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Elliptic Revan Index and its Exponential of Certain Networks

V. R. Kulli

Department of Mathematics, Gulbarga University, Gulbarga 585106, India

ARTICLE INFO	ABSTRACT
Published Online: 23 February 2024 Corresponding Author: **V. R. Kulli**	The reverse elliptic Sombor index is a recently introduced topological index. In this paper, we put forward the elliptic Revan index and its corresponding exponential of a graph and compute exact formulas for some families of networks such as silicate networks, rhombus silicate networks, oxide networks, rhombus oxide networks, hexagonal networks and honeycomb networks. Also we establish some properties of elliptic Revan index.

KEYWORDS: elliptic Revan index, elliptic Revan exponential, graph, network.

I. INTRODUCTION

Let $G = (V(G), E(G))$ be a finite, simple connected graph. The degree d_u is the number of vertices adjacent to u. Let $\Delta(G)$ ($\delta(G)$) denote the maximum (minimum) degree among the vertices of G. The Revan vertex degree of a vertex u in G is defined as $r_u = \Delta(G) + \delta(G) - d_u$. The Revan edge connecting the Revan vertices u and v will be denoted by uv. We refer [1] for undefined term and notation.

A molecular graph is a graph whose vertices correspond to the atoms and the edges to the bonds. Chemical graph theory has an important effect on the development of the Chemical Sciences. A single number that can be used to characterize some property of the graph of molecular is called a topological index. Numerous topological indices have been considered in Theoretical Chemistry see [2, 3].

The reverse elliptic Sombor index [4] of a graph G is defined as

$$ESO(G) = \sum_{uv \in E(G)} (d_u + d_v)\sqrt{d_u^2 + d_v^2}.$$

Recently, some Sombor indices were studied in [5-21].

The elliptic Revan index of a graph G is defined as

$$ER(G) = \sum_{uv \in E(G)} (r_u + r_v)\sqrt{r_u^2 + r_v^2}.$$

Recently, some elliptic indices were studied in [22-25].

We define the elliptic Revan exponential of a graph G as

$$ER(G, x) = \sum_{uv \in E(G)} x^{(r_u + r_v)\sqrt{r_u^2 + r_v^2}}.$$

We mention below some topological indices which are needed in this paper.

The second Revan Zagreb index [26] is defined as

$$RM_2(G) = \sum_{uv \in E(G)} r_u r_v.$$

The first and second Revan hyper Zagreb indices [27] are defined as

$$HRM_1(G) = \sum_{uv \in E(G)} (r_u + r_v)^2,$$

$$HRM_2(G) = \sum_{uv \in E(G)} (r_u r_v)^2.$$

The F-Revan index [28] is defined as

$$FR(G) = \sum_{uv \in E(G)} (r_u^2 + r_v^2).$$

We put forward the alpha Revan Gourava index of a graph G and it is defined as

$$\alpha RGO(G) = \sum_{uv \in E(G)} (r_u^2 + r_v^2) r_u r_v.$$

In this paper, we determine the elliptic Revan index and its corresponding exponential of some important networks. Also we establish some mathematical properties of elliptic Revan index.

II. MATHEMATICAL PROPERTIES

Proposion1. Let P be a path with $n \geq 3$ vertices. Then

$$ER(P) = 2\sqrt{2}n + 6\sqrt{5}. - 6\sqrt{2}.$$

Proof: Let P be a path with n□3 vertices. We obtain two partitions of the edge set of P as follows:

$E_1 = \{uv \in E(P) \mid d_u=1, d_v=2\}$, $|E_1| = 2$.
$E_2 = \{uv \in E(P) \mid d_u = d_v=2\}$, $|E_2| = n - 3$.

We have $\in (P) + \square(P) = 3$, $r_u = 3 - d_u$.

$RE_1= \{uv\in E(P) \mid r_u =2, r_v = 1\}$, $|RE_1| = 2$.
$RE_2 = \{uv\in E(P) \mid r_u = r_v = 1\}$, $|RE_2| = n - 3$.

$$ER(P) = \sum_{uv\in E(P)} (r_u + r_v)\sqrt{r_u^2 + r_v^2}$$

$$= 2(2+1)\sqrt{2^2+1^2} + (n-3)(1+1)\sqrt{1^2+1^2}$$
$$= 2\sqrt{2}n + 6\sqrt{5}. - 6\sqrt{2}.$$

Proposion2. Let G be an r-regular graph with n vertices, m edges and $r\geq 2$. Then

$$ER(G) = \sqrt{2}nr^3.$$

Proof: Let G be an r-regular graph with n vertices, $r\square 2$ and $m= \frac{nr}{2}$ edges. Every edge of G is incident with r edges. Also $\Delta=\delta= d_u=r$ for each vertex u in G. Thus $r_u= r - r + r=r$. Thus

$$ER(G) = \sum_{uv\in E(G)} (r+r)\sqrt{r^2+r^2}$$
$$= 2\sqrt{2}mr^2 = \sqrt{2}nr^3.$$

Corollary 2.1. Let C_n be a cycle with $n\square$ 3 vertices. Then
$ER(C_n) = 8\sqrt{2}n$.

Corollary 2.2. Let K_n be a complete graph with $n\square 3$ vertices. Then

$$ER(K_n) = \sqrt{2}n(n-1)^3.$$

Theorem 1. Let G be a simple connected graph. Then

$$ER(G) \geq \frac{1}{\sqrt{2}} HRM_1(G)$$

with equality if G is regular.

Proof: By the Jensen inequality, for a concave function *f(x)*,

$$f\left(\frac{1}{n}\sum x_i\right) \geq \frac{1}{n}\sum f(x_i)$$

with equality for a strict concave function if $x_1 = x_2 = \ldots = x_n$. Choosing $f(x) = \sqrt{x}$, we obtain

$$\sqrt{\frac{r_u^2+r_v^2}{2}} \geq \frac{(r_u+r_v)}{2}$$

thus

$$(r_u+r_v)\sqrt{r_u^2+r_v^2} \geq \frac{1}{\sqrt{2}}(r_u+r_v)^2.$$

Hence

$$\sum_{uv\in E(G)} (r_u+r_v)\sqrt{r_u^2+r_v^2} \geq \frac{1}{\sqrt{2}}\sum_{uv\in E(G)} (r_u+r_v)^2$$

Thus

$$ER(G) \geq \frac{1}{\sqrt{2}} HRM_1(G)$$

with equality if G is regular.

Corollary 1.1. Let G be a simple connected graph. Then

$$ER(G) \geq \frac{1}{\sqrt{2}}\left(FR(G) + 2RM_2(G)\right)$$

with equality if G is regular.

Proof: We have

$$\sum_{uv\in E(G)} (r_u+r_v)\sqrt{r_u^2+r_v^2} \geq \frac{1}{\sqrt{2}}\sum_{uv\in E(G)} (r_u+r_v)^2$$
$$\geq \frac{1}{\sqrt{2}}\sum_{uv\in E(G)} \left(r_u^2+r_v^2+2r_ur_v\right)$$
$$= \frac{1}{\sqrt{2}}\left(FR(G) + 2RM_2(G)\right).$$

Theorem 2. Let G be a simple connected graph. Then

$$ER(G) \leq \sqrt{2}\left(HRM_1(G) - \sqrt{\alpha RGO(G) + 2HRM_2(G)}\right).$$

Proof: It is known that for $1\leq x \leq y$,

$$f(x,y) = \left(x+y-\sqrt{xy}\right) - \sqrt{\frac{x^2+y^2}{2}}$$

$$f(x,y) = \left(x+y-\sqrt{xy}\right) - \sqrt{\frac{x^2+y^2}{2}}$$

is decreasing for each y. Thus $f(x,y) \geq f(y,y) = 0$. Hence

$$x+y-\sqrt{xy} \geq \sqrt{\frac{x^2+y^2}{2}}$$

or
$$\sqrt{\frac{x^2+y^2}{2}} \leq x+y-\sqrt{xy}.$$

Put $x=r_u$ and $y=r_v$, we get

$$\sqrt{\frac{r_u^2+r_v^2}{2}} \leq (r_u+r_v) - \sqrt{r_ur_v}$$

$$\frac{1}{\sqrt{2}}(r_u+r_v)\sqrt{r_u^2+r_v^2} \leq (r_u+r_v)^2 - (r_u+r_v)\sqrt{r_ur_v}$$

which implies

$$\frac{1}{\sqrt{2}}\sum_{uv\in E(G)} (r_u+r_v)\sqrt{r_u^2+r_v^2}$$

$$\leq \sum_{uv\in E(G)} (r_u + r_v)^2 - \sqrt{\sum_{uv\in E(G)} (r_u + r_v)^2 r_u r_v}$$

$$\leq \sum_{uv\in E(G)} (r_u + r_v)^2 - \sqrt{\sum_{uv\in E(G)} \left((r_u + r_v)^2 r_u r_v + 2r_u^2 r_v^2\right)}$$

Thus

$$\frac{1}{\sqrt{2}} ER(G)$$

$$\leq HRM_1(G) - \sqrt{\alpha RGO(G) + 2HRM_2(G)}.$$

Theorem 3. Let G be a simple connected graph. Then

$$ER(G) < HRM_1(G).$$

Proof: It is known that for $1 \leq x \leq y$,

$$\sqrt{x^2 + y^2} < x + y$$

$$(x+y)\sqrt{x^2 + y^2} < (x+y)^2$$

Setting $x = r_u$ and $y = r_v$, we get

$$(r_u + r_v)\sqrt{r_u^2 + r_v^2} < (r_u + r_v)^2.$$

Thus

$$\sum_{uv\in E(G)} (r_u + r_v)\sqrt{r_u^2 + r_v^2} < \sum_{uv\in E(G)} (r_u + r_v)^2.$$

Hence

$$ER(G) < HRM_1(G).$$

Theorem 4. Let G be a simple connected graph with n vertices. Then

$$ER(G) \leq \sqrt{HRM_1(G) FR(G)}.$$

Proof: By the Cauchy-Schwarz inequality,

$$\left(\sum a_i b_i\right)^2 \leq \left(\sum a_i^2\right)\left(\sum b_i^2\right)$$

with equality holds if and only if $a_i = g b_i$, $i = 1, 2, \ldots, n$, for some real number g.

Using this to *RES*, we obtain

$$(ER(G))^2 = \left(\sum_{uv\in E(G)} (r_u + r_v)\sqrt{r_u^2 + r_v^2}\right)^2$$

$$\leq \left(\sum_{uv\in E(G)} (r_u + r_v)^2\right)\left(\sum_{uv\in E(G)} (r_u^2 + r_v^2)\right)$$

$$= HRM_1(G) FR(G)$$

gives the desired result.

III. RESULTS FOR SILICATE NETWORKS

Silicates are obtained by fusing metal oxides or metal carbonates with sand. A silicate network in symbolized by SL_n where n is the number of hexagons between the center and boundary of SL_n. A 2-dimensional siliciate network is shown in Figure-1.

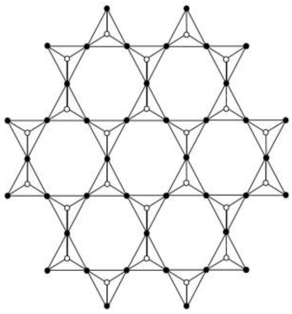

Figure 1. A 2-dimensional silicate network

Let G be the graph of silicate network SL_n with $15n^2+3n$ vertices and $36n^2$ edges. From Figure 1, it is easy to see that the vertices of SL_n are either of degree 3 or 6. In SL_n, by algebraic method, there are three types of edges based on the degree of the vertices of each edge as follows:

$E_1 = \{uv\in E(G) \mid d_u = d_v = 3\}$, $|E_1| = 6n$.

$E_2 = \{uv\in E(G) \mid d_u = 3, d_v = 6\}$, $|E_2| = 18n^2+6n$.

$E_3 = \{uv\in E(G) \mid d_u = d_v = 6\}$, $|E_3| = 18n^2 - 12n$.

Thus there are three types of Revan edges as follows:

We have $\Delta(G) + \delta(G) = 9$, $r_u = 9 - d_u$.

$RE_1 = \{uv\in E(G) \mid r_u = r_v = 6\}$, $|RE_1| = 6n$.

$RE_2 = \{uv\in E(G) \mid r_u = 6, r_v = 3\}$, $|RE_2| = 18n^2+6n$.

$RE_3 = \{uv\in E(G) \mid r_u = r_v = 3\}$, $|RE_3| = 18n^2 - 12n$.

Theorem 5. The elliptic Revan index of a silicate network SL_n is given by

$$ER(G)$$

$$= \left(324\sqrt{2} + 486\sqrt{5}\right)n^2 + \left(216\sqrt{2} + 162\sqrt{5}\right)n.$$

Proof: We have

$$ER(G) = \sum_{uv\in E(G)} (r_u + r_v)\sqrt{r_u^2 + r_v^2}$$

$$= 6n\left[(6+6)\sqrt{6^2 + 6^2}\right]$$

$$+ (18n^2 + 6n)\left[(6+3)\sqrt{6^2 + 3^2}\right]$$

$$+ (18n^2 - 12n)\left[(3+3)\sqrt{3^2 + 3^2}\right]$$

$$= \left(324\sqrt{2} + 486\sqrt{5}\right)n^2 + \left(216\sqrt{2} + 162\sqrt{5}\right)n.$$

Theorem 6. The elliptic Revan exponential of a silicate network SL_n is given by

$$ER(G,x) = 6nx^{72\sqrt{2}} + \left(18n^2 + 6n\right)x^{27\sqrt{5}}$$

$$+ \left(18n^2 - 12n\right)x^{18\sqrt{2}}.$$

Proof: We have

$$ER(G,x)=\sum_{uv\in E(G)} x^{(r_u+r_v)\sqrt{r_u^2+r_v^2}}$$
$$=6nx^{(6+6)\sqrt{6^2+6^2}}+\left(18n^2+6n\right)x^{(6+3)\sqrt{6^2+3^2}}$$
$$+\left(18n^2-12n\right)x^{(3+3)\sqrt{3^2+3^2}}$$
$$=6nx^{72\sqrt{2}}+\left(18n^2+6n\ \right)x^{27\sqrt{5}}$$
$$+\left(18n^2-12n\right)x^{18\sqrt{2}}.$$

IV. RESULTS FOR RHOMBUS SILICATE NETWORKS

We consider a family of rhombus silicate networks. A rhombus silicate network is symbolized by $RHSL_n$. A 3-dimensional rhombus silicate network is depicted in Figure 2.

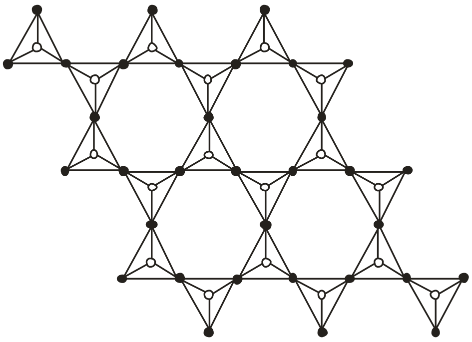

Figure 2. 3-dimensional rhombus silicate network

Let G be the graph of rhombus silicate network $RHSL_n$ with $5n^2+2n$ vertices and $12n^2$ edges. From Figure 2, it is easy to see that the vertices of $RHSL_n$ are either of degree 3 or 6. In $RHSL_n$, there are three types of edges as follows:

$E_1=\{uv\in E(G)\mid d_u=d_v=3\}$, $|E_1|=4n+2$.
$E_2=\{uv\in E(G)\mid d_u=3,\ d_v=6\}$, $|E_2|=6n^2+4n-4$.
$E_3=\{uv\in E(G)\mid d_u=d_v=6\}$, $|E_3|=6n^2-8n+2$.

We have $\Delta(G)+\delta(G)=9$, $r_u=9-d_u$.

Thus there are three types of Revan edges based on the degree of end Revan vertices of each Revan edge as follows:

$RE_1=\{uv\in E(G)\mid r_u=r_v=6\}$, $|RE_1|=4n+2$.
$RE_2=\{uv\in E(G)\mid r_u=6,\ r_v=3\}$, $|RE_2|=6n^2+4n-4$.
$RE_3=\{uv\in E(G)\mid r_u=r_v=3\}$, $|RE_3|=6n^2-8n+2$.

Theorem 7. The elliptic Revan index of a rhombus silicate network $RHSL_n$ is given by

$$ER(G)=\left(108\sqrt{2}+162\sqrt{5}\right)n^2+\left(144\sqrt{2}+108\sqrt{5}\right)n$$
$$+180\sqrt{2}-108\sqrt{5}.$$

Proof: We have

$$ER(G)=\sum_{uv\in E(G)}\left(r_u+r_v\right)\sqrt{r_u^2+r_v^2}$$
$$=(4n+2)\left[(6+6)\sqrt{6^2+6^2}\right]$$
$$+\left(6n^2+4n-4\right)\left[(6+3)\sqrt{6^2+3^2}\right]$$
$$+\left(6n^2 8n+2\right)\left[(3+3)\sqrt{3^2+3^2}\right]$$
$$=\left(108\sqrt{2}+162\sqrt{5}\right)n^2+\left(144\sqrt{2}+108\sqrt{5}\right)n$$
$$+180\sqrt{2}-108\sqrt{5}.$$

Theorem 8. The elliptic Revan exponential of a rhombus silicate network $RHSL_n$ is given by

$$ER(G,x)=(4n+2)x^{72\sqrt{2}}+\left(6n^2+4n-4\right)x^{27\sqrt{5}}$$
$$+\left(6n^2-8n+2\right)x^{18\sqrt{2}}.$$

Proof: We have

$$ER(G,x)=\sum_{uv\in E(G)} x^{(r_u+r_v)\sqrt{r_u^2+r_v^2}}$$
$$=(4n+2)x^{(6+6)\sqrt{6^2+6^2}}+\left(6n^2+4n-4\right)x^{(6+3)\sqrt{6^2+3^2}}$$
$$+\left(6n^2-8n+2\right)x^{(3+3)\sqrt{3^2+3^2}}$$
$$=(4n+2)x^{72\sqrt{2}}+\left(6n^2+4n-4\right)x^{27\sqrt{5}}$$
$$+\left(6n^2-8n+2\right)x^{18\sqrt{2}}.$$

V. RESULTS FOR OXIDE NETWORKS

An oxide network of n is symbolized by OX_n. These networks are of vital importance in the study of silicate networks. An oxide network of dimension five is shown in Figure 3.

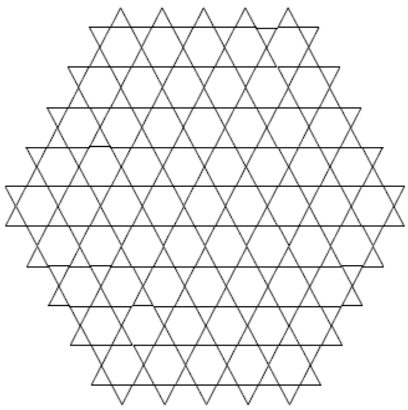

Figure 3. 5-dimensional oxide network

Let G be the graph of oxide network OX_n with $9n^2+3n$ vertices and $18n^2$ edges. From Figure 3, it is easy to see that the vertices of OX_n are either of degree 2 or 4. Thus $\Delta(G)=4$, $\delta(G)=2$. In G, there are two types of edges as follows:

$E_1=\{uv\in E(G)\mid d_u=2,\ d_v=4\}$, $|E_1|=12n$.
$E_2=\{uv\in E(G)\mid d_u=d_v=4\}$, $|E_2|=18n^2-12n$.

We have $r_u=\Delta(G)+\delta(G)-d_u=6-d_u$.

$RE_1=\{uv\in E(G)\mid r_u=4,\ r_v=2\}$, $|RE_1|=12n$.

$RE_2 = \{uv\in E(G) \mid r_u = r_v = 2\}$, $|RE_2| = 18n^2 - 12n$.

Theorem 9. The elliptic Revan index of an oxide network OX_n is given by

$$ER(G) = 144\sqrt{2}n^2 + \left(144\sqrt{5} - 96\sqrt{2}\right)n.$$

Proof: We have

$$ER(G) = \sum_{uv\in E(G)} (r_u + r_v)\sqrt{r_u^2 + r_v^2}$$

$$= 12n\left[(4+2)\sqrt{4^2+2^2}\right]$$

$$+ (18n^2 - 12n)\left[(2+2)\sqrt{2^2+2^2}\right]$$

$$= 144\sqrt{2}n^2 + \left(144\sqrt{5} - 96\sqrt{2}\right)n.$$

Theorem 10. The elliptic Revan exponential of an oxide network OX_n is given by

$$ER(G,x) = 12nx^{12\sqrt{5}} + (18n^2 - 12n)x^{8\sqrt{2}}.$$

Proof: We have

$$ER(G,x) = \sum_{uv\in E(G)} x^{(r_u+r_v)\sqrt{r_u^2+r_v^2}}$$

$$= 12nx^{(4+2)\sqrt{4^2+2^2}} + (18n^2 - 12n)\,x^{(2+2)\sqrt{2^2+2^2}}$$

$$= 12nx^{12\sqrt{5}} + (18n^2 - 12n)\,x^{8\sqrt{2}}.$$

VI. RESULTS FOR RHOMBUS OXIDE NETWORKS

We consider a family of rhombus oxide networks. A rhombus oxide network of dimension n is symbolized by $RHOX_n$. A 3- dimensional rhombus oxide network is depicted in Figure 4.

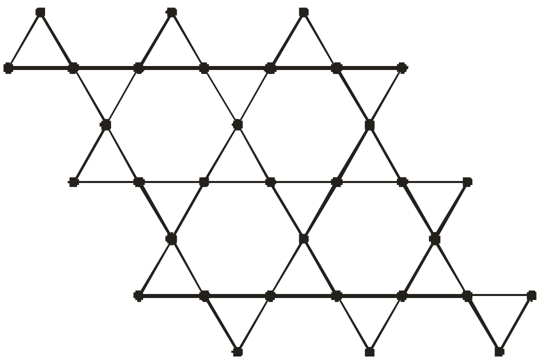

Figure 4. 3-dimensional rhombus oxide network

Let G be the graph of rhombus oxide network $RHOX_n$ with $3n^2 + 2n$ vertices and $6n^2$ edges. From Figure 4, it is easy to see that the vertices of $RHOX_n$ are either of degree 2 or 4. Thus $\Delta(G) = 4$, $\delta(G) = 2$. In G, there are three types of edges as follows:

$E_1 = \{uv\in E(G) \mid d_u = d_v = 2\}$, $|E_1| = 2$.

$E_2 = \{uv\in E(G) \mid d_u = 2,\ d_v = 4\}$, $|E_2| = 8n - 4$.

$E_3 = \{uv\in E(G) \mid d_u = d_v = 4\}$, $|E_2| = 6n^2 - 8n+2$.

We have $r_u = \Delta(G) + \delta(G) - d_u = 6 - d_u$.

$RE_1 = \{uv\in E(G) \mid r_u = r_v = 4\}$, $|RE_1| = 2$.

$RE_2 = \{uv\in E(G) \mid r_u = 4,\ r_v = 2\}$, $|RE_2| = 8n - 4$.

$RE_2 = \{uv\in E(G) \mid r_u = r_v = 2\}$, $|RE_2| = 6n^2 - 8n+2$.

Theorem 11. The elliptic Revan index of a rhombus oxide network $RHOX_n$ is given by

$$ER(G) = 48\sqrt{2}n^2 + \left(96\sqrt{5} - 64\sqrt{2}\right)n$$

$$+80\sqrt{2} - 48\sqrt{5}.$$

Proof: We have

$$ER(G) = \sum_{uv\in E(G)} (r_u + r_v)\sqrt{r_u^2 + r_v^2}$$

$$= 2\left[(4+4)\sqrt{4^2+4^2}\right]$$

$$+ (8n-4)\left[(4+2)\sqrt{4^2+2^2}\right]$$

$$+ (6n^2 - 8n + 2)\left[(2+2)\sqrt{2^2+2^2}\right]$$

$$= 48\sqrt{2}n^2 + \left(96\sqrt{5} - 64\sqrt{2}\right)n + 80\sqrt{2} - 48\sqrt{5}.$$

Theorem 12. The elliptic Revan exponential of a rhombus oxide network $RHOX_n$ is given by

$$ER(G,x) = 2x^{32\sqrt{2}} + (8n - 4)x^{12\sqrt{5}}$$

$$+ (6n^2 - 8n + 2)x^{8\sqrt{2}}.$$

Proof: We have

$$ER(G,x) = \sum_{uv\in E(G)} x^{(r_u+r_v)\sqrt{r_u^2+r_v^2}}$$

$$= 2x^{(4+4)\sqrt{4^2+4^2}} + (8n-4)\,x^{(4+2)\sqrt{4^2+2^2}}$$

$$+ (6n^2 - 8n + 2)\,x^{(2+2)\sqrt{2^2+2^2}}$$

$$= 2x^{32\sqrt{2}} + (8n-4)\,x^{12\sqrt{5}} + (6n^2 - 8n + 2)\,x^{8\sqrt{2}}.$$

VII. RESULTS FOR HEXAGONAL NETWORKS

Hexagonal network is symbolized by HX_n where n is the number of vertices in each side of hexagon. A hexagonal network of dimension six is shown in Figure 5.

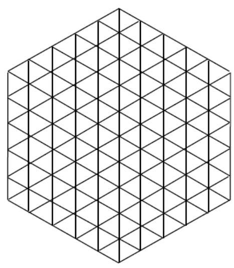

Figure 5. 6-dimensional hexagonal network

Let H be te graph of hexagonal network HX_n with $3n^2 - 3n + 1$ vertices and $9n^2 - 15n + 6$ edges. From Figure 5, it is easy to see that the vertices of HX_n are either of degree 3, 4 or 6. Thus $\Delta(H) = 6$, $\delta(H) = 3$. In H, by algebraic method, there are five types of edges as follows:

$E_1 = \{uv\in E(H) \mid d_u = 3,\ d_v = 4\}$, $|E_1| = 12$.

$E_2 = \{uv \in E(H) | \ d_u = 3, \ d_v = 6\}, |E_2| = 6.$
$E_3 = \{uv \in E(H) | \ d_u = d_v = 4\}, |E_3| = 6n - 18.$
$E_4 = \{uv \in E(H) | \ d_u = 4, \ d_v = 6\}, |E_4| = 12n - 24.$
$E_5 = \{uv \in E(H) | \ d_u = d_v = 6\}, |E_5| = 9n^2 - 33n + 30$

We have $r_u = \Delta\, H) + \delta(H) - d_u = 9 - d_u$.

$RE_1 = \{uv \in E(H) \mid r_u = 6, r_v = 5\}, |RE_{65}| = 12.$
$RE_2 = \{uv \in E(H) \mid r_u = 6, r_v = 3\}, |RE_{63}| = 6.$
$RE_3 = \{uv \in E(H) \mid r_u = r_v = 5\}, |RE_{55}| = 6n - 18.$
$RE_4 = \{uv \in E(H) \mid r_u = 5, r_v = 3\}, |RE_{53}| = 12n - 24.$
$RE_5 = \{uv \in E(H) \mid r_u = r_v = 3\}, |RE_{33}| = 9n^2 - 33n + 30$

Theorem 13. The elliptic Revan index of a hexagonal network HX_n is given by

$$ER(H) = 162\sqrt{2}n^2 + \left(96\sqrt{34} - 294\sqrt{2}\right)n + 120\sqrt{61} + 162\sqrt{5} - 360\sqrt{2} - 192\sqrt{34}.$$

Proof: We have

$$\begin{aligned} ER(H) &= \sum_{uv \in E(H)} (r_u + r_v)\sqrt{r_u^2 + r_v^2} \\ &= 12\left[(6+5)\sqrt{6^2+5^2}\right] + 6\left[(6+3)\sqrt{6^2+3^2}\right] \\ &\quad + (6n-18)\left[(5+5)\sqrt{5^2+5^2}\right] + (12n-24)\left[(5+3)\sqrt{5^2+3^2}\right] \\ &\quad + (9n^2-33n+30)\left[(3+3)\sqrt{3^2+3^2}\right] \\ &= 162\sqrt{2}n^2 + \left(96\sqrt{34} - 294\sqrt{2}\right)n + 120\sqrt{61} + 162\sqrt{5} - 360\sqrt{2} - 192\sqrt{34}. \end{aligned}$$

Theorem 14. The elliptic Revan exponential of a hexagonal network HX_n is given by

$$ER(H,x) = 12x^{10\sqrt{61}} + 6x^{27\sqrt{5}} + (6n-18)x^{50\sqrt{2}} + (12n-24)x^{8\sqrt{34}} + (9n^2-33n+30)x^{18\sqrt{32}}.$$

Proof: We have

$$\begin{aligned} ER(H,x) &= \sum_{uv \in E(H)} x^{(r_u+r_v)\sqrt{r_u^2+r_v^2}} \\ &= 12x^{(6+5)\sqrt{6^2+5^2}} + 6x^{(6+3)\sqrt{6^2+3^2}} \\ &\quad + (6n-18)x^{(5+5)\sqrt{5^2+5^2}} + (12n-24)x^{(5+3)\sqrt{5^2+3^2}} \\ &\quad + (9n^2-33n+30)x^{(3+3)\sqrt{3^2+3^2}} \\ &= 12x^{10\sqrt{61}} + 6x^{27\sqrt{5}} + (6n-18)x^{50\sqrt{2}} \\ &\quad + (12n-24)x^{8\sqrt{34}} + (9n^2-33n+30)x^{18\sqrt{32}}. \end{aligned}$$

VIII. RESULTS FOR HONEYCOMB NETWORK

Honeycomb networks are very useful in Chemistry and Computer Graphics. A honeycomb network of dimension n is symbolized by HC_n. A honeycomb network of dimension four is shown in Figure 6.

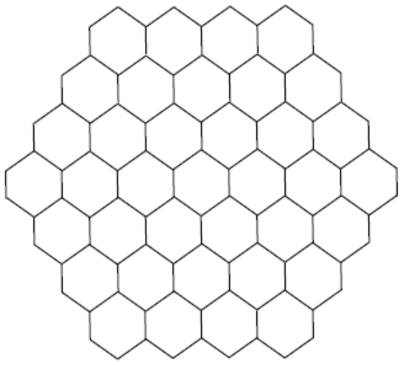

Figure 6. 4-dimensional honeycomb network

Let H be the graph of honeycomb network HC_n with $6n^2$ vertices and $9n^2 - 3n$ edges. From Figure 6, it is easy to see that the vertices of HX_n are either of degree 2 or 3. Thus $\Delta(H) = 3$, $\delta(H) = 2$. In H, there are three types of edges as follows:

$E_1 = \{uv \in E(H) | \ d_u = d_v = 2\}, |E_1| = 6.$
$E_2 = \{uv \in E(H) | \ d_u = 2, d_v = 3\}, |E_2| = 12n - 12.$
$E_3 = \{uv \in E(H) | \ d_u = d_v = 3\}, |E_3| = 9n^2 - 15n + 6.$

We have $r_u = \Delta(H) + \delta(H) - d_u = 5 - d_u$.

$RE_1 = \{uv \in E(H) \mid r_u = r_v = 3\}, |RE_1| = 6.$
$RE_2 = \{uv \in E(H) \mid r_u = 3, r_v = 2\}, |RE_2| = 12n - 12.$
$RE_3 = \{uv \in E(H) \mid r_u = r_v = 2\}, |RE_3| = 9n^2 - 15n + 6.$

Theorem 15. The elliptic Revan index of a honeycomb network HC_n is given by

$$ER(H) = 72\sqrt{2}n^2 + \left(60\sqrt{13} - 120\sqrt{2}\right)n + 156\sqrt{2} - 60\sqrt{13}.$$

Proof: We have

$$\begin{aligned} ER(H) &= \sum_{uv \in E(H)} (r_u + r_v)\sqrt{r_u^2 + r_v^2} \\ &= 6\left[(3+3)\sqrt{3^2+3^2}\right] \\ &\quad + (12n-12)\left[(3+2)\sqrt{3^2+2^2}\right] \\ &\quad + (9n^2-15n+6)\left[(2+2)\sqrt{2^2+2^2}\right] \\ &= 72\sqrt{2}n^2 + \left(60\sqrt{13} - 120\sqrt{2}\right)n + 156\sqrt{2} - 60\sqrt{13}. \end{aligned}$$

Theorem 16. The elliptic Revan exponential of a honeycomb network HC_n is given by

$$ER(H,x) = 6x^{18\sqrt{2}} + (12n-12)x^{5\sqrt{13}} + (9n^2-15n+6)x^{8\sqrt{2}}.$$

Proof: We have

$$\begin{aligned} ER(H,x) &= \sum_{uv \in E(H)} x^{(r_u+r_v)\sqrt{r_u^2+r_v^2}} \\ &= 6x^{(3+3)\sqrt{3^2+3^2}} + (12n-12)x^{(3+2)\sqrt{3^2+2^2}} \\ &\quad + (9n^2-15n+6)x^{(2+2)\sqrt{2^2+2^2}} \end{aligned}$$

$$= 6x^{18\sqrt{2}} + (12n - 12)\, x^{5\sqrt{13}}$$
$$+ (9n^2 - 15n + 6)\, x^{8\sqrt{2}}.$$

VI. CONCLUSION

In this study, we have determined the elliptic Revan index and its corresponding exponential of some networks which are appeared in chemical science.

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