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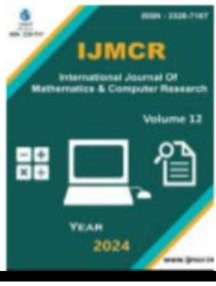

Nirmala Alpha Gourava and Modified Nirmala Alpha Gourava Indices of Certain Dendrimers

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ARTICLE INFO	ABSTRACT
Published Online: 24 May 2024 Corresponding Author: **V.R.Kulli**	In this paper, we introduce the Nirmalaalpha Gourava and modified Nirmala alpha Gourava indices and their corresponding exponentials of a graph. Also we compute theNirmala alpha Gourava and modified Nirmala alpha Gourava indices of four families of dendrimers.

KEYWORDS: Nirmala alpha Gourava index, modified Nirmala alpha Gourava index, dendrimer.

I. INTRODUCTION

The graph $G = (V(G), E(G))$ is a simple connected graph with vertex set $V(G)$ and edge set $E(G)$. For undefined term and notation, we refer the book [1].

A molecular graph is a graph in which the vertices correspond to the atoms and the edges to the bonds of a molecule. Several topological or graph indices [2] have been considered in Chemical Science and have found some applications, in *QSPR/QSAR* study, see [3, 4].

The Nirmala index [5] of a molecular graph G is defined as

$$N(G) = \sum_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)}.$$

Recently, some Nirmala indices were studied in [6-20].

The first alpha Gourava index was defined by Kulli in [21] as

$$AGO(G) = \sum_{uv \in E(G)} [d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)].$$

Recently, some Gourava indices were studied in [22-39].

Motivated by Nirmala and first alpha Gourava indices, we introduce the Nirmala alpha Gourava index of a graph G as follows:

The Nirmala alpha Gourava index of a molecular graph G is defined as

$$NAGO(G) = \sum_{uv \in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)}.$$

Considering the Nirmala alpha Gourava index, we define the Nirmala alpha Gourava exponential of a graph G as

$$NAGO(G, x) = \sum_{uv \in E(G)} x^{\sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)}}.$$

We define the modified Nirmala alpha Gourava index of a graph G as

$$^{m}NAGO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)}}.$$

Considering the modified Nirmala alpha Gourava index, we introduce the modified Nirmala alpha Gourava exponential of a graph G and defined it as

$$^{m}NAGO(G, x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)}}}.$$

Recently, some topological indices were studied in [40, 41, 42, 43].

In this study, we compute the Nirmala alpha Gourava and modified Nirmala alpha Gourava indices of four families of dendrimers.

2. RESULTS FOR PORPHYRIN DENDRIMER D_NP_N

We consider the family of porphyrin dendrimers. This family of dendrimers is denoted by D_nP_n. The molecular graph of D_nP_n is shown in Figure 1.

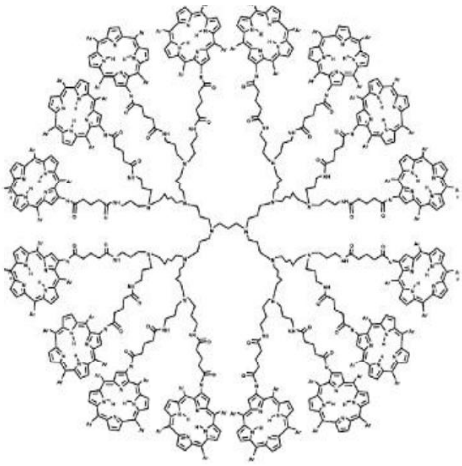

Figure 1. The molecular graph of D_nP_n

Let G be the molecular graph of D_nP_n. By calculation, we find that G has $96n - 10$ vertices and $105n - 11$ edges. In D_nP_n, there are six types of edges based on degrees of end vertices of each edge as given in Table 1.

Table 1. Edge partition of D_nP_n

$d_G(u), d_G(v) \backslash uv \in E(G)$	Number of edges
(1, 3)	$2n$
(1, 4)	$24n$
(2, 2)	$10n - 5$
(2, 3)	$48n - 6$
(3, 3)	$13n$
(3, 4)	$8n$

Theorem 1. Let D_nP_n be the family of porphyrin dendrimers. Then

$$NAGO(G) = 2n\sqrt{13} + 24n\sqrt{21} + 10n\sqrt{12}.$$
$$+48n\sqrt{19} + 13n\sqrt{27} + 8n\sqrt{37} - 5\sqrt{12} - 6\sqrt{19}.$$

Proof: From definition and by using Table 1, we deduce

$$NAGO(G)$$
$$= \sum_{uv \in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)}$$
$$= 2n\sqrt{1^2 + 3^2 + (1\times 3)} + 24n\sqrt{1^2 + 4^2 + (1\times 4)}$$
$$+(10n-5)\sqrt{2^2 + 2^2 + (2\times 2)}$$
$$+(48n-6)\sqrt{2^2 + 3^2 + (2\times 3)}$$
$$+13n\sqrt{3^2 + 3^2 + (3\times 3)} + 8n\sqrt{3^2 + 4^2 + (3\times 4)}$$
$$= 2n\sqrt{13} + 24n\sqrt{21} + 10n\sqrt{12} + 48n\sqrt{19}$$
$$+13n\sqrt{27} + 8n\sqrt{37} - 5\sqrt{12} - 6\sqrt{19}.$$

Theorem 2. Let D_nP_n be the family of porphyrin dendrimers. Then

$$NAGO(G,x) = 2nx^{\sqrt{13}} + 24nx^{\sqrt{21}}$$
$$+(10n-5)x^{\sqrt{12}} + (48n-6)x^{\sqrt{19}} + 13nx^{\sqrt{27}} + 8nx^{\sqrt{37}}.$$

Proof: From definition and by using Table 1, we deduce

$$NAGO(G,x) = \sum_{uv \in E(G)} x^{\sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)}}$$
$$= 2nx^{\sqrt{1^2+3^2+(1\times 3)}} + 24nx^{\sqrt{1^2+4^2+(1\times 4)}}$$
$$+(10n-5)x^{\sqrt{2^2+2^2+(2\times 2)}} + (48n-6)x^{\sqrt{2^2+3^2+(2\times 3)}}$$
$$+13nx^{\sqrt{3^2+3^2+(3\times 3)}} + 8nx^{\sqrt{3^2+4^2+(3\times 4)}}$$
$$= 2nx^{\sqrt{13}} + 24nx^{\sqrt{21}} + (10n-5)x^{\sqrt{12}}$$
$$+(48n-6)x^{\sqrt{19}} + 13nx^{\sqrt{27}} + 8nx^{\sqrt{37}}.$$

Theorem 3. The modified Nirmala alpha Gourava index of D_nP_n is

$$^{m}NAGO(G) = \frac{2n}{\sqrt{13}} + \frac{24n}{\sqrt{21}} + \frac{10n-5}{\sqrt{12}}$$
$$+ \frac{48n-6}{\sqrt{19}} + \frac{13n}{\sqrt{27}} + \frac{8n}{\sqrt{37}}.$$

Proof: We have

$$^{m}NAGO(G)$$
$$= \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)}}$$
$$= \frac{2n}{\sqrt{1^2+3^2+(1\times 3)}} + \frac{24n}{\sqrt{1^2+4^2+(1\times 4)}}$$
$$+ \frac{10n-5}{\sqrt{2^2+2^2+(2\times 2)}} + \frac{48n-6}{\sqrt{2^2+3^2+(2\times 3)}}$$
$$+ \frac{13n}{\sqrt{3^2+3^2+(3\times 3)}} + \frac{8n}{\sqrt{3^2+4^2+(3\times 4)}}.$$
$$= \frac{2n}{\sqrt{13}} + \frac{24n}{\sqrt{21}} + \frac{10n-5}{\sqrt{12}} + \frac{48n-6}{\sqrt{19}} + \frac{13n}{\sqrt{27}} + \frac{8n}{\sqrt{37}}.$$

Theorem 4. The modified Nirmala alpha Gourava exponential of D_nP_n is given by

$$^{m}NAGO(G,x) = 2nx^{\frac{1}{\sqrt{13}}} + 24nx^{\frac{1}{\sqrt{21}}} + (10n-5)x^{\frac{1}{\sqrt{12}}}$$
$$+(48n-6)x^{\frac{1}{\sqrt{19}}} + 13nx^{\frac{1}{\sqrt{27}}} + 8nx^{\frac{1}{\sqrt{37}}}.$$

Proof: We have

$$^{m}NAGO(G,x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)}}}$$

$$= 2nx^{\frac{1}{\sqrt{1^2+3^2+(1\times3)}}} + 24nx^{\frac{1}{\sqrt{1^2+4^2+(1\times4)}}}$$
$$+(10n-5)x^{\frac{1}{\sqrt{2^2+2^2+(2\times2)}}} + (48n-6)x^{\frac{1}{\sqrt{2^2+3^2+(2\times3)}}}$$
$$+13nx^{\frac{1}{\sqrt{3^2+3^2+(3\times3)}}} + 8nx^{\frac{1}{\sqrt{3^2+4^2+(3\times4)}}}$$
$$= 2nx^{\frac{1}{\sqrt{13}}} + 24nx^{\frac{1}{\sqrt{21}}} + (10n-5)x^{\frac{1}{\sqrt{12}}}$$
$$+(48n-6)x^{\frac{1}{\sqrt{19}}} + 13nx^{\frac{1}{\sqrt{27}}} + 8nx^{\frac{1}{\sqrt{37}}}.$$

3. RESULTS FOR PROPYL ETHER IMINE DENDRIMER *PETIM*

We consider the family of propyl ether imine dendrimers. This family of dendrimers is denoted by *PETIM*. The molecular graph of *PETIM* is depicted in Figure 2.

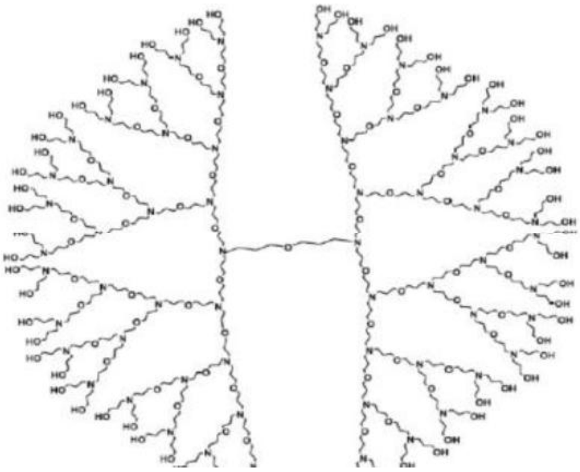

Figure 2. The molecular graph of *PETIM*

Let *G* be the molecular graph of *PETIM*. By calculation, we find that *G* has $24\times2^n - 23$ vertices and $24\times2^n - 24$ edges. In *PETIM*, there are three types of edges based on degrees of end vertices of each edge as given in Table 2.

Table 2. Edge partition of *PETIM*

$d_G(u)$, $d_G(v) \backslash uv\in E(G)$	Number of edges
(1, 2)	2×2^n
(2, 2)	$16 \times 2^n - 18$
(2, 3)	$6 \times 2^n - 6$

Theorem 5. Let *PETIM* be the family of propyl ether imine dendrimers. Then

$$NAGO(G) = 2\times2^n\sqrt{7} + 16\times2^n\sqrt{12}$$
$$+6\times2^n\sqrt{19} - 18\sqrt{12} - 6\sqrt{19}.$$

Proof: From definition and by using Table 2, we deduce

$$NAGO(G)$$
$$= \sum_{uv\in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)}$$
$$= 2\times2^n\sqrt{1^2+2^2+(1\times2)}$$
$$+(16\times2^n-18)\sqrt{2^2+2^2+(2\times2)}$$
$$+(6\times2^n-6)\sqrt{2^2+3^2+(2\times3)}$$
$$= 2\times2^n\sqrt{7} + 16\times2^n\sqrt{12}$$
$$+6\times2^n\sqrt{19} - 18\sqrt{12} - 6\sqrt{19}.$$

Theorem 6. Let *PETIM* be the family of propyl ether imine dendrimers. Then

$$NAGO(G,x) = 2\times2^n x^{\sqrt{7}} + 16\times2^n x^{\sqrt{12}}$$
$$+6\times2^n x^{\sqrt{19}} - 18x^{\sqrt{12}} - 6x^{\sqrt{19}}.$$

Proof: From definition and by using Table 2, we deduce

$$NAGO(G,x) = \sum_{uv\in E(G)} x^{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}$$
$$= 2\times2^n x^{\sqrt{1^2+2^2+(1\times2)}}$$
$$+(16\times2^n-18)x^{\sqrt{2^2+2^2+(2\times2)}}$$
$$+(6\times2^n-6)x^{\sqrt{2^2+3^2+(2\times3)}}$$
$$= 2\times2^n x^{\sqrt{7}} + 16\times2^n x^{\sqrt{12}}$$
$$+6\times2^n x^{\sqrt{19}} - 18x^{\sqrt{12}} - 6x^{\sqrt{19}}.$$

Theorem 7. The modified Nirmala alpha Gourava index of *PETIM* is

$${}^{m}NAGO(G) = \frac{2\times2^n}{\sqrt{7}} + \frac{16\times2^n}{\sqrt{12}}$$
$$+\frac{6\times2^n}{\sqrt{19}} - \frac{18}{\sqrt{12}} - \frac{6}{\sqrt{19}}.$$

Proof: We have

$${}^{m}NAGO(G)$$
$$= \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}$$
$$= \frac{2\times2^n}{\sqrt{1^2+2^2+(1\times2)}} + \frac{16\times2^n-18}{\sqrt{2^2+2^2+(2\times2)}}$$
$$+\frac{6\times2^n-6}{\sqrt{2^2+3^2+(2\times3)}}$$

$$= \frac{2\times2^n}{\sqrt{7}} + \frac{16\times2^n}{\sqrt{12}} + \frac{6\times2^n}{\sqrt{19}} - \frac{18}{\sqrt{12}} - \frac{6}{\sqrt{19}}.$$

Theorem 8. The modified Nirmala alpha Gourava exponential of *PETIM* is given by

$$ {}^{m}NAGO(G,x) = 2\times 2^{n} x^{\frac{1}{\sqrt{7}}} + (16\times 2^{n} - 18) x^{\frac{1}{\sqrt{12}}} + (6\times 2^{n} - 6) x^{\frac{1}{\sqrt{19}}}. $$

Proof: We have

$$ {}^{m}NAGO(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}} $$

$$ = 2\times 2^{n} x^{\frac{1}{\sqrt{1^2+2^2+(1\times 2)}}} + (16\times 2^{n} - 18) x^{\frac{1}{\sqrt{2^2+2^2+(2\times 2)}}} + (6\times 2^{n} - 6) x^{\frac{1}{\sqrt{2^2+3^2+(2\times 3)}}} $$

$$ = 2\times 2^{n} x^{\frac{1}{\sqrt{7}}} + (16\times 2^{n} - 18) x^{\frac{1}{\sqrt{12}}} + (6\times 2^{n} - 6) x^{\frac{1}{\sqrt{19}}}. $$

4. Results for Poly Ethylene Amide Amine Dendrimer *PETAA*

We consider the family of poly ethylene amide amine dendrimers. This family of dendrimers is denoted by *PETAA*. The molecular graph of *PETAA* is presented in Figure 3.

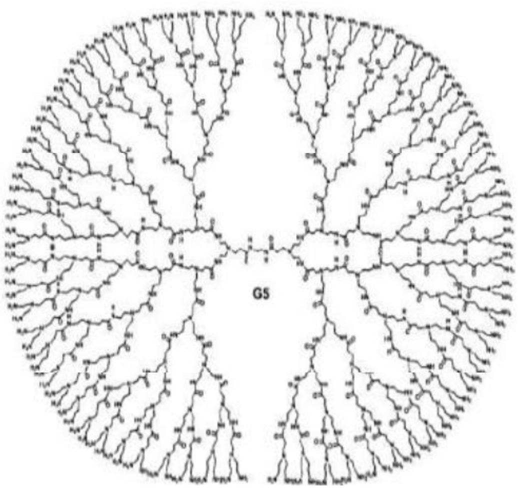

Figure 3. The molecular graph of *PETAA*

Let G be the molecular graph of *PETAA*. By calculation, we find that G has $44\times 2^n - 18$ vertices and $44\times 2^n - 19$ edges. In *PETAA*, there are three types of edges based on degrees of end vertices of each edge as given in Table 3.

Table 3. Edge partition of *PETAA*

$d_G(u), d_G(v) \backslash uv \in E(G)$	Number of edges
(1, 2)	4×2^n
(1, 3)	$4\times 2^n - 2$
(2, 2)	$16\times 2^n - 8$
(2, 3)	$20\times 2^n - 9$

Theorem 9. Let *PETAA* be the family of poly ethylene amide amine dendrimers. Then

$$ NAGO(G) = 4\times 2^{n}\sqrt{7} + 4\times 2^{n}\sqrt{13} + 16\times 2^{n}\sqrt{12} + 20\times 2^{n}\sqrt{19} - 2\sqrt{13} - 8\sqrt{12} - 9\sqrt{19}. $$

Proof: From definition and by using Table 3, we deduce

$$ NAGO(G) = \sum_{uv\in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)} $$

$$ = 4\times 2^{n}\sqrt{1^2+2^2+(1\times 2)} + (4\times 2^{n} - 2)\sqrt{1^2+3^2+(1\times 3)} + (16\times 2^{n} - 8)\sqrt{2^2+2^2+(2\times 2)} + (20\times 2^{n} - 9)\sqrt{2^2+3^2+(2\times 3)} $$

$$ = 4\times 2^{n}\sqrt{7} + 4\times 2^{n}\sqrt{13} + 16\times 2^{n}\sqrt{12} + 20\times 2^{n}\sqrt{19} - 2\sqrt{13} - 8\sqrt{12} - 9\sqrt{19}. $$

Theorem 10. Let *PETAA* be the family of poly ethylene amide amine dendrimers. Then

$$ NAGO(G,x) = 4\times 2^{n} x^{\sqrt{7}} + (4\times 2^{n} - 2) x^{\sqrt{13}} + (16\times 2^{n} - 8) x^{\sqrt{12}} + (20\times 2^{n} - 9) x^{\sqrt{19}}. $$

Proof: From definition and by using Table 3, we deduce

$$ NAGO(G,x) = \sum_{uv\in E(G)} x^{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}} $$

$$ = 4\times 2^{n} x^{\sqrt{1^2+2^2+(1\times 2)}} + (4\times 2^{n} - 2) x^{\sqrt{1^2+3^2+(1\times 3)}} + (16\times 2^{n} - 8) x^{\sqrt{2^2+2^2+(2\times 2)}} + (20\times 2^{n} - 9) x^{\sqrt{2^2+3^2+(2\times 3)}} $$

$$ = 4\times 2^{n} x^{\sqrt{7}} + (4\times 2^{n} - 2) x^{\sqrt{13}} + (16\times 2^{n} - 8) x^{\sqrt{12}} + (20\times 2^{n} - 9) x^{\sqrt{19}}. $$

Theorem 11. The modified Nirmala alpha Gourava index of *PETAA* is

$$ {}^{m}NAGO(G) = \frac{4\times 2^{n}}{\sqrt{7}} + \frac{4\times 2^{n} - 2}{\sqrt{13}} + \frac{16\times 2^{n} - 8}{\sqrt{12}} + \frac{20\times 2^{n} - 9}{\sqrt{19}}. $$

Proof: We have

$$ {}^{m}NAGO(G) $$

$$= \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)}}$$

$$= \frac{4\times 2^n}{\sqrt{1^2+2^2+(1\times 2)}} + \frac{4\times 2^n - 2}{\sqrt{1^2+3^2+(1\times 3)}}$$

$$+ \frac{16\times 2^n - 8}{\sqrt{2^2+2^2+(2\times 2)}} + \frac{20\times 2^n - 9}{\sqrt{2^2+3^2+(2\times 3)}}$$

$$= \frac{4\times 2^n}{\sqrt{7}} + \frac{4\times 2^n - 2}{\sqrt{13}} + \frac{16\times 2^n - 8}{\sqrt{12}} + \frac{20\times 2^n - 9}{\sqrt{19}}.$$

Theorem 12. The modified Nirmala alpha Gourava exponential of *PETAA* is given by

$$^{m}NAGO(G,x) = 4\times 2^n x^{\frac{1}{\sqrt{7}}} + (4\times 2^n - 2)x^{\frac{1}{\sqrt{13}}}.$$

$$+(16\times 2^n - 8)x^{\frac{1}{\sqrt{12}}} + (20\times 2^n - 9)x^{\frac{1}{\sqrt{19}}}.$$

Proof: We have

$$^{m}NAGO(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}}$$

$$= 4\times 2^n x^{\frac{1}{\sqrt{1^2+2^2+(1\times 2)}}} + (4\times 2^n - 2)x^{\frac{1}{\sqrt{1^2+3^2+(2\times 3)}}}$$

$$+(16\times 2^n - 8)x^{\frac{1}{\sqrt{2^2+2^2+(2\times 2)}}}$$

$$+(20\times 2^n - 9)x^{\frac{1}{\sqrt{2^2+3^2+(2\times 3)}}}$$

$$= 4\times 2^n x^{\frac{1}{\sqrt{7}}} + (4\times 2^n - 2)x^{\frac{1}{\sqrt{13}}}$$

$$+(16\times 2^n - 8)x^{\frac{1}{\sqrt{12}}} + (20\times 2^n - 9)x^{\frac{1}{\sqrt{19}}}.$$

5. RESULTS FOR ZINC PROPHYRIN DENDRIMER DPZ_N

We consider the family of zinc prophyrindendrimers. This family of dendrimers is denoted by DPZ_n, where n is the steps of growth in this type of dendrimers. The molecular graph of DPZ_n is shown in Figure 4.

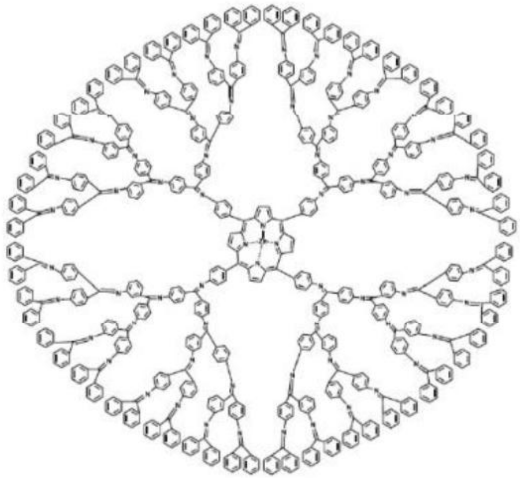

Figure 4. The molecular graph of DPZ_n

Let G be the molecular graph of DPZ_n. By calculation, we obtain that G has $56\times 2^n - 7$ vertices $64\times 2^n - 4$ edges. In DPZ_n, there are four types of edges based on degrees of end vertices of each edge as given in Table 4.

Table 4. Edge partition of DPZ_n

$d_G(u)$, $d_G(v)\backslash uv\in E(G)$	Number of edges
(2, 2)	16×2^n -4
(2, 3)	$40\times 2^n - 16$
(3, 3)	8×2^n +12
(3, 4)	4

Theorem 13. Let DPZ_n be the family of zinc prophyrindendrimers. Then

$$NAGO(G) = 16\times 2^n\sqrt{12} + 40\times 2^n\sqrt{19} + 8\times 2^n\sqrt{27}$$
$$-4\sqrt{12} - 16\sqrt{19} + 12\sqrt{27} + 4\sqrt{37}.$$

Proof: From definition and by using Table 4, we deduce

$$NAGO(G)$$

$$= \sum_{uv\in E(G)} \sqrt{d_G(u)^2 + d_G(v)^2 + d_G(u)d_G(v)}$$

$$= (16\times 2^n - 4)\sqrt{2^2+2^2+(2\times 2)}$$

$$+(40\times 2^n - 16)\sqrt{2^2+3^2+(2\times 3)}$$

$$+(8\times 2^n + 12)\sqrt{3^2+3^2+(3\times 3)}$$

$$+4\sqrt{3^2+4^2+(3\times 4)}$$

$$= 16\times 2^n\sqrt{12} + 40\times 2^n\sqrt{19} + 8\times 2^n\sqrt{27}$$

$$-4\sqrt{12} - 16\sqrt{19} + 12\sqrt{27} + 4\sqrt{37}.$$

Theorem 14. Let DPZ_n be the family of zinc prophyrin dendrimers. Then

$$NAGO(G,x) = (16\times 2^n - 4)x^{\sqrt{12}}$$

$$+(40\times 2^n - 16)x^{\sqrt{19}} + (8\times 2^n + 12)x^{\sqrt{27}} + 4x^{\sqrt{37}}.$$

Proof: From definition and by using Table 4, we deduce

$$NAGO(G,x)=\sum_{uv\in E(G)} x^{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}$$

$$=(16\times 2^n-4)x^{\sqrt{2^2+2^2+(2\times 2)}}$$

$$+(40\times 2^n-16)x^{\sqrt{2^2+3^2+(2\times 3)}}$$

$$+(8\times 2^n+12)x^{\sqrt{3^2+3^2+(3\times 3)}}+4x^{\sqrt{3^2+4^2+(3\times 4)}}$$

$$=(16\times 2^n-4)x^{\sqrt{12}}+(40\times 2^n-16)x^{\sqrt{19}}$$

$$+(8\times 2^n+12)x^{\sqrt{27}}+4x^{\sqrt{37}}.$$

Theorem 15. The modified Nirmala alpha Gourava index of DPZ_n is

$$^{m}NAGO(G)=\frac{16\times 2^n-4}{\sqrt{12}}+\frac{40\times 2^n-16}{\sqrt{19}}$$

$$+\frac{8\times 2^n+12}{\sqrt{27}}+\frac{4}{\sqrt{37}}.$$

Proof: We have

$$^{m}NAGO(G)$$

$$=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}$$

$$=\frac{16\times 2^n-4}{\sqrt{2^2+2^2+(2\times 2)}}+\frac{40\times 2^n-16}{\sqrt{2^2+3^2+(2\times 3)}}$$

$$+\frac{8\times 2^n+12}{\sqrt{3^2+3^2+(3\times 3)}}+\frac{4}{\sqrt{3^2+4^2+(3\times 4)}}$$

$$=\frac{16\times 2^n-4}{\sqrt{12}}+\frac{40\times 2^n-16}{\sqrt{19}}+\frac{8\times 2^n+12}{\sqrt{27}}+\frac{4}{\sqrt{37}}.$$

Theorem 16. The modified Nirmala alpha Gourava exponential of DPZ_n is given by

$$^{m}NAGO(G,x)=(16\times 2^n-4)x^{\frac{1}{\sqrt{12}}}$$

$$+(40\times 2^n-16)x^{\frac{1}{\sqrt{19}}}+(8\times 2^n+12)x^{\frac{1}{\sqrt{27}}}+4x^{\frac{1}{\sqrt{37}}}.$$

Proof: We have

$$^{m}NAGO(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{\sqrt{d_G(u)^2+d_G(v)^2+d_G(u)d_G(v)}}}$$

$$=(16\times 2^n-4)x^{\frac{1}{\sqrt{2^2+2^2+(2\times 2)}}}$$

$$+(40\times 2^n-16)x^{\frac{1}{\sqrt{2^2+3^2+(2\times 3)}}}$$

$$+(8\times 2^n+12)x^{\frac{1}{\sqrt{3^2+3^2+(3\times 3)}}}$$

$$+4x^{\frac{1}{\sqrt{3^2+4^2+(3\times 4)}}}$$

$$=(16\times 2^n-4)x^{\frac{1}{\sqrt{12}}}+(40\times 2^n-16)x^{\frac{1}{\sqrt{19}}}$$

$$+(8\times 2^n+12)x^{\frac{1}{\sqrt{27}}}+4x^{\frac{1}{\sqrt{37}}}.$$

6. CONCLUSION

In this paper, a novel invariant is considered which is the Nirmala alpha Gourava index. The Nirmala alpha Gourava and modified Nirmala alpha Gourava indices of four families of dendrimers are determined.

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