[Volume 3 issue 1 January 2015]
Page No.829-836 ISSN :2320-7167

INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTER RESEARCH

Rainbow and Strong Rainbow criticalness of some standard graphs.

K.Srinivasa Rao[#1], R.Murali[#2], S.K.Rajendra[#3].

srinivas.dbpur@gmail.com, muralir2968@gmail.com , rajendra.drait@gmail.com

#1 Department of Mathematics, Shri Pillappa College of Engineering, Bangalore,

#2 Department of Mathematics, Dr. Ambedkar Institute of Technology, Bangalore,

#3 Department of Industrial Engg.& Management, Dr. Ambedkar Institute of Technology, Bangalore

ABSTRACT

A path in an edge−colored graph is said to be a rainbow path,if every edge in the path has different color. An edge colored graph is rainbow connected if there exists a rainbow path between every pair of vertices. The rainbow connection of a graph G, denoted by $rc(G)$, is the smallest number of colors required to color the edges of graph such that the graph is rainbow connected. Given two arbitrary vertices u and v in G, a rainbow $u-v$ geodesic in G is a rainbow $u-v$ path of length d(u, v), where d(u, v) is the distance between u and v. The graph G is strongly rainbow connected if there exists a rainbow $u-v$ geodesic for any two vertices u and v in G. The strong rainbow connection number of G, denoted by $src(G)$, is the minimum number of colors needed to make G strongly rainbow connected. Deletion of any edge in G if the property alters, then that graph is called critical graph with respect to that property.

In this paper we find the rainbow connection number of Cartesian product of pan graph, Stacked Book Graph, their criticalness with respect to rainbow coloring and strong rainbow connection of Book graph.

Keywords: Edge-coloring, Rainbow path, Rainbow connectionnumber,Criticality.

AMS Subject Classification 2010: 05C15

1.Introduction

The concept connectivity is perhaps the most fundamental graph-theoretic subject. There are many elegant and powerful results on connectivity in graph theory. There are also many ways to strengthen the connectivity concept, such as requiring hamiltonicity,k-connectivity, imposing bounds on the diameter, and so on. An interesting way to strengthen the connectivity requirement, the rainbow connection, a natural and interesting quantifiable way to strengthen the connectivity requirementwas introduced by Chartrand, Johns, McKeon and Zhang [1] in 2008, which is restated as follows for thefinite undirected and simple graphs.

Edge coloring of a graph is a function from its edge set to the set of natural numbers. Let G be a nontrivial connected graph with an edge coloring $c : E(G) \to \{1, 2 ,3,\ldots\ldots, k\},\ k \in N$, where adjacent edges may be colored the same. A path of G is called rainbow pathif no two edges of it are colored the same. An edge colored graph G is said to be rainbow connected if for any two vertices of G there is a rainbow path of G connecting them. Clearly, if a graph is rainbow connected, it must be connected. Conversely, any connected graph has a trivial edge coloring that makes it rainbow connected, i.e., the coloring such that each edge has a distinct color.

In [2] K.Srinivasa Rao and R.Murali have determined the rainbow connection number of Cartesian product graphs like Grid graph, Prism graph of odd and even vertices.

In this paper we compute the critical property with respect to rainbow connection number and Strong rainbow connection number of Cartesian product of some graphs.

Definition 1.1: Agraph G is said to be Rainbow critical and strong rainbow critical if removal of any edge of G increases the rainbow connection number or strong rainbow connection of G. i.e. If $rc(G)=k$ for some positive integer, then $rc(G-e)>k$ for any edge e in G.Similarly if $src(G)=k$ for some positive integer, then $src(G-e)>k$ for any edge e in G.

Definition 1.2: The m-pan graph T_{m+1} is the graph obtained by joining a cycle graph C_n to a singleton graph K_1 with a bridge.

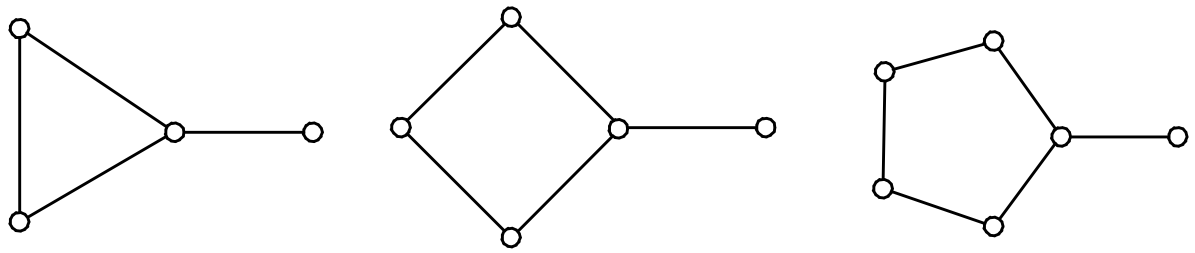

Fig 1. 3-Pan, 4-Pan,5-Pan graphs

Definition 1.3: The $m \times n$ graph is the Cartesian product of $T_{m+1} \times P_n$, having the vertex set $V=\{v_{i,j} : 1 \le i \le m, 1 \le j \le n\}$ with two vertices $v_{i,j}$ *and* $v_{i',j'}$ being adjacent if $i=i'$ & $|j-j'|=1$ or if $j=j'$ & $|i-i'|=1$.

Definition 1.4: The Stacked book graph $B_{m,n}$ is defined as the graph Cartesian product $S_{m+1} \times P_n$, where S_m is a star graph and P_n is the path graph on two vertices. Where $m \ge 3, n \ge 2$

2. Some preliminary result:

Proposition 2.1: [1] For n > 3, the strong rainbow connection number of the wheel W_n is $src(W_n)=\left\lceil \frac{n}{3} \right\rceil$.

Theorem 2.2: [4]For each integer n, the rainbow connection of G is $rc(G)=4$ where $G \cong G_n$ *with* $n>4$, *or* $G \cong B_n$ *with* $n>3$.

3. Main Result

Theorem3.1: If $G_1 = T_m$ *and* $G_2 = P_n$ $m,n>1$, and $G=G_1 \times G_2$, then $rc(G)=src(G)=\left\lfloor \frac{m}{2} \right\rfloor + n$.

Proof: Consider the graph G, for $m>4$, and $n \ge 2$ where m and n be any integer.

G is the Cartesian product of $G_1 = T_{m+1}$ *and* $G_2 = P_n$. Let the vertices of G be denoted by $v_{i,j}$ *where* $1 \le i \le m, 1 \le j \le n$.

Let P_i denote m-pan in G given by $P_i : v_{1i} - v_{2i} - v_{3i} - - - - - v_{m-1i} - v_{mi}$ and let P_j' denote n paths in G given by $P_j' : v_{j1} - v_{j2} - v_{j3} - - - - - v_{jn-1} - v_{jn}$. Since the diameter of G is $\left\lfloor \frac{m}{2} \right\rfloor + n$, implies

$$rc(G) \geq src(G) \geq \left\lfloor \frac{m}{2} \right\rfloor + n \text{ -------------------------- (1).}$$

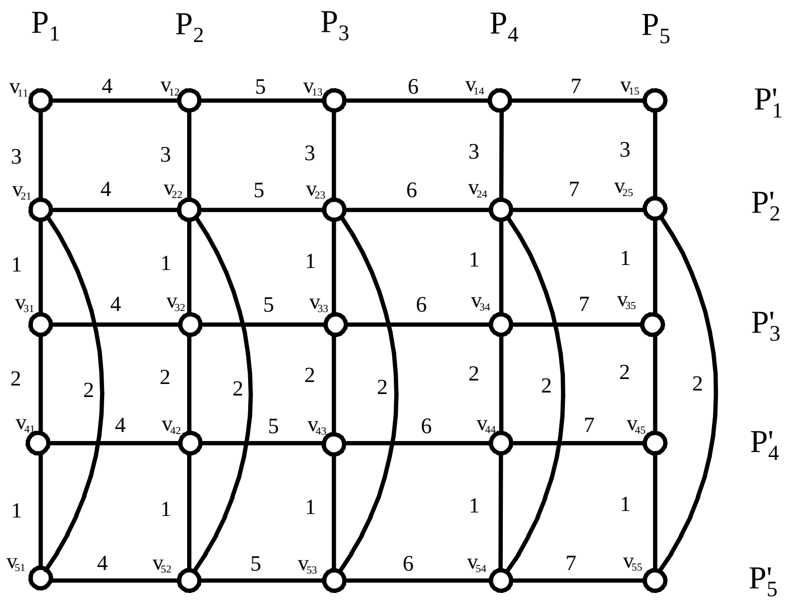

Fig 2. Cartesian product of 4-Pan and Path graph $T_5 \times P_5$

Show $rc(G) \leq src(G) \leq \left\lfloor \frac{m}{2} \right\rfloor + n$

Consider the pan graph $P_1 : v_{11} - v_{21} - v_{31} - - - - - v_{m-11} - v_{m1} - v_{11}$, by definition pan graph is the graph obtained by adding pendent edge to the cycle. Since G has $n+1$ vertices, let $m = n+1$. We know that $rc(C_m) = \left\lfloor \frac{m}{2} \right\rfloor$. For a complete assignment of rainbow coloring to G we requireanother color for the pendent edge other than $\left\lfloor \frac{m}{2} \right\rfloor$ colors that already assigned to the cycle.

$$\therefore \; rc(T_m) = \left\lfloor \frac{m}{2} \right\rfloor + 1 \text{.----------------------(a)}$$

Now consider the path $P_1' : v_{11} - v_{12} - v_{13} - - - - - v_{1n-1} - v_{1n}$ in G. Since P_1' is a path with n vertices and $n-1$ edges, assign $n-1$ different colors. Assign the same colors to the edges of the paths $P_2', P_3', \ldots\ldots\ldots P_j'$ for all j. $\therefore rc(P_j') = src(P_j') = n-1$ ----------- (b)

Combining (a) and (b), it is clear that

$rc(G) \le src(G) \le \left\lfloor \frac{m}{2} \right\rfloor + 1 + n - 1$

$rc(G) \le src(G) \le \left\lfloor \frac{m}{2} \right\rfloor + n$ -- (2)

Combining (1) and (2) $rc(G) = src(G) = \left\lfloor \frac{m}{2} \right\rfloor + n$.

□

Lemma3.2: If $G_1 = T_{m+1}$ *and* $G_2 = P_n$ $m,n > 1$, and $G = G_1 \times G_2$, then $rc(G-e) = src(G-e) = \left\lfloor \frac{m}{2} \right\rfloor + n + 1$ .That is G is a rainbow critical graph.

Proof: In G, it clearly observes that any edge is an edge of cycle of length is 4. Consider an edge $e \in E(G)$. Let $e \in E(P_k')$ *where* $k = 1,2,.........i$, i.e. $e = \{v_{ij}, v_{i(j+1)}\}$ such that $d(v_{ij}, v_{i(j+1)}) = 1$, deletion of the edge e in G that is $G - e$ results $d(v_{ij}, v_{ij+1}) = 3$. Because the set of vertices $v_{ij}, v_{i(j+1)}, v_{(i+1)j}, v_{(i+1)(j+1)}$ in G forms a cycle of length 4, deletion of any edge in the cycle results a path of length 3.

From the assignment of colors in theorem 3.1, it is clearly observed that, to get a rainbow path between the vertices v_{ij}, v_{ij+1}, we require one more color apart from $\left\lfloor \frac{m}{2} \right\rfloor + n$ colors that already assigned for coloring the graph G. This holds $\forall\, e \in E(G)$. $rc(G-e) = src(G-e) = \left\lfloor \frac{m}{2} \right\rfloor + n + 1$.

This proves G is Rainbow critical graph.

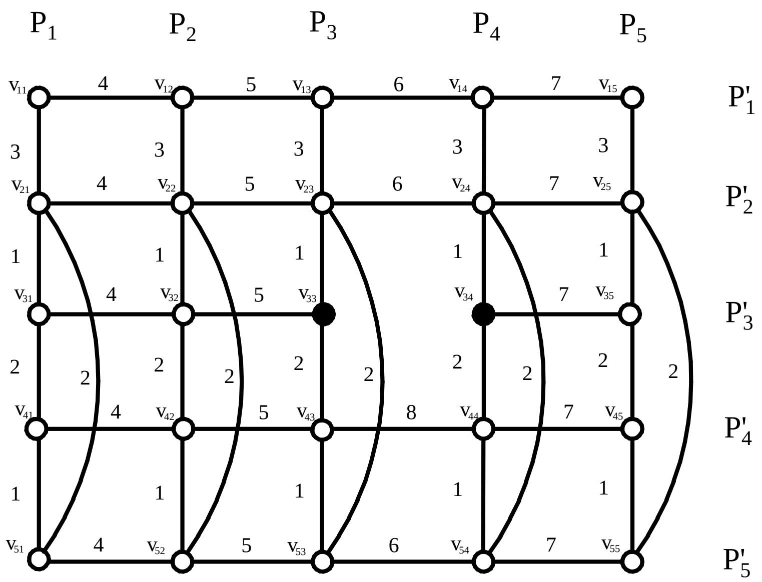

Fig 3. Rainbow critical graph $T_5 \times P_5$

□

Theorem3.3: Let G is the Cartesian product graph $S_{m+1} \times P_2$ (Book Graph), then Strong rainbow connection of G i.e., $src(G) = m + 1$.

Proof: The graph G consists of two star graphs say S_{m+1} & S'_{m+1} with $2(m+1)=n$ vertices. Let us divide the vertex set of G into two sets V & V' such that $V=\{v_1,v_2,v_3,........\}$ & $V'=\{v'_1,v'_2,v'_3,.......\}$. Since G is the Cartesian product graph $S_{m+1}\times P_2$, there is an edge between the every vertices of V to V'.

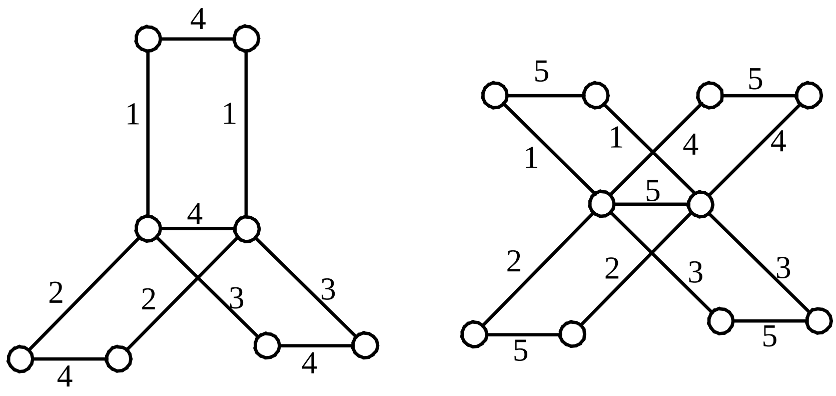

Fig 4. Book graph B_3 & B_4

Now define the colors c to the edges of S_{m+1} , $c:e(S_{m+1})\to\{1,2,3,.......m\}$ such that $c(e_i)=i$ where $1\le i\le m$ and assign the same color to the edges of S'_{m+1}. Assign $m+1$ color to the connected edges of the vertices V & V'. Hence $src(G)=m+1$.

□

Remark:

In theorem 3.3 $src(G)=m+1$, deletion of any edge in G alters the Strong rainbow connection number, i.e., $src(G-e)=m+2$. This shown to be G is Strong rainbow critical graph. But if we assign the colors c to the edges of G in the following manner,clears that G is not strong rainbow critical graphs.

If m is even:

Define the colors c to the edges of S_{m+1} , $c:e(S_{m+1})\to\{1,2,3,.......m\}$ such that

$$c(pp_i)=\begin{cases}2i-1 & for \quad 1\le i\le \frac{m}{2}\\ 2i-m & for \quad \frac{m}{2}<i\le m\end{cases}\qquad c(qq_i)=\begin{cases}2i & for \quad 1\le i\le \frac{m}{2}\\ 2i-(m+1) & for \quad \frac{m}{2}<i\le m\end{cases}$$

and $c(p_iq_i)=m+1$

If m is odd:

Define the colors c to the edges of S_{m+1} , $c:e(S_{m+1})\to\{1,2,3,.......m\}$ such that

$$c(pp_i)=\begin{cases}2i-1 & for \quad 1\le i\le \left\lceil\frac{m}{2}\right\rceil\\ 2i-(m+1) & for \quad \left\lceil\frac{m}{2}\right\rceil<i\le m\end{cases}\qquad c(qq_i)=\begin{cases}2i & for \quad 1\le i\le \left\lfloor\frac{m}{2}\right\rfloor\\ 2i-m & for \quad \left\lfloor\frac{m}{2}\right\rfloor<i\le m\end{cases}$$

and $c(p_i q_i) = m+1$

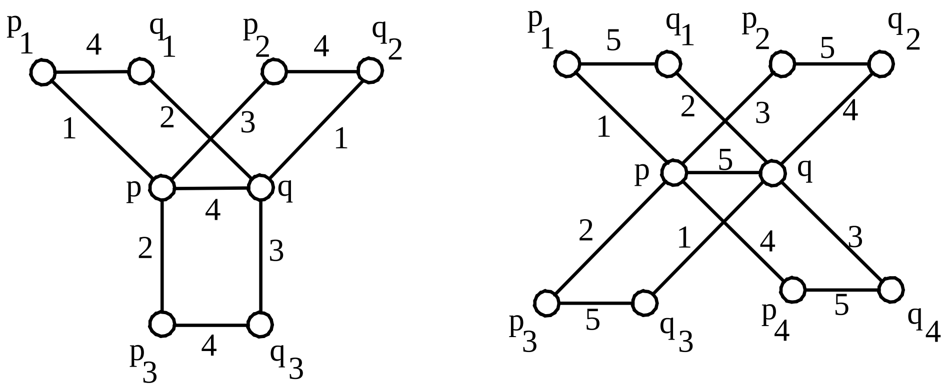

Fig 5. Book graph

B_3 & B_4

□

Theorem3.4: If $G_1 = S_{m+1}$ and $G_2 = P_n$ $m \geq 3, n \geq 2$, and $G = G_1 \times G_2$, then $rc(G) = src(G) = m+n-1$.

Proof: The graph G consists of n star graphs say nS_{m+1}. Define the colors c to the edges of S_{m+1} ,in such a way that $c : e(S_{m+1}) \to \{1,2,3,\ldots\ldots m\}$

Such that $c(e_i) = i$ where $1 \leq i \leq m$.

Since G is the Cartesian product of $S_{m+1} \times P_n$, the joining edges between the vertices of all-star graphs forms a path. Since $rc(P_n) = n-1$, it requires n-1 colors for getting rainbow path between any two vertices in G. Hence $rc(G) = src(G) = m+n-1$.

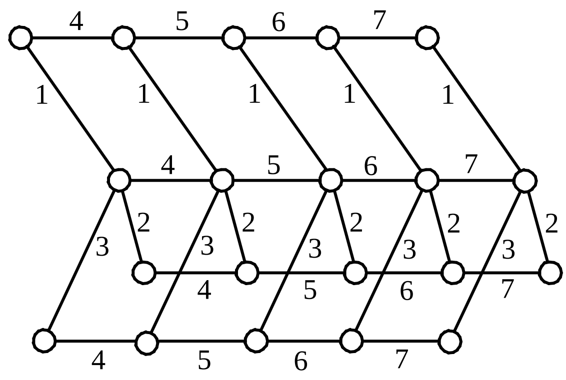

Fig 6. Stacked Book graph $B_{3,5}$

□

Lemma 3.5: If $G_1 = S_{m+1}$ and $G_2 = P_n$ $m \geq 3, n \geq 2$, and $G = G_1 \times G_2$, then $rc(G-e) = src(G-e) = m+n$.

Proof: In G, if we consider any edge, that edge belongs to the cycle of length 4. Deletion of that edge forms a path of length 3 between the deleted vertices. So to get a rainbow path between these vertices, it requires one more color other than $(m+n-1)^{th}$ colors. That is $rc(G-e) = src(G-e) = m+n$.

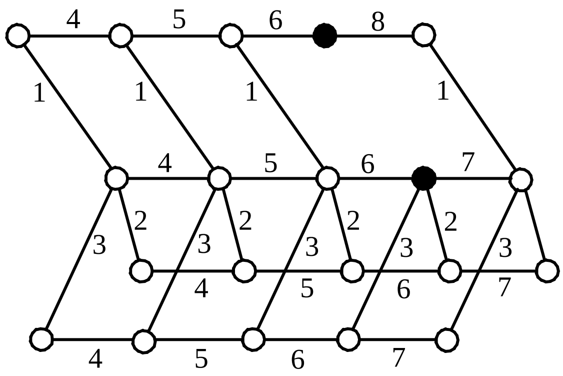

Fig 5. Rainbow critical Stacked Book graph $B_{3,5}$

□

ACKNOWLEDGMENT

The first author is thankful to the management and the staff of the Department of Mathematics, Shri Pillappa College of Engineering, Bangalore,for their support and encouragement. The authors are also thankful to the management and R&D center, Department of Mathematics, Dr. Ambedkar Institute of Technology, Bangalore.

REFERENCES

[1] Chartrand, G., Johns, G., McKeon, K., and Zhang, P. (2008). "Rainbow connection in graphs". Math. Bohem, 133(1):85–98.

[2] K.Srinivasa Rao and R.Murali., (2014),"Rainbow critical graphs" International Journal of Computer Application, Vol 4, Issue 4, pp.252-259.

[3] SyafrizalSy, GemaHistaMedika, LyraYulianti, (2013), "Rainbow Connection Numbers of fan and sun" Vol7 , No.64, pp.3155-3159.

[4] SyafrizalSy, Reni Wijaya, Surahmat, (2014), "Rainbow Connection Numbers of Some Graphs" Vol8 , No.94, pp.4693-4696.