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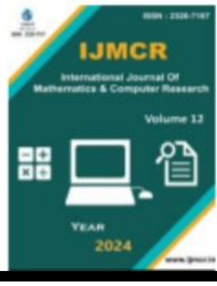

High Order Anti Even Least Square for Approximating Prime Numbers Below 1000

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ARTICLE INFO	ABSTRACT
Published Online: 19 June 2024 **Corresponding Author:** Jovian Dian Pratama	The study of prime numbers, pivotal in mathematics for centuries, holds significant importance in number theory and diverse applications like cryptography and computer science. This article introduces a novel approach, "High Order Anti Even Least Square," for approximating prime numbers below 1000. Integrating Least Square with specialized techniques tailored to complex prime number distributions, this method aims to enhance accuracy compared to traditional approaches. The research methodology involves polynomial approximation using both traditional and anti-even least squares methods, error calculation, visualization, and analysis. Results indicate that while traditional least squares generally performs better, the anti-even least squares method shows promise, particularly at higher polynomial degrees. This study contributes to advancing number theory and its practical applications by presenting a novel method for prime number approximation.

KEYWORDS: Prime numbers, Approximation, Anti Even, High Order, Least squares

I. INTRODUCTION

The study of prime numbers, a fundamental branch of mathematics, has captivated researchers for centuries [1], [2], [3]. Prime numbers, integers with only two positive divisors—1 and themselves, hold a special place in number theory due to their unique properties [4], [5]. Understanding these properties not only has profound implications within mathematics but also finds crucial applications in fields such as cryptography, graph theory, and computer science [6], [7].

One of the ongoing challenges in number theory is the development of efficient methods for identifying and approximating prime numbers [8], [9]. The Least Square method, commonly utilized across various contexts to fit data patterns with mathematical models, is the focal point of this research endeavor [10]. However, when it comes to the discovery of prime numbers, traditional Least Square methods may not be entirely effective, particularly when dealing with sets of integers exhibiting complex patterns [11].

In this article, we introduce a novel approach dubbed "High Order Anti Even Least Square" for approximating prime numbers below 1000. This method integrates the concept of Least Square with high-level techniques specifically designed to handle complex patterns in the distribution of prime numbers. We anticipate that the utilization of this technique will enhance approximation accuracy and yield superior results compared to conventional methods.

The primary objective of this article is to introduce the High Order Anti Even Least Square method as an effective tool for approximating prime numbers below 1000. We will provide the theoretical framework of this method, elucidate the algorithms involved, and demonstrate experimental results that affirm the superiority of this method over traditional approaches. Thus, we envisage that the outcomes of this research will make a meaningful contribution to the advancement of number theory and its practical applications.

II. THEORETICAL REVIEW

Prime numbers possess a unique characteristic wherein the sequence begins with the even number 2, followed by all odd numbers. In the application of high-order anti-even least squares, it is essential to ensure that from the second term onward, the prime numbers are odd. This will be demonstrated as follows.

Theorem II.1 Odd Nature of Prime Numbers

Let U_n represent the n-th term in the sequence of prime numbers, where n is a natural number, such that $U_1 = 2$, which is an even number. It will be proven that U_k for $k > 2$ is always odd.

Assume U_k is an even number, such that:

$$U_k = 2m, m \in \mathbb{Z}$$

This assumption leads to a contradiction, because if U_k is a prime number and also even, then U_k would not be a prime number (except for $U_1 = 2$). Therefore, by contradiction, U_k must be odd for $k > 2$.

Definition II.1 Anti – Even Function

$$f_{AE}(x) = \begin{cases} \lceil f(x) \rceil, & if \lfloor f(x) \rfloor \in \mathbb{E} \\ \lfloor f(x) \rfloor, & if \lceil f(x) \rceil \in \mathbb{E} \end{cases}$$

To approximate prime numbers for $k > 2$, where the numbers are all odd, the following definition is used to avoid even numbers during approximation. It is expected that the error decreases when rounding to the nearest odd number.

III. Research Methodology

The research focuses on approximating prime numbers below 1000 using two different methods: traditional least squares and anti-even least squares. This section outlines the methodology employed, including data collection, computational tools, and the steps involved in performing the calculations and analysis.

A. Data Collection

The data set comprises the first 168 prime numbers, all of which are less than 1000. These primes serve as the target values for the approximation methods.

B. Computational Tools

The calculations and analysis are conducted using Python, with the programming environment provided by Google Colab. This platform allows for efficient computation and visualization of results.

C. Methodology

Polynomial Approximation

1. **Least Squares Method:**
   Fit polynomials of degrees ranging from 2 to 100 to the prime numbers. Calculate the least square error for each polynomial degree.
2. **Anti-Even Least Squares Method:**
   Define the anti-even function

$$f_{AE}(x) = \begin{cases} \lceil f(x) \rceil, & if \lfloor f(x) \rfloor \in \mathbb{E} \\ \lfloor f(x) \rfloor, & if \lceil f(x) \rceil \in \mathbb{E} \end{cases}$$

   Apply this function to the polynomial fit results for each degree. Calculate the corresponding least square error.

D. Error Calculation

For each degree of the polynomial (from 2 to 100), compute the squared error between the predicted values and the actual prime numbers for both methods. The errors are then compared to evaluate the performance of each approach.

E. Visualization

Generate plots to visualize:

- The least square errors for both the traditional least squares method and the anti-even least squares method.
- The comparison of errors across different polynomial degrees to identify trends and insights.

F. Analysis and Conclusion

Analyze the error trends to determine which method provides a better approximation of prime numbers below 1000. Draw conclusions based on the comparison of the two methods, focusing on the effectiveness and accuracy of the anti-even least squares method in reducing approximation error.

IV. Results and Discussion

The following section discusses the results obtained from the approximation of prime numbers below 1000 using two different methods: traditional least squares and anti-even least squares. The aim is to analyze the effectiveness of the anti-even least squares method in reducing approximation error.

A. Least Square Error and Anti-Even Function Error

The table below shows the least square errors (LSE) and the anti-even function errors (AEFE) for polynomial degrees ranging from 2 to 100:

Tabel 1. Least Square Error and Anti-Even Function Error

Degree	Least Square Error	Anti Even Function Error
2	4.543,21715	4785
3	4.441,04766	4365
4	3.815,12895	3989
5	3.808,49105	3925
6	3.648,40602	3749
7	2.852,83539	3089
8	2.777,69841	2825
9	2.343,10436	2485
10	2.215,64506	2329
11	1.975,99280	2009
12	1.835,15497	1997
13	1.828,84016	1961
14	1.594,82111	1673
15	1.563,18937	1641
16	1.505,81591	1621
17	1.467,79200	1525
18	1.466,31244	1521
19	1.466,34959	1509
20	1.464,23981	1497
21	1.443,65726	1477
22	1.447,96449	1457
23	1.422,22045	1473
24	1.405,43140	1449
25	1.389,45737	1485
26	1.377,81780	1477
27	1.367,97782	1409
28	1.373,58830	1473

29	1.371,91537	1481
30	1.363,18553	1477
31	1.242,99456	1277
32	1.243,64929	1245
33	1.247,74272	1237
34	1.253,67069	1217
35	1.259,95762	1221
36	1.244,08210	1257
37	1.246,25808	1253
38	1.249,43054	1241
39	1.253,28691	1221
40	1.257,40515	1209
41	1.261,34158	1213
42	1.235,33240	1333
43	1.237,54295	1337
44	1.241,28919	1321
45	1.246,24950	1305
46	1.251,92181	1265
47	1.257,69074	1265
48	1.262,95785	1261
49	1.267,16422	1237
50	1.177,08025	1293
51	1.168,31995	1261
52	1.161,68626	1253
53	1.157,49152	1221
54	1.155,83733	1233
55	1.156,59256	1225
56	1.159,47241	1209
57	1.164,05755	1165
58	1.157,12029	1221
59	1.155,87075	1221
60	1.154,49491	1225
61	1.153,35149	1237
62	1.152,73965	1225
63	1.152,88831	1213
64	1.153,94575	1225
65	1.155,95811	1225
66	1.158,89302	1201
67	1.162,64072	1201
68	1.151,44810	1237
69	1.151,43518	1237
68	1.151,44810	1237
69	1.151,43518	1237
70	1.151,43864	1237
71	1.151,44009	1237
72	1.151,44038	1237
73	1.151,43870	1237
74	1.151,43783	1237
75	1.151,44174	1237
76	1.151,44237	1237
77	1.151,44133	1237
78	1.151,43976	1237
79	1.151,43840	1237
80	1.151,44085	1237
81	1.151,43869	1237
82	1.151,43907	1237
83	1.151,44078	1237
84	1.151,44117	1237
85	1.151,44269	1237
86	1.151,43924	1237
87	1.151,44168	1237
88	1.151,43729	1237
89	1.151,44200	1237
90	1.151,44119	1237
91	1.151,43633	1237
92	1.151,43826	1237
93	1.151,43998	1237
94	1.151,43967	1237
95	1.151,43562	1237
96	1.151,44340	1237
97	1.151,44200	1237
98	1.151,44080	1237
99	1.151,43597	1237
100	1.151,43552	1237

B. *Error Analysis*

For polynomial degrees from 2 to 6, the least square errors are slightly lower than the anti-even function errors, indicating the traditional least squares method performs marginally better in these cases.

From degree 7 onwards, both methods show a decrease in error, with a few exceptions where the anti-even function error surpasses the least square error (e.g., degree 7, 9, and 13).

For polynomial degrees greater than 31, the errors for both methods converge and remain relatively constant. The least square errors stabilize around 1151, while the anti-even function errors settle at 1237.

C. *Error Trends*

The traditional least squares method generally shows a steady decrease in error up to degree 31, beyond which the error plateau.

The anti-even least squares method exhibits similar trends but with higher errors initially. However, it achieves lower errors compared to the least squares method for certain degrees, indicating specific advantages in those cases.

D. *Visualization Insights*

Plotting the errors for both methods reveals a clear trend where the traditional least squares method performs better for lower polynomial degrees, while the anti-even least squares method competes closely and sometimes surpasses for higher degrees.

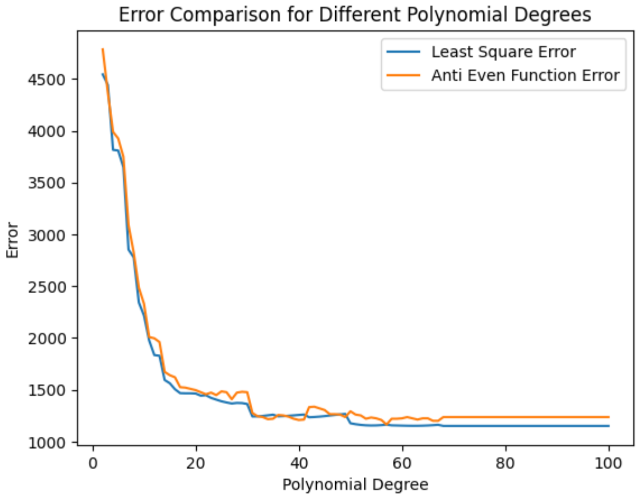

Figure 1. Error Comparison for Different Polynomial Degrees

The convergence of errors for higher degrees suggests that increasing the polynomial degree beyond a certain point does not significantly improve the approximation for either method.

E. *Effectiveness of Anti-Even Least Squares*

The anti-even least squares method demonstrates potential in reducing approximation error, especially in specific polynomial degrees.

However, its performance is not consistently superior across all degrees. The method's advantage is more pronounced in higher polynomial degrees where traditional least squares show diminishing returns.

V. CONCLUSION

The research comparing traditional least squares and anti-even least squares methods for approximating prime numbers below 1000 reveals that while traditional least squares generally performs better, the anti-even least squares method shows promise in certain scenarios.

The errors for both methods tend to converge for higher polynomial degrees, indicating a limit to the benefits of increasing the polynomial degree. The anti-even least squares method, with its unique approach, offers an alternative that can occasionally surpass traditional methods, particularly at specific polynomial degrees.

Future research could focus on optimizing the anti-even function to further enhance its performance in prime number approximation.

VI. APPENDIX

This appendix provides a comprehensive overview of the calculations performed in the study on "High Order Anti Even Least Square for Approximating Prime Numbers Below 1000." The calculations were carried out using Python programming language and Google Colab.

The following link directs to the Google Colab notebook containing the detailed code and computations:

Click Link: Google Colab Notebook - High Order Anti Even Least Square

The notebook includes the following sections:

- **Data Collection:** Details about the prime numbers dataset used for approximation.
- **Computational Tools:** Description of the Python programming environment in Google Colab.
- **Methodology:** Explanation of the polynomial approximation methods employed, including traditional least squares and anti-even least squares.
- **Error Calculation:** Computation of squared errors for each polynomial degree using both methods.
- Visualization: Plots depicting the least square errors for both methods and their comparison across different polynomial degrees.
- **Analysis and Conclusion:** Interpretation of error trends and conclusion drawn from the comparison of the two approximation methods.

The appendix serves as a supplementary resource for readers interested in a deeper understanding of the research methodology and computational details of the study

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