2n+4)x^{(2+2)\sqrt{2^2+2^2}} + (4m+4n-4)x^{(2+3)\sqrt{2^2+3^2}} + (6mn+m-5n-4)x^{(3+3)\sqrt{3^2+3^2}}$$

$$= (2n+4)x^{8\sqrt{2}} + (4m+4n-4)x^{5\sqrt{13}}$$
$$+ (6mn+m-5n-4)x^{18\sqrt{2}}.$$

Theorem 15. The modified elliptic Sombor index of a jagged rectangle benzenoid system $B_{m,n}$ is given by

$$^{m}ESO(G)$$
$$= \frac{1}{6\sqrt{2}}p^2 + \left(\frac{12}{5\sqrt{13}} - \frac{1}{6\sqrt{2}}\right)p + \frac{1}{\sqrt{2}} - \frac{16}{5\sqrt{13}} + \frac{2}{9\sqrt{2}}.$$

Proof: We have

$$^{m}ESO(G) = \sum_{uv\in E(G)} \frac{1}{(d_u+d_v)\sqrt{d_u^2+d_v^2}}$$
$$= \frac{2n+4}{(2+2)\sqrt{2^2+2^2}} + \frac{4m+4n-4}{(2+3)\sqrt{2^2+3^2}}$$
$$+ \frac{6mn+m-5n-4}{(3+3)\sqrt{3^2+3^2}}$$
$$= \frac{1}{6\sqrt{2}}p^2 + \left(\frac{12}{5\sqrt{13}} - \frac{1}{6\sqrt{2}}\right)p + \frac{1}{\sqrt{2}} - \frac{16}{5\sqrt{13}} + \frac{2}{9\sqrt{2}}.$$

Theorem 16. The modified elliptic Sombor exponential of a jagged rectangle benzenoid system $B_{m,n}$ is given by

$$^{m}ESO(G,x) = (2n+4)\,x^{\frac{1}{8\sqrt{2}}} + (4m+4n-4)\,x^{\frac{1}{5\sqrt{13}}}$$
$$+ (6mn+m-5n-4)\,x^{\frac{1}{18\sqrt{2}}}.$$

Proof: We have

$$^{m}ESO(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{(d_u+d_v)\sqrt{d_u^2+d_v^2}}}$$
$$= (2n+4)x^{\frac{1}{(2+2)\sqrt{2^2+2^2}}} + (4m+4n-4)x^{\frac{1}{(2+3)\sqrt{2^2+3^2}}}$$
$$+ (6mn+m-5n-4)x^{\frac{1}{(3+3)\sqrt{3^2+3^2}}}$$
$$= (2n+4)x^{\frac{1}{8\sqrt{2}}} + (4m+4n-4)x^{\frac{1}{5\sqrt{13}}}$$
$$+ (6mn+m-5n-4)x^{\frac{1}{18\sqrt{2}}}.$$

VI. CONCLUSION

In this study, we have determined the elliptic Sombor and modified elliptic Sombor indices and their corresponding exponentials of some families of benzenoid systems which are appeared in chemical science.

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