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Probabilistic Analysis of a Three-Unit Series-Parallel System with Common-Cause, Human Error and Environmental State Failures

Dr. D. Sarada Devi
Associate Professor, Department of Mathematics, Government Degree College for Women, Begumpet, Hyderabad-500016, Telangana – India

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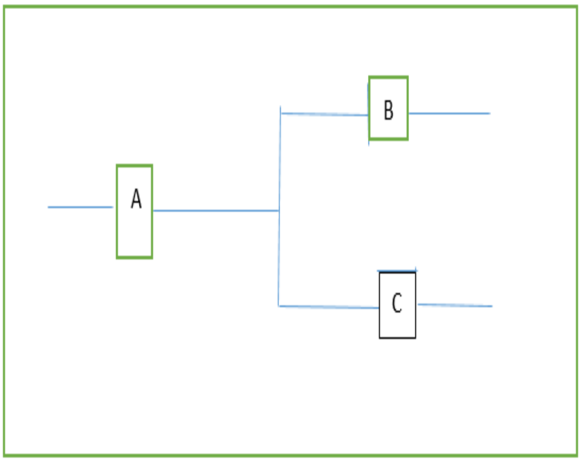
Corresponding Author: **Dr. D. Sarada Devi**

ABSTRACT
This paper presents a mathematical model for performing reliability and availability analyses of Series-Parallel system with constant human error and common-cause failure rates. The method of Integro-differential equation was used to develop equations for the model to obtain the general expression. The Laplace transform technique was used to obtain the system reliability with constant human error failure rate of the system.

KEYWORDS: Series –Parallel system, Laplace Transform, Inverse Laplace Transform, Reliability

INTRODUCTION

Consider a human system, the heart and the kidneys should function properly for the human being to survive (assuming that the remaining parts of the body are all operative). However, this cannot be modelled as a simple series or parallel system comprising two sub-systems with one or two components in each subsystem. While the heart does appear as a single component in the first sub- system, the second sub-system is representing the two components, each one corresponding to one of the kidneys. This is necessitated by the fact that the proper functioning of any one of the two kidneys ensures the survival of the human being. The Picture of series- parallel system is given below.



ASSUMPTIONS

0 The state of the system with all the three components functioning state

1 The state of the system with the component A and one of the components B or C is in functioning state

2 The failed state of the system corresponding to the failure of A from state 0

3 The failed state of the system due to the failure of the component A from state 1

4 The failure state of the system corresponding to the failure of both the components B and C while A is functioning

5 The failed state of the system due to human error from state 0

6 The failed state of the system due to common cause error

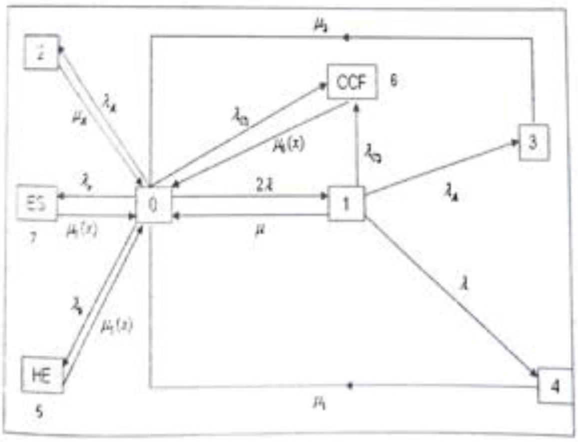
7 The failed state of the system due to environmental state

NOTATIONS

t = Time
s = Laplace Transform Variable
λ =Constant failure rate of the units A & B
λ_{c1}= Constant common cause failure rate of the system from the state 0
λ_{c2}= Constant common cause failure rate of the system from the state 1
λ_h= Constant human error failure rate of the system
λ_A= Constant failure rate of the unit A
μ_A= Constant repair rate of the unit A

μ = Constant repair rate of the unit
λ_e= Constant environmental failure rate of the system
μ_1= Constant repair rate of the system from the failed state 4
μ_2= Constant repair rate of the system from the failed state 3
$P_k(x,t)$= Probability density (With respect to repair time) that the failed system is in state k and has an elapsed repair time of x for k= 5,6,7

FORMULATION OF MATHEMATICAL MODEL:



With the above notation, the transition diagram of the system is given by The system of Integro -differential equations associated with this model are given by

$$\frac{d}{dt}P_0(t) + (2\lambda + \lambda_{c1} + \lambda_h + \lambda_A + \lambda_e)P_0(t) = \mu P_1(t) + \mu_A P_2(t) + \mu_1 P_4(t) + \mu_2 P_3(t) + \int_0^\infty P_5(x,t)\mu_5(x)dx + \int_0^\infty P_6(x,t)\mu_6(x)dx + \int_0^\infty P_7(x,t)\mu_7(x)dx$$

$$\frac{d}{dt}P_1(t) + (\lambda_A + \lambda + \lambda_{c2} + \mu)P_1(t) = 2\lambda P_0(t)$$
$$\frac{d}{dt}P_2(t) + \mu_A P_2(t) = \lambda_A P_0(t)$$
$$\frac{d}{dt}P_3(t) + \mu_2 P_3(t) = \lambda_A P_1(t)$$
$$\frac{d}{dt}P_4(t) + \mu_1 P_4(t) = \lambda P_1(t)$$
$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_5(x)]P_5(x,t) = 0$$
$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_6(x)]P_6(x,t) = 0$$
$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + \mu_7(x)]P_7(x,t) = 0$$

$A_1 = \frac{2\lambda(s_1+A_2)}{(s_1-s_2)(s_1-s_3)}$, $B_1 = \frac{2\lambda(s_2+A_2)}{(s_2-s_1)(s_2-s_3)}$,$C_1 = \frac{2\lambda(s_3+A_2)}{(s_3-s_1)(s_3-s_2)}$

The system reliability is given by

$R(t) = P_0(t)+P_1(t)=(A_0 + A_1)e^{s_1t}+(B_0+B_1)e^{s_2t} + (C_0 + C_1)e^{s_3t}$

$\mu_k(x), q_k(x)$ = Repair rate and probability density function of repair time respectively when the failed system is in state k and has an elapsed repair time of x for k=5,6,7
β = The shape parameter of the Gamma probability density function

$P_5(0,t) = \lambda_h P_0(t)$
$P_6(0,t) = \lambda_{c1}P_0(t) + \lambda_{c2}P_1(t)$
$P_7(0,t) = \lambda_e P_0(t)$

The initial conditions are given by

$P_0(0) = 1$ and $P_j(0) = 0$ for j=1,2,3,4,$P_k(x,0) = 0$ for k=5,6,7

Assume that $\mu_1 = \mu_2 = \mu_5 = \mu_6 = \mu_7 = 0$

$$\frac{d}{dt}P_0(t) + (2\lambda + \lambda_{c1} + \lambda_h + \lambda_A + \lambda_e)P_0(t) = \mu P_1(t) + \mu_A P_2(t)$$
$$\frac{d}{dt}P_1(t) + (\lambda_A + \lambda + \lambda_{c2} + \mu)P_1(t) = 2\lambda P_0(t)$$
$$\frac{d}{dt}P_2(t) + \mu_A P_2(t) = \lambda_A P_0(t)$$
$$\frac{d}{dt}P_3(t) = \lambda_A P_1(t)$$
$$\frac{d}{dt}P_4(t) = \lambda P_1(t)$$
$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t}]P_5(x,t) = 0$$
$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t}]P_6(x,t) = 0$$
$$[\frac{\partial}{\partial x} + \frac{\partial}{\partial t}]P_7(x,t) = 0$$

The initial conditions are given by

$P_0(0) = 1$ and $P_j(0) = 0$ for j=1,2,3,4,$P_k(x,0) = 0$ for k=5,6,7 By taking Laplace transformation and solving, we get

$P_0(s)=\frac{(s+A_1)(s+A_2)}{(s+A_9)(s+A_1)(s+A_2)-2\lambda\mu(s+A_2)-\lambda_A\mu_A(s+A_1)}$

$P_1(s)=\frac{2\lambda(s+A_2)}{Y_2(s)}$, $P_2(s)=\frac{\lambda_A(s+A_1)}{Y_2(s)}$, $P_3(s)=\frac{2\lambda\lambda_A(s+A_2)}{sY_2(s)}$, $P_4(s)=\frac{2\lambda^2(s+A_2)}{sY_2(s)}$

Where $Y_2(s) = (s + A_9)(s + A_1)(s + A_2) - 2\lambda\mu(s + A_2) - \lambda_A\mu_A(s + A_1)$

Taking Inverse Laplace transforms for the above equations, we get

$P_0(t)=A_0e^{s_1t}+B_0e^{s_2t} + C_0e^{s_3t}$
$P_1(t)=A_1e^{s_1t}+B_1e^{s_2t} + C_1e^{s_3t}$

Where

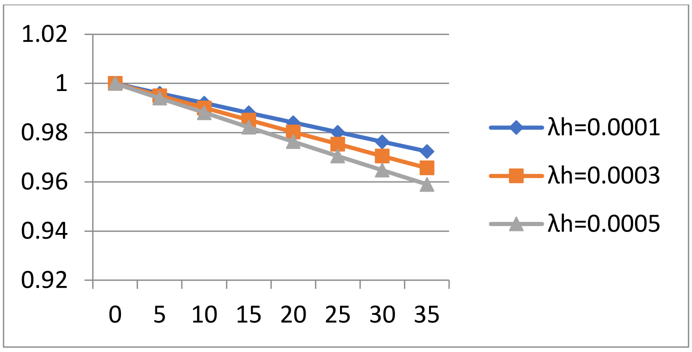
$A_0 = \frac{(s_1+A_1)(s_1+A_2)}{(s_1-s_2)(s_1-s_3)}$, $B_0 = \frac{(s_2+A_1)(s_2+A_2)}{(s_2-s_1)(s_2-s_3)}$,$C_0 = \frac{(s_3+A_1)(s_3+A_2)}{(s_3-s_1)(s_3-s_2)}$

Forλ =0.0002; μ=0.0003; $\mu_A = 0.0001$; $\lambda_{c_1} = 0.0004$; $\lambda_{c_2} = 0.0005$; $\lambda_e = 0.0002$
$\mu_A = 0.0003$;$\mu_1 = 0$; $\mu_2 = 0$; $\mu_5 = 0$; $\mu_6 = 0$; $\mu_7 = 0$

Time	λh =0.0001	λh =0.0003	λh =0.0005
0	1	1	1

5	0.995946	0.995016	0.994021
10	0.991972	0.990058	0.988082
15	0.988014	0.985127	0.982183
20	0.984072	0.980223	0.976324
25	0.980147	0.975346	0.970504
30	0976238	0.970496	0.964724
35	0.972346	0.965673	0.958982

Reliability



Time

CONCLUSION

It is observed that

i) The reliability of the system decreases with time.

ii) As the human error increases, the reliability of the system decreases with different values of λ_h

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