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Some New Results on Prime Graphs

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Abstract:

A graph $G=(V,E)$ with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceeding n such that the label of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph. And a graph G is said to be a strongly prime graph if for any vertex v of G there exists a prime labeling f satisfying $f(v)=1$. In this paper we investigate prime labeling for some graphs related to Star, Cycle , $C_n(C_n)$ graph , H - graph and also we prove that Crown graph, $G \odot S_2$ where G is a Cycle, are strongly prime Graphs.

Keywords*:* Prime Labeling, Prime Graph, Strongly Prime Graph .

1.0 Introduction:

In this paper, we consider only simple, finite, undirected and non trivial graph $G=(V(G),E(G))$ with the vertex set $V(G)$ and the edge set $E(G)$. The set of vertices adjacent to a vertex u of $V(G)$ is denoted by N(u). For notations and terminology we refer to Bondy and Murthy [1].

The notion of a prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout.A (1982 P 365-368) [7] Two integers a and b are said to be relatively prime if their greatest common divisor is 1. Relatively prime numbers play an important role in both analytic and algebraic number theory. Many researchers have studied prime graph. For example Fu.H (1994 P 181-186) [3] have proved that path P_n on n vertices is a prime graph. Deresky.T (1991 P 359-369) [2] have proved that the C_n on n vertices is a prime graph. Lee.S (1998 P 59-67) [5] have proved that wheel W_n is a prime graph iff n is even. Around 1980 Roger Etringer conjectured that all trees having prime labeling which is not settled till today. In [8] S.K.Vaidya and K.K.Kanani have proved the *Prime Labeling For Some Cycle Related Graph*. In [9] S.K.Vaidya and Udayan M.Prajapati have introduced Strongly prime graph and has proved the C_n, P_n and $K_{1,n}$ are Strongly prime graphs and W_n is a Strongly prime graph for every even integer $n \geq 4$, in *Some New Results On Prime Graph* (2012 P 99-104).

In [6] S.Meena and K.Vaithiligam have proved some results on *Prime Labeling For Some Helm Related Graphs* (2013 P 1075-1085).In [10] R.Vasuki and A.Nagarajan have proved *Some Results On Super Mean Graphs* Vol.3 (2009), 82-96.For latest *Dynamic Survey On Graph Labeling* we refer to [4] (Gallian .J.A., 2009). Vast amount of literature is available on different types of graph labeling more than 1000 research papers have been published so far in last four decades.

Definition 1.1:

Let $G=(V(G),E(G))$ be a graph with p vertices. A bijection $f:V(G)\rightarrow\{1,2,......p\}$ is called a prime labeling if for each edge $e=uv, \gcd\{f(u),f(v)\}=1$. A graph which admits prime labeling is called a prime graph.

Definition 1.2:

A graph G is said to be a strongly prime graph if for any vertex v of G there exists a prime labeling f satisfying $f(v)=1$.

Definition 1.3:

A graph H is called a super subdivision of a graph G, if every edge uv of G is replaced by $K_{2,m}$ ['m' may vary for each edge] by identifying u and v with the two vertices in $K_{2,m}$ that form one of the two partite sets.

Definition 1.4:

The corona of a graph G on p vertices $v_1,v_2,...v_p$ is the graph obtained from G by adding p new vertices $u_1,u_2,...u_p$ and the new edges u_iv_i for $1\le i\le p$,denoted by $G\odot K_1$ .For a graph G, the 2-Corona of G is the graph obtained from G by identifying the center vertex of the star at S_2 at each vertex of G, denoted by $G\odot S_2$.

Definition 1.5:

The H graph of a path P_n is the graph obtained from two copies of P_n with vertices $u_1,u_2,...u_n$ and $v_1,v_2,...v_n$ by joining the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ if n is odd and the vertices $u_{\frac{n}{2}+1}$ and $v_{\frac{n}{2}}$ if n is even.

Definition 1.6:

Let $G=(V,E)$ be a graph. Let $e=uv$ be an edge of G and w is not a vertex of G. The edge e is subdivided when it is replaced by edges $e=uw$ and $e=wv$. Let $G=(V,E)$ be a graph. If every edge of graph G is subdivided, then the resulting graph is called barycentric subdivision of graph G. In other words barycentric subdivision is the graph obtained by inserting a vertex of degree two into every edge of original graph. Consider barycentric subdivision of cycle and join each newly inserted vertices of incident edges by an edge. We denote the new graph by $C_n(C_n)$ as it look like C_n inscribed in C_n.

Definition 1.7:

The Crown graph C_n^* is obtained from a cycle C_n by attaching a pendent edge at each vertex of the n-cycle.

2.0 Prime Graphs:

Theorem 2.1:

The graph H obtained by Super subdivision of a star graph G where every edge uv of G is replaced by $K_{2,2}$ then H is a prime graph.

Proof:

Let H be the graph obtained by the super subdivision of a star graph G where every edge uv of G is replaced by $K_{2,2}$. Now let G be the star graph with vertices $u_0,u_1,u_2,...u_n$ where u_0 is the centre vertex. Let every edge u_0u_i of G be replaced by v_i and w_i by joining v_iu_0, v_iu_i and w_iu_0, w_iu_i for $1\le i\le n$. Then the required graph H has edge set, $E(H)=\{u_0v_i, u_0w_i /1\le i\le n\}\cup\{u_1v_i, u_1w_i /1\le i\le n\}$

and the vertex set $V(H)=\{u_0,u_1,u_2,...u_n,v_1,v_2,...v_n,w_1,w_2,...w_n\}$

here $|V(H)|=3n+1$ where n is a positive integer.

Define a labeling $f: V(H)\to\{1,2,...3n+1\}$ as follows:

$f(u_0)=1$,

$f(u_i)=3i$, for $1\le i\le n$,

$f(v_i)=3i-1$, for $1\le i\le n$,

$f(w_i)=3i+1$, for $1\le i\le n$,

here $\gcd(f(u_0), f(v_i))=\gcd(1, f(v_i))=1$, for $1\le i\le n$,

$\gcd(f(u_0), f(w_i))=\gcd(1, f(w_i))=1$, for $1\le i\le n$,

$\gcd(f(v_i), f(u_i))=\gcd(3i-1,3i)=1$, for $1\le i\le n$,

and $\gcd(f(w_i), f(u_i))=\gcd(3i+1,3i)=1$. for $1\le i\le n$,

Since both are consecutive numbers.

Clearly vertex labels are distinct.

Thus labeling defined above gives a prime labeling for H. Thus H is a prime graph.

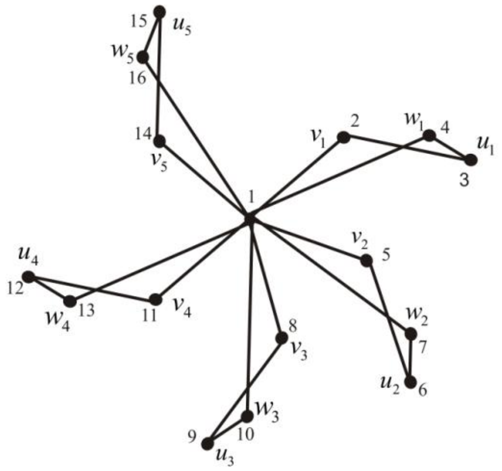

Figure 1: prime labeling of super subdivision of $K_{1,5}$ by $K_{2,2}$

Theorem 2.2:

The graph H obtained by Super subdivision of a star graph G where every edge uv of G is replaced by $K_{2,3}$ is a prime graph.

Proof:

Let H be the graph obtained by the super subdivision of a star graph G where every edge uv of G is replaced by $K_{2,3}$. In G let the vertices be $u_0,u_1,u_2,...u_n$ with u_0 as the centre vertex and every edge u_0u_i of G be replaced by $v_iw_ix_i$ by joining $v_iu_0,v_iu_i,w_iu_0,w_iu_i$, x_iu_0, and x_iu_i for $1\le i\le n$. Then we get the required graph H whose vertex set is $V(H)=\{u_0,u_1,u_2,...u_n,v_1,v_2,...v_n,w_1,w_2,...w_n,x_1,x_2,...x_n\}$ and the edge set is $E(H)=\{u_0v_i,u_0w_i,u_0x_i\,/\,1\le i\le n\}\cup\{u_1v_i,u_1w_i,u_1x_i\,/\,1\le i\le n\}$.

Now $|V(H)|=4n+1$ where n is a positive integer.

Define a labeling $f : V(H) \to \{1, 2, ... 4n+1\}$ as follows:

$f(u_0) = 1$,

$f(u_i) = 4i - 1$, for $1 \le i \le n$,

$f(v_i) = 4i - 2$, for $1 \le i \le n$,

$f(w_i) = 4i$, for $1 \le i \le n$,

$f(x_i) = 4i + 1$, for $1 \le i \le n$,

here $\gcd(f(u_0), f(v_i)) = \gcd(1, f(v_i)) = 1$, for $1 \le i \le n$,

$\gcd(f(u_0), f(w_i)) = \gcd(1, f(w_i)) = 1$, for $1 \le i \le n$,

$\gcd(f(u_0), f(x_i)) = \gcd(1, f(x_i)) = 1$, for $1 \le i \le n$,

$\gcd(f(v_i), f(u_i)) = \gcd(4i - 2, 4i - 1) = 1$, for $1 \le i \le n$,

and $\gcd(f(w_i), f(u_i)) = \gcd(4i, 4i - 1) = 1$, for $1 \le i \le n$,

Since both are consecutive numbers.

$\gcd(f(x_i), f(u_i)) = \gcd(4i + 1, 4i - 1) = 1$, for $1 \le i \le n$,

Since it is a consecutive odd numbers.

Clearly vertex labels are distinct.

Thus labeling defined above gives a prime labeling for H . Thus H is a prime graph.

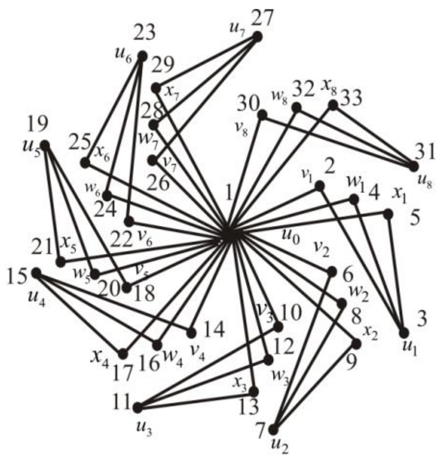

Figure 2: Prime labeling of super subdivision of $K_{1,8}$ by $K_{2,3}$

Theorem 2.3:

The graph G obtained by attaching $K_{1,3}$ at each vertex of a cycle C_n is a prime graph.

Proof:

Let C_n be the cycle $u_1, u_2, ... u_n, u_1$. Let v_i, x_i, y_i, z_i be the vertices of ith copy of $K_{1,3}$ in which v_i is the central vertex. Identify z_i with u_i , $1 \le i \le n$. Let the resultant graph be G.

Now the vertex set of G is $\{u_1, u_2, ... u_n, v_1, v_2, ... v_n, x_1, x_2, ... x_n, y_1, y_2, ... y_n\}$.

The edge set of G is $E(G)=\{u_iu_{i+1}/1\le i\le n-1\}\cup\{u_nu_1\}\cup\{u_iv_i, x_iv_i, y_iv_i /1\le i\le n\}$

here $|V(G)|=4n$

Define a function $f: \mathrm{V}(G)\to\{1,2,3,\ldots 4\mathrm{n}\}$ by

$f(\mathrm{u}_i)=4i-3$, for $1\le i\le n$,

$f(v_i)=4i-1$, for $1\le i\le n$,

$f(x_i)=4i-2$, for $1\le i\le n$,

$f(y_i)=4i$, for $1\le i\le n$,

here $\gcd(f(u_i), f(u_{i+1}))=\gcd(4i-3,4i+1)=1$, for $1\le i\le n-1$,

as these two numbers are odd and their difference is 4

$\gcd(f(u_i), f(v_i))=\gcd(4i-3,4i-1)=1$, for $1\le i\le n-1$

Since it is consecutive odd numbers.

$\gcd(f(v_i), f(x_i))=\gcd(4i-1,4i-2)=1$, for $1\le i\le n-1$,

$\gcd(f(v_i), f(y_i))=\gcd(4i-1,4i)=1$, for $1\le i\le n-1$,

Since both are consecutive positive integers.

$\gcd(f(u_1), f(u_n))=\gcd(1, f(u_n))=1.$

Clearly vertex labels are distinct.

Thus the labeling defined above gives a prime labeling for G. Thus G is a prime graph.

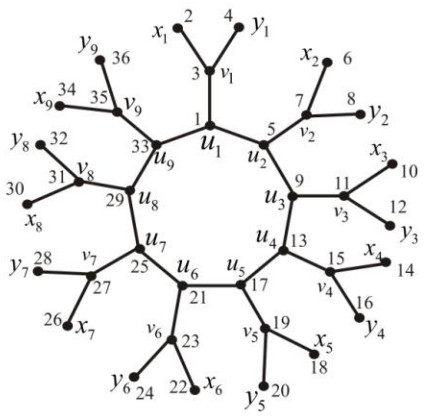

Figure 3: Prime labeling of C_9 with $K_{1,3}$

Theorem 2.4:

The graph $G\odot K_1$ is a prime graph where G is a $C_n(C_n)$ graph.

Proof:

Let G be the graph with vertices $u_1,u_2,\ldots u_n,\mathrm{v}_1,\mathrm{v}_2,\ldots\mathrm{v}_n$. Let $u_1^{'},u_2^{'},\ldots u_n^{'}$ and $\mathrm{v}_1^{'},\mathrm{v}_2^{'},\ldots\mathrm{v}_n^{'}$ be the corresponding new vertices, join $u_iu_i^{'}$ and $v_iv_i^{'}$ in G. we get the graph G_1 i.e., $G\odot K_1$ where $G=C_n(C_n)$.

Now the vertex set of G_1 is $\{u_1,u_2,\ldots u_n,v_1,v_2,\ldots v_n,u_1^{'},u_2^{'},\ldots u_n^{'},v_1^{'},v_2^{'},\ldots v_n^{'}\}$.

The edge set $E(G_1) = \{v_i v_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i, u_i u_i^{'}, v_i v_i^{'} / 1 \le i \le n\} \cup \{v_i u_{i+1} / 1 \le i \le n-1\}$.

Then $|V(G_1)| = 4n$.

Define a labeling $f : V(G_1) \to \{1, 2, 3, \ldots 4n\}$ by

$f(u_1) = 4n - 1$,

$f(u_1^{'}) = 4n$,

$f(u_i) = 4i - 5$, for $2 \le i \le n$,

$f(u_i^{'}) = 4i - 4$, for $2 \le i \le n$,

$f(v_i) = 4i - 3$, for $1 \le i \le n$,

$f(v_i^{'}) = 4i - 2$, for $1 \le i \le n$.

here $\gcd(f(v_i), f(v_i^{'})) = \gcd(4i-3, 4i-2) = 1$, for $1 \le i \le n-1$,

$\gcd(f(u_i), f(u_i^{'})) = \gcd(4i-5, 4i-4) = 1$, for $2 \le i \le n-1$,

Since these are consecutive positive integers.

$\gcd(f(v_i), f(v_{i+1})) = \gcd(4i-3, 4i+1) = 1$, for $1 \le i \le n-1$,

as these two numbers are odd and their difference is 4

$\gcd(f(v_i), f(u_{i+1})) = \gcd(4i-3, 4i-1)$, for $1 \le i \le n-1$,

$\gcd(f(u_i), f(v_i)) = \gcd(4i-5, 4i-3)$, for $2 \le i \le n-1$,

$\gcd(f(v_n), f(u_1)) = 1$,

Since these are consecutive odd integers.

Clearly vertex labels are distinct.

Thus labeling defined above gives a prime labeling for G_1. Thus $G \odot K_1$ is a prime graph.

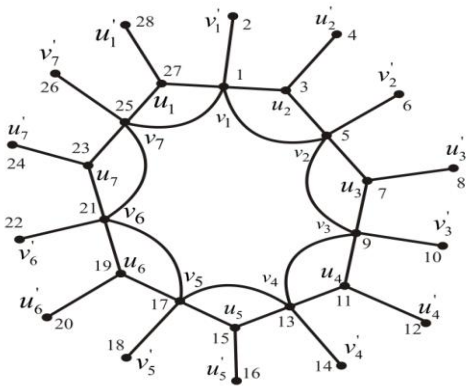

Figure 4: : Prime labeling of $G_1 = C_7(C_7) \odot K_1$

Theorem 2.5:

The graph $G\square\, S_2$ is a prime graph, where G is a H-graph.

Proof:

Let G be the graph with vertices $u_1,u_2,...u_n,\mathrm{v}_1,\mathrm{v}_2,...\mathrm{v}_n$. Let $u_1^{'},u_2^{'},...u_n^{'}\ u_1^{''},u_2^{''},...u_n^{''}$, and $v_1^{'},v_2^{'},...v_n^{'},v_1^{''},v_2^{''},...v_n^{''}$ be the corresponding new vertices, join $u_iu_i^{'}$, $u_iu_i^{''}$ and $v_iv_i^{'}$, $v_iv_i^{''}$ in G. we get the graph G_1 i.e., $G\square\, S_2$ where G is a H-graph Now the vertex set of G_1 is $\{u_1,u_2,...u_n,v_1,v_2,...v_n,u_1^{'},u_2^{'},...u_n^{'},u_1^{''},u_2^{''},...u_n^{''},v_1^{'},v_2^{'},...v_n^{'},v_1^{''},v_2^{''},...v_n^{''}\}$.

The edge set $E(G_1)=\{\mathrm{u}_i\, u_{i+1}, v_iv_{i+1} / 1\le i\le n-1\}\cup\{\mathrm{u}_i\, u_i^{'},\mathrm{u}_i\, u_i^{''},\mathrm{v}_i\, v_i^{'},\mathrm{v}_i\, v_i^{''} / 1\le i\le n\}$

$$\cup\left\{u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}\ \ if\ \ n\ is\ odd\right\}\text{ (or) }\cup\left\{u_{\frac{n}{2}+1}v_{\frac{n}{2}}\ \ if\ \ n\ is\ even\right\}.$$

here $V(G_1)=6n$.

Define a labeling $f:\mathrm{V}(G_1)\to\{1,2,3,...6\,\mathrm{n}\}$ by considering the following cases:

Case (i): When n is odd.

$f(\mathrm{u}_i)=3i+2$, for $1\le i\le n$,

$f(u_i^{'})=3i+1$, for $1\le i\le n$,

$f(u_i^{''})=3i+3$, for $1\le i\le n$,

$f(v_i)=3n+3i+2$, for $1\le i<\frac{n+1}{2}$,

$f(v_i)=3n+3i-1$, for $\frac{n+1}{2}<i\le n$,

$f(v_i^{'})=3n+3i+1$, for $1\le i<\frac{n+1}{2}$,

$f(v_i^{'})=3n+3i-2$, for $\frac{n+1}{2}<i\le n$,

$f(v_i^{''})=3n+3i+3$, for $1\le i<\frac{n+1}{2}$,

$f(v_i^{''})=3n+3i$, for $\frac{n+1}{2}<i\le n$,

$$f\left(v_{\frac{n+1}{2}}\right)=1,$$

$$f\left(v_{\frac{n+1}{2}}^{'}\right)=2,$$

$$f\left(v_{\frac{n+1}{2}}^{''}\right)=3,$$

here $\gcd(f(u_i),f(u_i^{'}))=\gcd(3i+2,3i+1)=1$, for $1\le i\le n$,

$\gcd(f(u_i), f(u_i'')) = \gcd(3i+2, 3i+3) = 1$, for $1 \le i \le n$,

Since these are consecutive positive integers.

$$\gcd\left(f\left(v_{\frac{n+1}{2}}\right), f\left(u_{\frac{n+1}{2}}\right)\right) = \gcd\left(1, f\left(u_{\frac{n+1}{2}}\right)\right) = 1,$$

$$\gcd\left(f\left(v_{\frac{n+1}{2}}\right), f\left(v_{\frac{n-1}{2}}\right)\right) = \gcd\left(1, f\left(v_{\frac{n-1}{2}}\right)\right) = 1,$$

$$\gcd\left(f\left(v_{\frac{n+1}{2}}\right), f\left(v_{\frac{n+3}{2}}\right)\right) = \gcd\left(1, f\left(v_{\frac{n-1}{2}}\right)\right) = 1,$$

$\gcd(f(u_i), f(u_{i+1})) = \gcd(3i+2, 3i+5) = 1$, for $1 \le i \le n-1$,

$\gcd(f(v_i), f(v_{i+1})) = \gcd(3n+3i+2, 3n+3i+5) = 1$, for $1 \le i < \frac{n-3}{2}$,

$\gcd(f(v_i), f(v_{i+1})) = \gcd(3n+3i-1, 3n+3i+2) = 1$, for $\frac{n+3}{2} < i \le n-1$,

as one of these numbers is even then the other number is odd. Also the difference of these two numbers is 3.

$\gcd(f(v_i), f(v_i')) = 1$, for $1 \le i \le n$,

$\gcd(f(v_i), f(v_i'')) = 1$, for $1 \le i \le n$,

Since they are consecutive integers.

Clearly vertex labels are distinct.

Thus labeling defined above gives a prime labeling for a graph G_1.

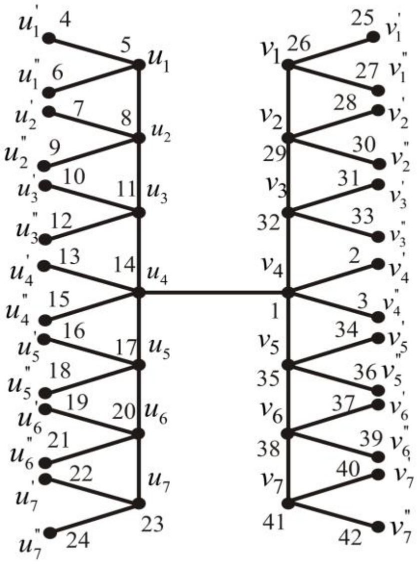

Figure 5: Prime labeling of $G_1 = H_7 \odot S_2$

Case (ii): When n is even.

If n is even then we join the vertices $u_{\frac{n}{2}+1}$ and $v_{\frac{n}{2}}$ in two copies of path P_n .Then let f_2 be the labeling obtained from f in case (i) by changing the labels $f\left(v_{\frac{n}{2}}\right)=1, f\left(v'_{\frac{n}{2}}\right)=2, f\left(v''_{\frac{n}{2}}\right)=3$,

$f(v_i) = 3n + 3i + 2$, for $1 \le i < \frac{n}{2}$,

$f(v_i) = 3n + 3i - 1$, for $\frac{n}{2} < i \le n$,

$f(v'_i) = 3n + 3i + 1$, for $1 \le i < \frac{n}{2}$,

$f(v'_i) = 3n + 3i - 2$, for $\frac{n}{2} < i \le n$,

$f(v''_i) = 3n + 3i + 3$, for $1 \le i < \frac{n}{2}$,

$f(v''_i) = 3n + 3i$, for $\frac{n}{2} < i \le n$,

and for all other remaining vertices $f_2(v) = f(v)$. Then the resulting labeling f_2 is a prime labeling.

Thus labeling defined above gives a prime labeling for a graph G_1 .

Thus $G\square S_2$ is a prime graph.

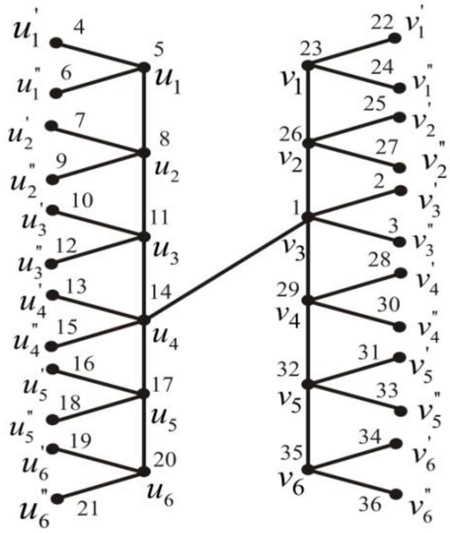

Figure 6: Prime labeling of $G_1 = H_6\square S_2$

3.0 Strongly Prime Graphs:

Theorem 3.1:

The Crown graph C_n^* is a strongly prime graph.

Proof:

Let C_n^* be the crown graph with vertices $v_1, v_2, ... v_n, v'_1, v'_2, ... v'_n$. Let $E(C_n^*)$ be the edges of the crown graph where, $E(C_n^*) = \{v_i v'_i / 1 \le i \le n\} \cup \{v_i v_{i+1} / 1 \le i \le n-1\} \cup \{v_1 v_n\}$.

Here $V\left|C_n^*\right| = 2n$, where n is a positive integer.

If v is any arbitrary vertex of C_n^* then we have the following possibilities.

Case (i): When v is of degree 3.

If $v = v_j$ for some $j \in \{1,2,3,...n\}$ then the function $f : V(C_n^*) \to \{1,2,3,...2n\}$ defined by

$$f(v_i) = \begin{cases} 2n + 2i - 2j + 1, & if \ \ i = 1,2,...j-1; \\ 2i - 2j + 1, & if \ \ i = j+1, j+2,...n, \end{cases}$$

$f(v_j) = 1$,

$$f(v_i') = \begin{cases} 2n + 2i - 2j + 2, & if \ \ i = 1,2,...j-1; \\ 2i - 2j + 2, & if \ \ i = j+1, j+2,...n, \end{cases}$$

$f(v_j') = 2$.

is a prime labeling for C_n^* with $f(v) = f(v_j) = 1$. Thus f admits prime labeling as well as it is possible to assign label 1 to any arbitrary vertex of degree 3 in C_n^*.

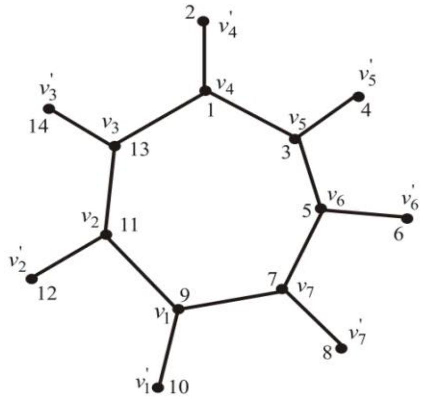

Figure 7: Strongly Prime labeling of C_7^* when $j = 4$, $v_j = 1$

Case (ii): When v is of degree 1.

Let $v = v_j'$ for some $j \in \{1,2,3,...n\}$ .Then let f_2 be the labeling obtained from f in case (i) by interchanging the labels $f(v_j)$ and $f(v_j')$ and for all other remaining vertices $f_2(v) = f(v)$. Then the resulting labeling f_2 is a prime labeling as well as it is possible to assign label 1 to any arbitrary vertex of C_n^*. Thus from all the cases described above C_n^* is a strongly prime graph.

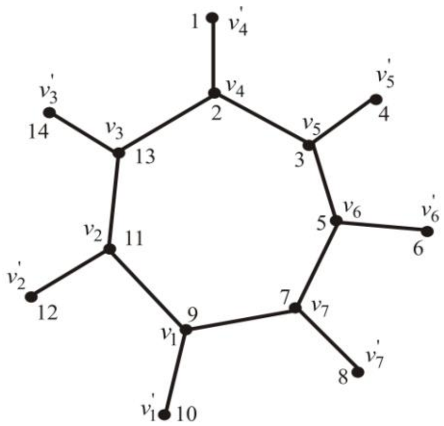

Figure 8: case(ii) Strongly Prime labeling of C_7^* when $j=4, v_j'=1$

Theorem 3.2:

The graph $G \odot S_2$ is a Strongly prime graph where G is a cycle with n vertices.

Proof:

Let G be the graph with vertices $v_1, v_2, \ldots v_n$. Let $v_1', v_2', \ldots v_n'$ and $v_1'', v_2'', \ldots v_n''$ be the corresponding new vertices, join $v_i v_i'$ and $v_i v_i''$ in G. we get the graph G_1 ie., $G \odot S_2$ graph. Now the vertex set of G_1 is $\{v_1, v_2, \ldots v_n, v_1', v_2', \ldots v_n', v_1'', v_2'', \ldots v_n''\}$. The edge set is, $E(G_1) = \{v_i v_i' / 1 \le i \le n\} \cup \{v_i v_i'' / 1 \le i \le n\} \cup \{v_i v_{i+1} / i = 1, 2, \ldots n-1\} \cup \{v_n v_1\}$.

Here $|V(G_1)| = 3n$. If v is any arbitrary vertex of G_1 then we have the following possibilities.

Case (i): When n is even in G_1.

Sub case (i): If $v = v_j'$ for some $j \in \{1, 2, 3, \ldots n\}$ then the function $f : V(G_1) \to \{1, 2, 3, \ldots 3n\}$ defined by

$$f(v_i) = \begin{cases} 3n + 3i - 3j + 2, & \text{if } i = 1, 2, \ldots j-1; \\ 3i - 3j + 2, & \text{if } i = j+1, j+2, \ldots n, \end{cases}$$

$$f(v_i') = \begin{cases} 3n + 3i - 3j + 1, & \text{if } i = 1, 2, \ldots j-1; \\ 3i - 3j + 1, & \text{if } i = j+1, j+2, \ldots n, \end{cases}$$

$$f(v_i'') = \begin{cases} 3n + 3i - 3j + 3, & \text{if } i = 1, 2, \ldots j-1; \\ 3i - 3j + 3, & \text{if } i = j+1, j+2, \ldots n, \end{cases}$$

$$f(v_j) = 2,$$

$$f(v_j') = 1,$$

$$f(v_j'') = 3,$$

is a prime labeling for G_1 with $f(v) = f(v_j') = 1$. Thus f admits prime labeling as well as it is possible to assign label 1 to any arbitrary vertex of v_j' in G_1.

Subcase (ii): If $v = v''_j$ for some $j \in \{1,2,3,...n\}$.Then let f_2 be the labeling obtained from f in subcase (i) by interchanging the labels $f(v'_j)$ and $f(v''_j)$ and for all other remaining vertices $f_2(v) = f(v)$. Then f_2 is a prime labeling as well as it is possible to assign label 1 to any arbitrary vertex of v''_j in G_1.

Subcase (iii): If $v = v_j$ for some $j \in \{1,2,3,...n\}$.Then let f_3 be the labeling obtained from f in subcase (i) by interchanging the labels $f(v'_j)$ and $f(v_j)$ and for all other remaining vertices $f_3(v) = f(v)$. Then f_3 is a prime labeling as well as it is possible to assign label 1 to any arbitrary vertex of v_j in G_1.Thus from all the cases described above gives G_1 is a strongly prime graph when n is even.

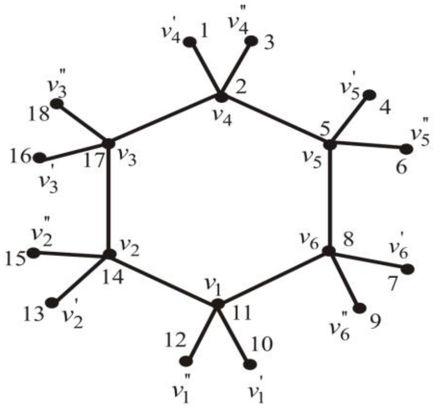

Figure 8: Subase(i) Strongly Prime labeling of $G \square S_2$ where $G = C_6$ here $j = 4, v'_j = 1$

Case (ii): When n is odd in G_1.

If $v = v'_j$ for some $j \in \{1,2,3,...n\}$. Then let f_4 be the labeling obtained from f in subcase (i) by interchanging the labels $f(v_{j-1})$ and $f(v''_{j-1})$ and for all other remaining vertices $f_4(v) = f(v)$.

If $v = v''_j$ for some $j \in \{1,2,3,...n\}$. Then let f_5 be the labeling obtained from f_2 in subcase (ii) by interchanging the labels $f(v_{j-1})$ and $f(v''_{j-1})$ and for all other remaining vertices $f_5(v) = f_2(v)$.

If $v = v_j$ for some $j \in \{1,2,3,...n\}$ then label the vertices as same as subcase (iii).

Thus from all the cases described above G_1 is a strongly prime graph for all n

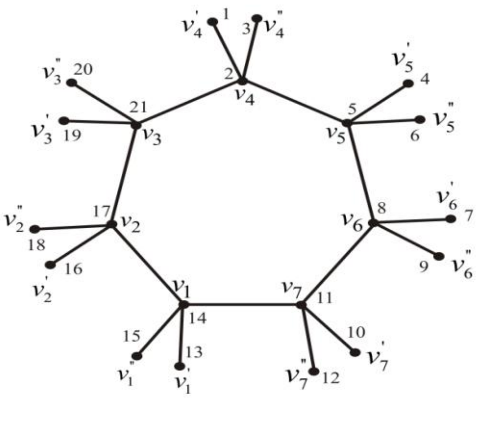

Figure 9: Case(ii) Strongly Prime labeling of $G \square S_2$ where $G = C_7$ here $j = 4, v'_j = 1$

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