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Magneto Rotatory Double –Diffusive Kuvshiniski Viscoelastic Fluid through a Porous Medium

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ARTICLE INFO	ABSTRACT
Published Online: 20 February 2024 **Corresponding Author:** Mahinder Singh	The effect of magnetic field on an incompressible (Kuvshiniski – Type) viscoelastic rotating fluid heated and soluted from below is considered. For the case of stationary convection medium permeability and stable solute gradient have destabilizing and stabilizing effect on the thermosolutal convection. Magnetic field and stable solute gradient have stabilizing effect, where as medium permeability has destabilizing effect in absence of rotation and having both stabilizing as well as destabilizing effect in the thermosolutal kuvshiniski viscoelastic rotating fluid. It is also found that rotation, magnetic field and stable solute gradient introduce oscillatory modes in the system, where as in their absence principal of exchange of stabilities is satisfied. Graphs also have been plotted by giving some numerical values to the parameters.

KEYWORDS: Thermosolutal Instability, Kuvshiniski – Type Viscoelastic Incompressible Fluid, Effect of Rotation, Magnetic Field and Porous Medium.

INTRODUCTION

The thermal instability of a fluid layer heated from below plays an important role in geophysics, oceanography, atmospheric physics, etc., and has been investigated by many authors, e.g. Benard (1), Rayleigh (2) and Jeffreys (3). A detailed account of the theoretical and experimental studies of so called Benard convection in Newtonian Fluids has been given by Chandrasekhar (4). The Boussinesq approximation, which states that the density can be treated as a constant in all terms of the equation of motion except the external force term, has been used throughout.

There has been considerable interest in recent years in the study of the breakdown of the stability of a layer of fluid subject to a vertical temperature gradient in a porous medium and the possibility of convective flow. The stability of flow of a single component fluid through a porous medium taking into account the Darcy resistance has been considered by Lapwood (5) and Wooding (6). The Darcy equation describes the incompressible flow of a Newtonian fluid of viscosity μ through a macroscopically homogeneous and isotropic porous medium of permeability k_1. If $\vec{v}$ is the filter velocity of the fluid, the resistance term $-\left(\frac{\mu}{k_1}\right)\vec{v}$ replaces the usual viscous term in the equation of fluid motion. There is mounting evidence, both theoretical and experimental, that suggests the Darcy's equation sometimes provides an unsatisfactory description of the hydrodynamic conditions, particularly near boundaries of a porous medium, Beavers et. al. (7) demonstrated experimentally the existence of shear within the porous medium near a surface where the porous medium is exposed to a freely flowing fluid, thus forming a zone of shear – induced fluid flow.

Since viscoelastic fluids play an important role in polymers and electrochemical industry, the studies on waves and stability in different viscoelastic fluid dynamical configurations has been carried out by several researchers. Chaudhary and Singh (8) considered the flow of a dusty viscoelastic (Kuvshiniski Type) fluid down an inclined plan. The effect of a magnetic field on the flow of a dusty viscoelastic (Kuvshiniski Type) fluid down an inclined plan studied by Johari and Gupta (9). Varshney and Dwivedi (10) studied the unsteady effect on MHD free convection and mass transfer flow of a Kuvshiniski fluid through a porous medium with constant suction, heat and mass flux. Kumar and Singh (11) studied a viscoelastic fluid heated from below in a porous medium, Kumar and Singh (12) also studied thermal instability of a Kuvshiniski viscoelastic fluid with fine dust through porous medium.

Prakash et. al. (13) studied MHD free convective flow of a flat plate under the influence of radiative heat transfer moving with velocity decreasing exponentially with time. Kumar (14) studied magneto – rotatory stability of a two stratified fluid layers of a Kuvshiniski viscoelastic superposed fluid in a porous medium. Kumar and Kumar (15) also studied the effect of the magnetic field on an incompressible (Kuvshiniski - Type) viscoelastic rotating fluid heated from below through a porous medium. Singh (16) have studied thermal instability of Kuvshiniski fluid with suspended particles saturated in a porous medium in the presence of a magnetic field.

Keeping in mind the importance and applications of non – Newtonian fluids in modern technology and industries and owing to the importance of variable magnetic field, rotation and porous medium in chemical engineering and geophysics, the present paper attempts to study the thermosolutal instability ofa Kuvshiniski viscoelastic fluid through a porous medium.

PERTURBATION EQUATIONS

Consider an infinite horizontal layer of Kuvshiniski viscoelastic fluid of depth d in porous medium, heated and solute concentrated from below and acted on by gravity force $\vec{g}(0, 0, -g)$.

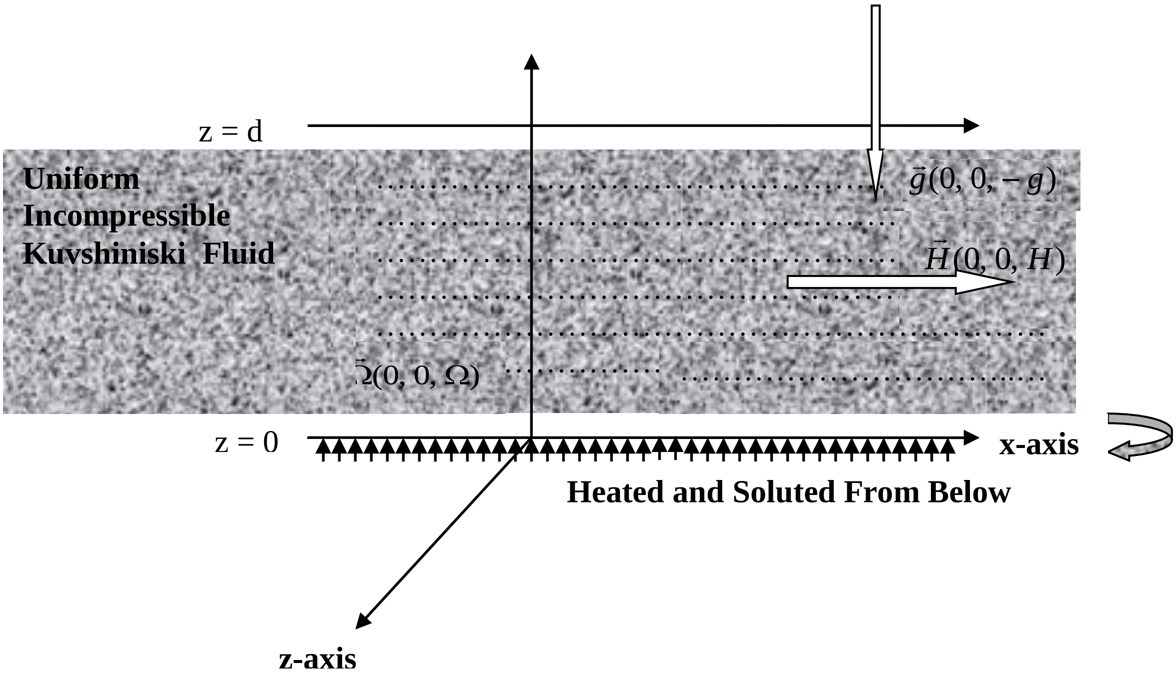

Figure: Basic figure of Kuvshiniski Magneto Rotatory Fluid Through Porous Layer

Let $\delta\rho, \delta p, q(u, v, w), \theta$ *and* γ denote respectively the perturbations in density ρ, pressure p, filter velocity (zero initially), temperature T and solute concentration C. Then the linearized thermosolutal perturbations equations through porous medium, following Boussinesq approximation are [in study of Kuvshiniski (16) and Mandal et. al. (17)].

$$\frac{1}{\in}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial q}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p + g\frac{\delta\rho}{\rho_0} + \frac{\upsilon}{\in}\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\nabla^2 - \frac{\in}{k_1}\right)q, \quad (1)$$

$$\nabla.q = 0, \quad (2)$$

$$\left(E\frac{\partial}{\partial t} - \kappa\nabla^2\right)\theta = \beta w, \quad (3)$$

$$\left(E'\frac{\partial}{\partial t} - \kappa'\nabla^2\right)\gamma = \beta' w. \quad (4)$$

Here $\upsilon\left(=\mu/\rho_0\right), \kappa, \kappa', \beta\left(=\left|\frac{dC}{dz}\right|\right)$ stand for the kinematic viscosity, the thermal diffusivity, the solute diffusivity, uniform temperature gradient and uniform solute concentration gradient, respectively. $E=\in+(1-\in)\frac{\rho_s c_s}{\rho_0 c_v}$ where $\rho, c_v, \rho_s \; and \; c_s$ stand for density, specific heat of fluid and solid matrix, respectively. E' is an analogous solute constant.

The equation of state

$$\rho=\rho_0\left[1-\alpha\left(T-T_0\right)+\alpha'\left(C-C_0\right)\right], \tag{5}$$

contains a thermal coefficient of expansion α and an analogous solvent coefficient α'. The suffix zero refers to value at the reference level z = 0. The change in density $\delta\rho$, caused by the perturbations $\theta \; and \; \gamma$ in temperature and concentration, is given by

$$\delta\rho=-\rho_0\left(\alpha\theta-\alpha'\gamma\right). \tag{6}$$

Eliminating δp between the three component equations of (1) and using (2), we obtain

$$\frac{1}{\in}\nabla^2\frac{\partial w}{\partial t}-g\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\alpha\theta-\alpha'\gamma\right)-\frac{\upsilon}{\in}\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\nabla^2-\frac{\in}{k_1}\right)\nabla^2 w=0. \tag{7}$$

Let us assume both the boundaries to be free. The case of two free boundaries is a little artifical except in stellar atmospheres (Spiegel [18]). However, this assumption allows us to obtain the analytical solution without affecting the essential feature of the problem. The boundary conditions appropriate for the problem are (Chandrasekhar [4], Lapwood [5]):

$$w=\frac{\partial^2 w}{\partial z^2}=\theta=0 \; at \; z=0 \; and \; z=d. \tag{8}$$

DISPERSION RELATION AND DISCUSSION

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$\left(w, \theta, \gamma\right)=\left[W(z), \Theta(z), \Gamma(z)\right]\exp(ik_x x+ik_y y-nt), \tag{9}$$

where k_x, k_y are wave numbers along x- and y-directions, respectively, $k\left(=\sqrt{k_x{}^2+k_y{}^2}\right)$ is the resultant wave number and n is , in general, a complex constant.

Assume that x, y, z stand for the coordinates in the new unit of length d and letting $a=kd, \sigma=nd^2/\upsilon, F=\lambda\upsilon/d^2, p_1=\upsilon/\kappa, q=\upsilon/\kappa', p_l=k_1/d^2 \; and \; D=d/dz$, equations (3), (4) and (7) using expression (9), in non-dimensional form become

$$(1+F\sigma)\left[\frac{\sigma}{\in}+\frac{1}{p_l}\right]\left(D^2-a^2\right)W+\frac{gd^2a^2}{\upsilon}\left(\alpha\Theta-\alpha'\Gamma\right)-\frac{1}{\in}(1+F\sigma)\left(D^2-a^2\right)^2W=0, \tag{10}$$

$$\left(D^2-a^2-Ep_1\sigma\right)\Theta=-\left(\frac{\beta d^2}{\kappa}\right)\mathrm{W}, \tag{11}$$

$$\left(D^2-a^2-E'q\sigma\right)\Gamma=-\left(\frac{\beta' d^2}{\kappa'}\right)W. \tag{12}$$

Operating equation (10) by $(D^2 - a^2 - Ep_1\sigma)(D^2 - a^2 - E'q\sigma)$ and using (11) and (12), thus eliminating Θ *and* Γ, we obtain

$$(D^2 - a^2 - Ep_1\sigma)(D^2 - a^2 - E'q\sigma)(D^2 - a^2)(1 + F\sigma)\left[\frac{\sigma}{\in} + \frac{1}{p_l} - \frac{1}{\in}(D^2 - a^2)\right]W$$
$$= a^2\left[R(D^2 - a^2 - E'q\sigma) - S(D^2 - a^2 - Ep_1\sigma)\right]W, \quad (13)$$

where $R = \frac{g\alpha\beta d^4}{\upsilon\kappa}$ is the Rayleigh number and $S = \frac{g\alpha'\beta' d^4}{\upsilon\kappa'}$ is the analogous solute Rayleigh number.

The boundary condition (8) transform to

$$W = D^2W = \Theta = \Gamma = 0 \text{ at } z = 0 \text{ and } 1. \quad (14)$$

Using the above boundary conditions, it can be shown that all the even order derivatives of W must vanish for z = 0 and 1 and hence the proper solution of equation (13) characterizing the lowest mode is

$$W = W_0 \sin \pi z, \quad (15)$$

where W_0 is constant.

Substituting (15) in equation (13) and letting $R_1 = R/\pi^4,\ S_1 = S/\pi^4,\ i\sigma_1 = \sigma/\pi^2,$ $x = a^2/\pi^2$ *and* $P = \pi^2 p_l$, we obtain the dispersion relation

$$R_1 = \frac{(1 + x + iEp_1\sigma_1)(1 + x)(1 + i\pi^2F\sigma_1)}{x}\left[\frac{1}{\in}(i\sigma_1 + 1 + x) + \frac{1}{P}\right] + S\frac{(1 + x + iEp_1\sigma_1)}{(1 + x + iE'q\sigma_1)}. \quad (16)$$

For the stationary convection $\sigma = 0$ and equation (16) reduces to

$$R_1 = \frac{(1 + x)^2}{x}\left[\frac{1}{P} + \frac{(1 + x)}{\in}\right] + S_1. \quad (17)$$

Thus for stationary convection, the stress relaxation time parameter F vanish with σ and the Kuvshiniski fluid behaves like an ordinary Newtonian fluid. Equation (17) gives

$$\frac{dR_1}{dP} = -\frac{(1 + x)^2}{xP^2}, \quad (18)$$

and

$$\frac{dR_1}{dS_1} = +1, \quad (19)$$

meaning thereby that medium permeability and stable solute gradient have destabilizing and stabilizing effects, respectively, on the thermosolutal convection for the stationary case.

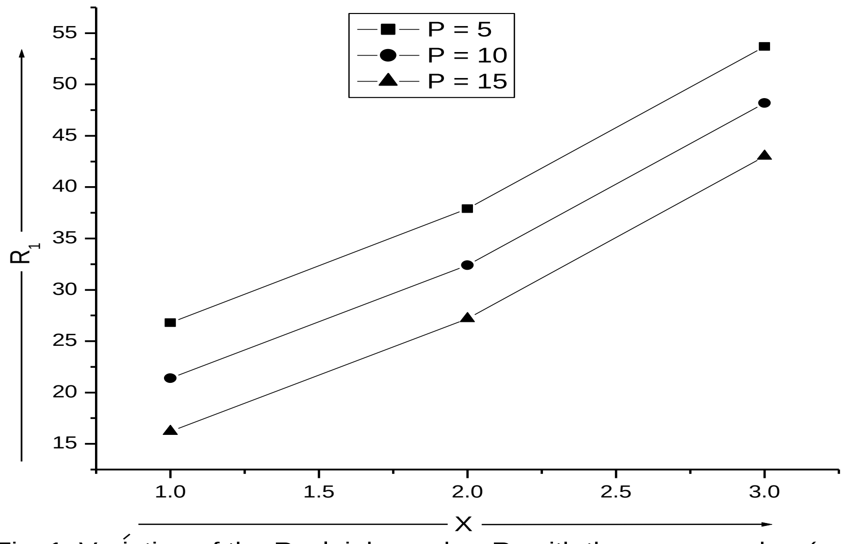

Fig. 1: Variation of the Rayleigh number R_1 with the wave number (x= 1,2,3), when ε = 0.5, S_1= 10 and P = 5, 10, 15

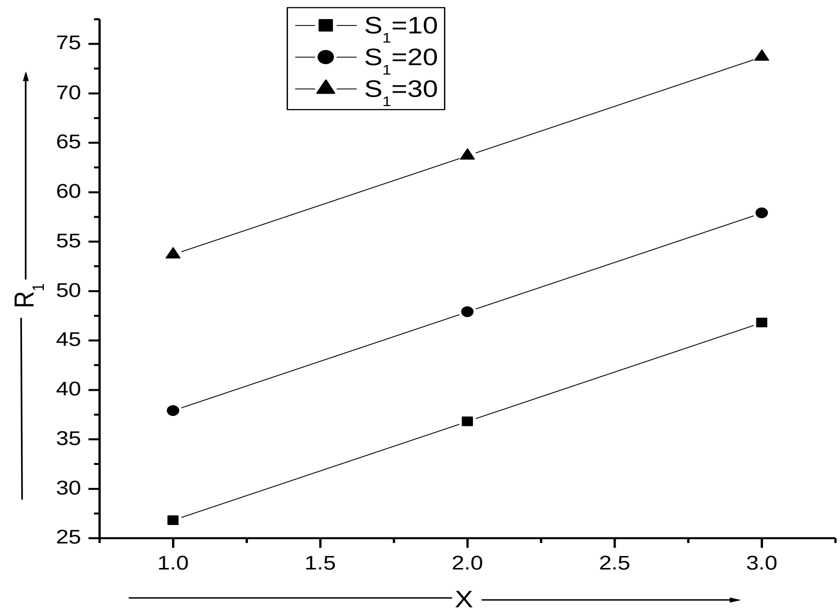

Fig.2: Variation of the Rayleigh number R_1, with the Wave Number (x = 1, 2, 3), e = 0.5, P = 5 and S_1 = 10, 20, 30

In the figure 1: It is clear that as the value of medium permeability parameter P increased value of the variation of Rayleigh number R_1 decrease, shows that medium permeability have destabilizing effect on the system.

In the figure 2: It is clear that as the value of Stable Solute Gradient parameter S_1 increased value of the variation of Rayleigh number R_1 increase, shows that Stable Solute Gradient have stabilizing effect on the system.

EFFECT OF ROTATION

Here the problem is considered to be the same as described in section 2 except that the fluid is in a state of uniform rotation $\vec{\Omega}(0, 0, \Omega)$. The linearized perturbed equation of motion becomes

$$\frac{1}{\in}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial q}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p-g(\alpha\theta-\alpha'\gamma)+\left(1+\lambda\frac{\partial}{\partial t}\right)\left[2\left(q\times\vec{\Omega}\right)+\frac{\upsilon}{\in}\nabla^2-\frac{\upsilon}{k_1}\right]q. \quad (20)$$

Equations (2)-(4) remain unaltered. Let $\varsigma=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ stand for the z-component of vorticity and express

$$\varsigma = Z(z)\exp(ik_x x+ik_y y+nt). \quad (21)$$

Equations (2)-(4) and (20), using expression (9) and (21) yield the dimensionless equations

$$(1+F\sigma)\left[\sigma\in\left(D^2-a^2\right)w+\frac{2\Omega d^3}{\in\upsilon}DZ\right]+\frac{gd^2a^2}{\upsilon}(\alpha\Theta-\alpha'\Gamma)$$
$$=(1+F\sigma)\left(D^2-a^2-\frac{\in}{p_l}\right)\left(D^2-a^2\right)W, \quad (22)$$

$$\left[\sigma\in-\left(D^2-a^2-\frac{\in}{p_l}\right)\right]Z=\frac{2\Omega d}{\upsilon}DW, \quad (23)$$

together with (11) and (12). The boundary conditions in addition to (14) are

$$DZ=0 \ at \ z=0 \ and \ 1. \quad (24)$$

Eliminating Θ, Γ *and* Z between equations (11), (12), (22) and (23) and using the proper solution (15), we obtain

$$R_1=\frac{(1+x)(1+x+iEp_1\sigma_1)\left[i\in\sigma_1+\left(1+x+\frac{\in}{P}\right)\right]}{x}$$
$$+S_1\frac{(1+x+iEp_1\sigma_1)}{(1+x+E'q\sigma_1)}+T_{A_1}\frac{(1+x+iEp_1\sigma_1)}{x\left[i\in\sigma_1+\left(1+x+\frac{\in}{P}\right)\right]}, \quad (25)$$

where $T_{A_1}=T_A/\pi^4$.

THE STATIONARY CONVECTION

For the stationary convection, $\sigma=0$ and equation (25) reduces to

$$R_1=S_1+\frac{(1+x)^2\left(1+x+\frac{\in}{P}\right)}{x}+T_{A_1}\frac{(1+x)}{x\left(1+x+\frac{\in}{P}\right)}, \quad (26)$$

Equation (26) yields

$$\frac{dR_1}{dT_{A_1}}=\frac{(1+x)}{x\left(1+x+\frac{\in}{P}\right)}, \quad (27)$$

which imply that the uniform rotation has stabilizing effect on the system.

$$\frac{dR_1}{dS_1} = +1, \tag{28}$$

which imply that stable solute gradient have stabilizing effect on the system.

$$\frac{dR_1}{dP} = \left(\frac{1+x}{x}\right)\frac{\in}{P^2}\left[\frac{T_{A_1}}{\left(1+x+\frac{\in}{P}\right)} - (1+x)\right], \tag{29}$$

It is clear from equation (29), that the medium permeability has a destabilizing effect in the absence of rotation. It still has a destabilizing effect if $T_{A_1} < (1+x)\left(1+x+\frac{\in}{P}\right)^2$. The medium permeability has a stabilizing effect if $T_{A_1} > (1+x)\left(1+x+\frac{\in}{P}\right)^2$. Thus the medium permeability has both stabilizing and destabilizing effect on the system.

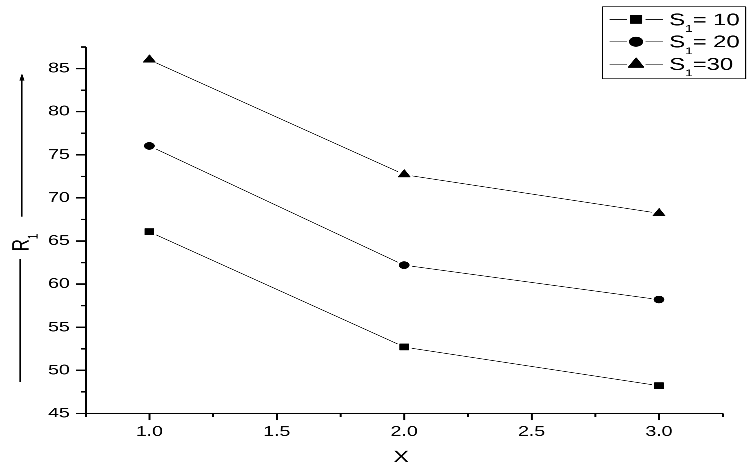

Fig.3: Variation of the Rayleigh number R_1, with the wave number(x = 1,2,3), P= 5, T_{A_1}= 50, ε= 0.5 and S_1=10, 20, 30

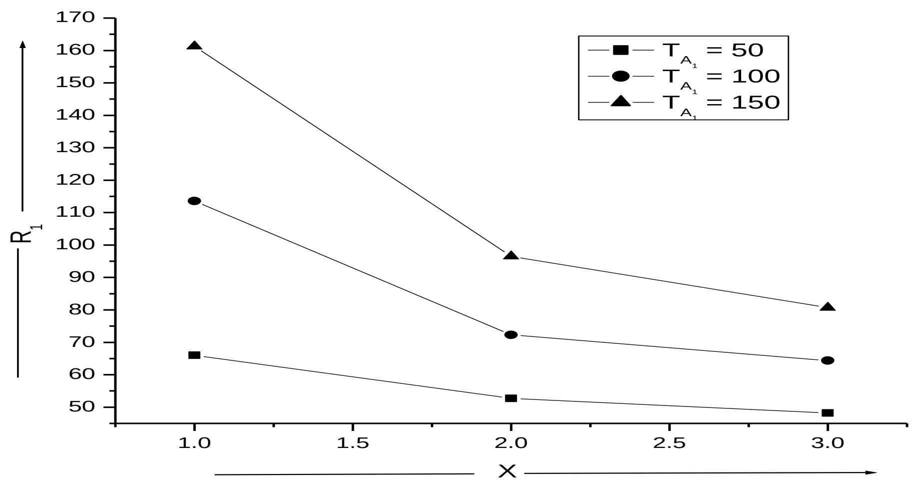

Fig. 4: Variation of the Rayleigh number R_1, with the wave number (x= 1, 2, 3), for P = 5, S_1 = 10, ε = 0.5, & T_{A_1}= 50, 100, 150

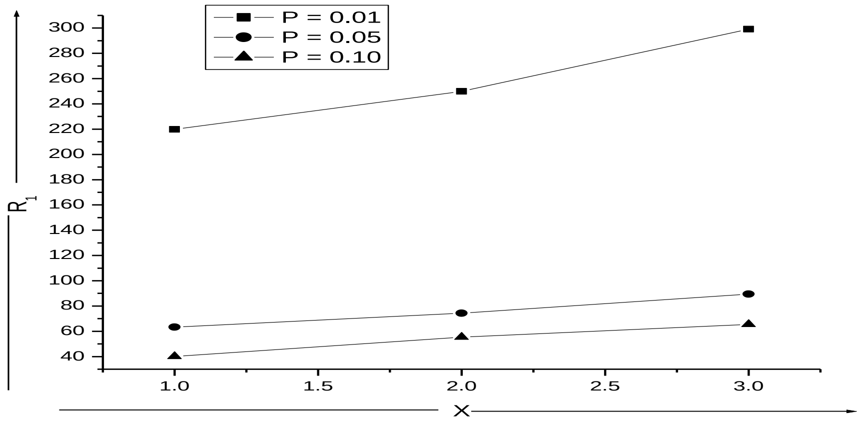

Fig. 5: Variation of Rayleigh number R_1, with the wave number (x = 1,2,3), for S_1=10,T_{A_1} = 50, ε = 0.5 and P = 0.01, 0.05. 0.10

In the figure 3: It is clear that as the value of Stable Solute Gradient parameter S_1 increased value of the variation of Rayleigh number R_1 increase , shows that the stable solute gradient have stabilizing effect on the system in presence of rotation.

In the figure 4: It is clear that as the value of Uniform rotation parameter T_{A_1} increased value of the variation of Rayleigh number R_1 increase , shows that the uniform rotation have stabilizing effect on the system.

In the figure 5: It is clear that as the value of medium permeability parameter P increased value of the variation of Rayleigh number R_1 increasing as well as decreasing , shows that the medium permeability have stabilizing as well as destabilizing effect on the system in presence of rotation.

The Oscillatory Modes

Here we discuss the possibility of oscillatory modes, if any, coming into play due to the presence of rotation. Multiplying Equation (22) by W*, the complex conjugate of W, integrating over the range of z and making use of (11), (12) and (23) together with the boundary conditions (14) and (24), we obtain

$$\left(\sigma+\frac{1}{P_l}\right)I_1+\frac{g\alpha'\kappa'a^2}{\upsilon\beta'}\left(I_4+E'q\sigma^*I_5\right)+d^2\left[\sigma^*+\frac{1}{P_l}\left(1+F\sigma^*\right)\right]I_6+\frac{d^2}{\in}\left(1+F\sigma^*\right)I_7+\frac{I_8}{\in}$$

$$=\frac{g\alpha\kappa a^2}{\upsilon\beta}\left(I_2+EP_1\sigma^*I_3\right),. \qquad (30)$$

Where

$$\left.\begin{aligned}
&I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz, \qquad I_2=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,\\
&I_3=\int_0^1|\Theta|^2dz, \qquad I_4=\int_0^1\left(|D\Gamma|^2+a^2|\Gamma|^2\right)dz,\\
&I_5=\int_0^1|\Gamma|^2dz, \qquad I_6=\int_0^1|Z|^2dz\,,\\
&I_7=\int_0^1\left(|DZ|^2+a^2|Z|^2\right)dz,\ I_8=\int_0^1\left(|D^2W|^2+2a^2|DW|^2+a^2|W|^2\right)dz
\end{aligned}\right\} \qquad (31)$$

Which all are positive definite. Putting $\sigma=\sigma_r+i\sigma_i$ in equation (30) and then equating real and imaginary parts, we obtain

$$\sigma_r\left[\begin{aligned}&\left\{1+\frac{\left(2F+\sigma_rF^2\right)}{\left(1+F\sigma_r\right)^2+F^2\sigma_i^2}\frac{1}{P_l}\right\}I_1+\frac{g\alpha'\kappa'a^2}{\upsilon\beta'}E'qI_5+d^2\left(1+\frac{F}{P_l}\right)I_6+\frac{d^2F}{\in}I_7\\&+\frac{\left(2F+\sigma_rF^2\right)}{\left(1+F\sigma_r\right)^2+F^2\sigma_i^2}\frac{I_8}{\in}-\frac{g\alpha\kappa a^2}{\upsilon\beta}Ep_1I_3\end{aligned}\right]=$$

$$-\left[\frac{\left(1+F^2+\sigma_i^2\right)}{(1+F\sigma_r)^2+F^2\sigma_i^2}\frac{I_1}{P_l}+\frac{g\alpha'\kappa' a^2}{\upsilon\beta'}I_4+\frac{d^2}{P_l}I_6+\frac{d^2}{\in}I_7+\frac{\left(1+F^2\sigma_i^2\right)}{(1+F\sigma_r)^2+F^2\sigma_i^2}\frac{I_8}{\in}-\frac{g\alpha\kappa a^2}{\upsilon\beta}I_2\right].$$

(32)

And

$$\sigma_i\left[I_1-\frac{g\alpha'\kappa' a^2}{\upsilon\beta'}E'qI_5-d^2\left(1+\frac{F}{P_l}\right)I_6-\frac{Fd^2}{\in}I_7+\frac{g\alpha\kappa a^2}{\upsilon\beta}Ep_1I_3\right]=0 \qquad (33)$$

It is clear from equation (32) that σ_r may be positive or negative implying thereby that there may be stability or instability in the presence of rotation, stable solute gradient, viscoelasticity and porosity on thermosolutal convection in Kuviniski viscoelastic fluid in porous medium which is also true in their absence.

Equation (33) implies that $\sigma_i = 0 \; or \; \sigma_i \neq 0$ which means that the modes may be non – oscillatory or oscillatory . In the absence of stable solute gradient and rotation, equation (33) reduces to

$$\sigma_i\left[I_1+\frac{g\alpha\kappa a^2}{\upsilon\beta}Ep_1I_3\right]=0,$$

which yields $\sigma_i = 0$ implying thereby that the oscillatory modes are not allowed and the principal of exchange of stabilities is satisfied for the porous medium in the absence of rotation and stable solute gradient.

The rotation and the stable solute gradient thus, introduce oscillatory modes in the system which were non – existent in their absence.

EFFECT OF MAGNETIC FIELD

Here the problem is considered to be the same as described in section 2 except that the fluid is finitely (electrically) conducting and is acted on by a uniform magnetic field H(0, 0, H). The linearized perturbed equations are

$$\frac{1}{\in}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial q}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p-g(\alpha\theta-\alpha'\gamma)+\left(1+\lambda\frac{\partial}{\partial t}\right)\left[\frac{\mu_e}{4\pi\rho_0}(\nabla\times h)\times H\right]$$
$$+\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\frac{\upsilon}{\in}\nabla-\frac{\upsilon}{k_1}\right)q, \qquad (34)$$

$$\nabla . h=0, \qquad (35)$$

$$\in\frac{\partial h}{\partial t}=(H.\nabla)q+\in\eta\nabla^2h, \qquad (36)$$

Together with equations (2)-(4), $\mu_e, \eta \; and \; h(h_x, h_y, h_z)$ denote respectively, the magnetic permeability, the resistivity and the perturbation in magnetic field H. Substituting $p_2=\frac{\upsilon}{\eta}$ and

$$h_z=K(z)\exp(ik_x x+ik_y y+nt), \qquad (37)$$

Equations (2)-(4) and (34)-(36), using expression (9) and (37), yield the dimensionless equations

$$(1+F\sigma)\left[\sigma-\frac{1}{\in}\left(D^2-a^2-\frac{\in}{p_l}\right)\right](D^2-a^2)W-\frac{\mu_e Hd}{4\pi\rho_0\upsilon}(1+F\sigma)(D^2-a^2)DK$$
$$+\frac{gd^2a^2}{\upsilon}(\alpha\Theta-\alpha'\Gamma)=0, \qquad (38)$$

$$(D^2-a^2-p_2\sigma)K=-\frac{Hd}{\in\eta}DW, \qquad (39)$$

together with (11) and (12), the boundary condition to (14) for free, electrically non-conducting boundaries are

DK = 0 at z = 0 and 1. (40)

Eliminating Θ, Γ *and* K from equations (12), (13), (38) and (39) and using the proper solution (15), we obtain

$$R_1 = \frac{(1+x)(1+x+iEp_1\sigma_1)}{x}\left[1+x+\in\left(\frac{1}{P}+i\sigma_1\right)\right]+S_1\frac{(1+x+iEp_1\sigma_1)}{(1+x+iE'q\sigma_1)}+Q_1\left(\frac{1+x}{x}\right)\times\frac{(1+x+iEp_1\sigma_1)}{(1+x+iEp_2\sigma_1)},$$

(41)

where $Q_1 = \dfrac{\mu_e H^2 d^2}{4\pi\rho_0 \upsilon\eta}$.

THE STATIONARY CONVECTION

For stationary convection, put $\sigma_1 = 0$ in equation (41), we obtain

$$R_1 = \frac{(1+x)^2}{x}\left(1+x+\frac{\in}{P}\right)+S_1+Q_1\frac{(1+x)}{x}. \tag{42}$$

The Kuvniski viscoelastic fluid, thus behaves like a Newtonian viscous fluid for the stationary convection. Equation (42) yields

$$\frac{dR_1}{dQ_1} = \frac{1+x}{x}, \tag{43}$$

$$\frac{dR_1}{dS_1} = +1, \tag{44}$$

$$\frac{dR_1}{dP} = -\frac{(1+x)^2\in}{xP^2}. \tag{45}$$

The magnetic field and stable solute gradient have stabilizing effects whereas the medium permeability has a destabilizing effect on the system.

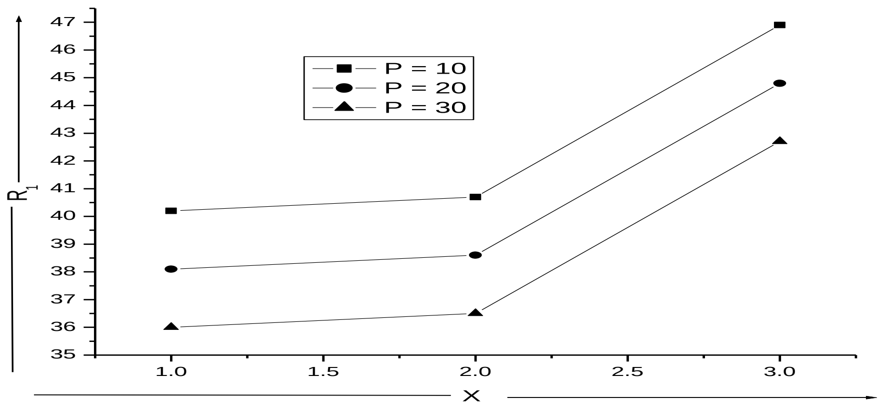

Fig. 6: Variation of Rayleigh number R_1, with the wave number (x = 1,2,3), for S_1 = 10, Q_1 = 10, ε = 0.5 and P = 10, 20, 30

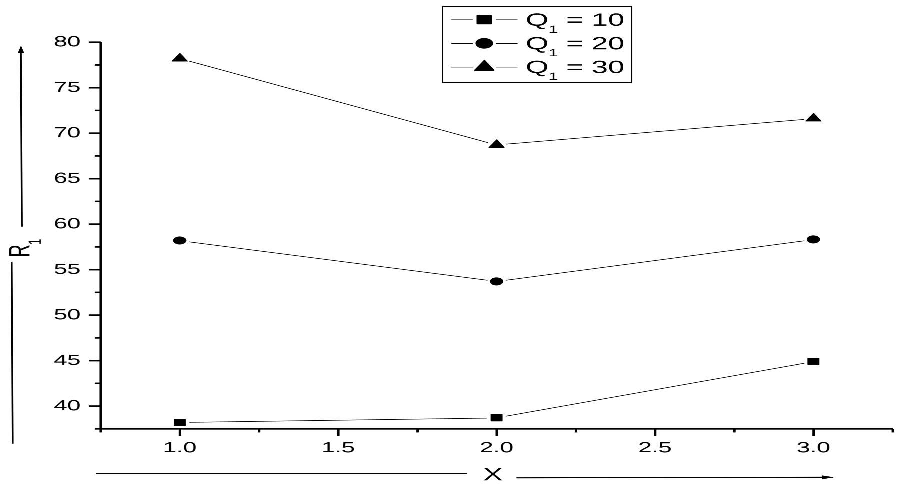

Fig. 7: Variation of the Rayleigh number R_1, with the wave number (x = 1,2, 3), for S_1= 10, P = 10, ε = 0.5 and Q_1 = 10, 20, 30

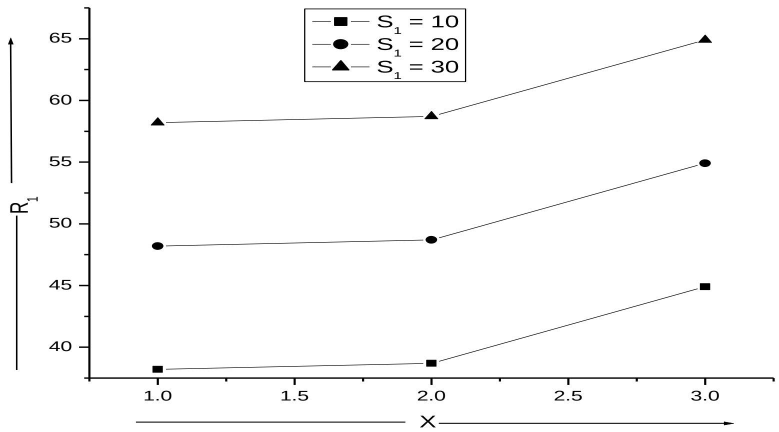

Fig. 8: Variation of the Rayleigh number R_1, with the wave number (x = 1,2,3), for Q_1 = 10, P = 10 , ε = 0.5 and S_1 = 10, 20, 30

In the figure 6: It is clear that as the value of medium permeability parameter P increased value of the variation of Rayleigh number R_1 decrease , shows that the medium permeability have destabilizing effect on the system in presence of magnetic field.

In the figure 7: It is clear that as the value of magnetic field parameter Q_1 increased value of the variation of Rayleigh number R_1 increase , shows that the magnetic field have stabilizing effect on the system .

In the figure 8: It is clear that as the value of Stable Solute Gradient parameter S_1 increased value of the variation of Rayleigh number R_1 increase , shows that the stable solute gradient have stabilizing effect on the system in presence of magnetic field as well as rotation.

THE OSCILLATORY MODES

Here we examine the possibility of oscillatory modes if any, coming into play due to the presence of magnetic filed. Multiplying equation (38) by W*, the complex conjugate of W, integrating over the range of z and making use of (12), (13) and (39) together with the boundary conditions (14) and (40), we obtain

$$\left(\sigma+\frac{1}{P_l}\right)I_1+\frac{g\alpha'\kappa'a^2}{\upsilon\beta'}\left(I_4+E'q\sigma^*I_5\right)+\frac{I_8}{\in}+\frac{\mu_e\eta\in}{4\pi\rho_0\upsilon}\left(I_9+p_2\sigma^*I_{10}\right)$$

$$=\frac{g\alpha\kappa a^2}{\upsilon\beta}\left(I_2+Ep_1\sigma^*I_3\right), \quad (46)$$

where I_1-I_5 *and* I_8 are given by equation (31) and

$$I_9 = \int_0^1 \left(\left| D^2 K \right|^2 + 2a^2 \left| DK \right|^2 + a^4 \left| K \right|^2 \right) dz, \qquad I_{10} = \int_0^1 \left(\left| DK \right|^2 + a^2 \left| K \right|^2 \right) dz \Bigg\} \tag{47}$$

which are all positive definite. Putting $\sigma = \sigma_r + i\sigma_i$, in equation (46) and then equating real and imaginary parts, we obtain

$$\sigma_r \left[I_1 + \frac{2F + \sigma_r F^2}{(1 + F\sigma_r)^2 + F^2 \sigma_i^2} \left(\frac{I_1}{P_l} + \frac{I_8}{\in} \right) + \frac{g\alpha' \kappa' a^2}{\upsilon \beta'} E' q I_5 + \frac{\mu_e \eta \in}{4\pi \rho_0 \upsilon} p_2 I_{10} - \frac{g\alpha \kappa a^2}{\upsilon \beta} E p_1 I_3 \right] =$$

$$-\left[\frac{1 + F^2 \sigma_i^2}{(1 + F\sigma_r)^2 + F^2 \sigma_i^2} \left(\frac{I_1}{P_l} + \frac{I_8}{\in} \right) + \frac{g\alpha' \kappa' a^2}{\upsilon \beta'} I_4 - \frac{g\alpha \kappa a^2}{\upsilon \beta} I_2 \frac{\mu_e \eta \in}{4\pi \rho_0 \upsilon} I_9 \right], \tag{48}$$

and

$$\sigma_i \left[I_1 - \frac{g\alpha' \kappa' a^2}{\upsilon \beta'} E' q I_5 - \frac{\mu_e \eta \in}{4\pi \rho_0 \upsilon} p_2 I_{10} + \frac{g\alpha \kappa a^2}{\upsilon \beta} E p_1 I_3 \right] = 0. \tag{49}$$

In the absence of stable solute gradient and magnetic field, equation (49) reduces to

$$\sigma_i \left[I_1 + \frac{g\alpha \kappa a^2}{\upsilon \beta} E p_1 I_3 \right] = 0, \tag{50}$$

which gives $\sigma_i = 0$ and hence the oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for the porous medium in the absence of stable solute gradient and magnetic field.

The stable solute gradient and magnetic field, thus introduce oscillatory modes in the system which were non – existent in their absence.

CONCLUSIONS

With the growing importance of non – Newtonian fluids, rotation and magnetic field in chemical technology and industries, investigation on (Kuvshiniski– Type) viscoelastic fluid are desirable. In the present paper, I have investigated the effect of uniform magnetic field on a (Kuvshiniski – Type) viscoelastic rotating fluid heated and soluted from below in presence of porous medium. Dispersion relation governing the effects of rotation, magnetic field and medium permeability is derived. The main results from the analysis of the paper are as follow:

(1): In case of stationary convection, a Kuvshiniski viscoelastic fluid behaves like an ordinary Newtonian fluid.

(2): Medium permeability and stable solute gradient have destabilizing and stabilizing effect as are evident from the equations (18), (19) and figure’s (1) and (2) for the permissible range of values of various parameters.

(3): From equation (27) and figure (4), it shows that uniform rotation have stabilizing effect on the system.

(4): Medium permeability has a destabilizing effect in the absence of rotation and stabilizing as well as destabilizing effect in presence of rotation by equation (29) and figure (5).

(5): The rotation and stable solute gradient introduce oscillatory modes in the system which were non – existent in their absence by equation (33).

(6): The magnetic field and stable solute gradient have stabilizing effect whereas the medium permeability has a destabilizing effect on the system by equation’s (43), (44) & (45) and figure’s (6), (7) & (8).

(7): The stable solute gradient and magnetic field introduce oscillatory modes in the system in their absence principle of exchange of stabilities is satisfied in the system.

Nomenclature

$\vec{g}$ - acceleration due to gravity, [m s^{-2}]

K - Stokes’ drag coefficient, [kg s^{-1}]

k - wave number of the distance, [m^{-1}]

k_x, k_y - horizontal wave numbers, [m^{-1}]

k_1 - medium permeability, [m^2]

P - fluid pressure, [Pa]

δp - perturbation in the pressure p, [-]

$\delta \rho$ - perturbation in density ρ, [-]

C_v - fluid at constant volume, [-]

R - is the Rayleigh number, [-]

S - is the analogous Rayleigh number, [-]

λ - coefficient of viscoelasticity, [-]

Ω - rotation vector having components $(0, 0, \Omega)$, [-]

d - depth of the fluid layer, [m]

H - magnetic field intensity vector having components (0, 0, H), [G]

Greek Letters

ρ - density, [kg m^{-3}]

ν - kinematic viscosity, [$m^2 s^{-1}$]

ε - medium porosity, [$m^0 s^0 k^0$]

θ - perturbation in temperature, [K]

γ - perturbation in concentration C, [-]

$\beta\left(=\left|\frac{dT}{dz}\right|\right)$ - uniform temperature gradient, [k m^{-1}]

$\beta'\left(=\left|\frac{dC}{dz}\right|\right)$ - uniform solute concentration gradient, [-]

α' - analogous coefficient

μ_e - magnetic permeability, [H m^{-1}]

η - electrically resistivity, [-]

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