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Kepler Banhatti and Modified Kepler Banhatti Indices

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ARTICLE INFO	ABSTRACT
Published Online: **18 June 2024** Corresponding Author: **V.R.Kulli**	We introduce a novel vertex degree based topological index, called Kepler Banhatti index. Also we put forward the modified Kepler Banhatti index of a graph. We propose the Kepler Banhatti and modified Kepler Banhatti exponentials of a graph. In this study, we determine the newly defined the Kepler Banhatti indices and their corresponding exponentials for certain dendrimers. Furthermore, we establish some properties of the Kepler Banhatti index.

KEYWORDS: Kepler Banhatti index, modified Kepler Banhatti index, graph, dendrimer.

1. INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree d_u of a vertex u is the number of vertices adjacent to u. We refer [1] for undefined notations and terminologies.

A graph index is a numerical parameter mathematically derived from the graph structure. Several graph indices have been considered in Theoretical Chemistry and many graph indices were defined by using vertex degree concept [2]. The Zagreb, Banhatti, Revan, Gourava indices are the most degree based graph indices in Chemical Graph Theory. Graph indices have their applications in various disciplines in Science and Technology [3, 4, 5].

In applications, Zagreb indices are among the best applications to recognize the physical properties. The first Zagreb index $M_1(G)$ and the second Zagreb index $M_2(G)$ were introduced by Gutman et al. in [6, 7]. They are defined as

$$M_1(G)=\sum_{uv\in E(G)}(d_u+d_v)=\sum_{u\in V(G)}d_u^2$$

$$M_2(G)=\sum_{uv\in E(G)}d_ud_v$$

The reciprocal Randic index was introduced in [8, 9] and it is defined as

$$RR(G)=\sum_{uv\in E(G)}\sqrt{d_ud_v}$$

The Kepler expression was proposed in [10, 11]

$$\pi(r_1+r_2)$$

where $r_1=\sqrt{a^2+b^2}$, $r_2=\frac{1}{\sqrt{2}}(a+b)$,

$a=d_u, b=d_v, a\geq b.$

The Kepler expression motivates us to introduce a new index, defined as

$$KB(G)=\sum_{uv\in E(G)}[(d_u+d_v)+\sqrt{d_u^2+d_v^2}]$$

which we propose to be named as Kepler Banhatti index.

Considering the Kepler Banhatti index, we introduce the Kepler Banhatti exponential of a graph G and defined it as

$$KB(G,x)=\sum_{uv\in E(G)}x^{(d_u+d_v)+\sqrt{d_u^2+d_v^2}}.$$

We define the modified Kepler Banhatti index of a graph G as

$$^mKB(G)=\sum_{uv\in E(G)}\frac{1}{(d_u+d_v)+\sqrt{d_u^2+d_v^2}}.$$

Considering the modified Kepler Banhatti index, we introduce the modified Kepler Banhatti exponential of a graph G and defined it as

$$^mKB(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{(d_u+d_v)+\sqrt{d_u^2+d_v^2}}}.$$

Recently, some graph indices were studied in [12, 13, 14, 15, 16].

2. MATHEMATICAL PROPERTIES

Proposion1. Let P be a path with $n\geq 3$ vertices. Then

$$KB(P)=\left(4+2\sqrt{2}\right)n+2\sqrt{5}-6\sqrt{2}-6.$$

Proof: Let P be a path with $n\geq 3$vertices. We obtain two partitions of the edge set of P as follows:

$E_1 = \{uv \in E(P) \mid d_u=1, d_v=2\}, |E_1| = 2.$
$E_2 = \{uv \in E(P) \mid d_u = d_v=2\}, |E_2| = n - 3.$

$$KB(P)=\sum_{uv\in E(G)}[\left(d_u+d_v\right)+\sqrt{d_u^2+d_v^2}]$$
$$=2[(1+2)+\sqrt{1^2+2^2}]+(n-3)[(2+2)+\sqrt{2^2+2^2}]$$
$$=\left(4+2\sqrt{2}\right)n+2\sqrt{5}-6\sqrt{2}-6.$$

Proposion2. Let G be an r-regular graph with n vertices, m edges and $r\geq 2$. Then

$$KB(G)=\left(1+\frac{1}{\sqrt{2}}\right)nr^2.$$

Proof: Let G be an r-regular graph with n vertices, $r\geq 2$ and $m=\frac{nr}{2}$ edges. Every edge of G is incident with r edges. Thus

$$KB(G)=\sum_{uv\in E(G)}[(r+r)+\sqrt{r^2+r^2}]$$
$$=\left(2+\sqrt{2}\right)rm$$
$$=\left(1+\frac{1}{\sqrt{2}}\right)nr^2$$

Corollary 2.1. Let C_n be a cycle with $n\geq 3$ vertices. Then

$$KB(C_n)=\left(1+\frac{1}{\sqrt{2}}\right)4n.$$

Corollary 2.2. Let K_n be a complete graph with $n\geq 3$ vertices. Then

$$KB(K_n)=\left(1+\frac{1}{\sqrt{2}}\right)n\left(n-1\right)^2.$$

Theorem 1. Let G be a simple connected graph. Then

$$KB(G)\geq\left(1+\frac{1}{\sqrt{2}}\right)M_1(G)$$

with equality if G is regular.

Proof: By the Jensen inequality, for a concave function $f(x)$,

$$f\left(\frac{1}{n}\sum x_i\right)\geq\frac{1}{n}\sum f(x_i)$$

with equality for a strict concave function if $x_1 = x_2 = \ldots = x_n$. Choosing $f(x) = \sqrt{x}$, we obtain

$$\sqrt{\frac{d_u^2+d_v^2}{2}}\geq\frac{\left(d_u+d_v\right)}{2}$$

thus

$$[\left(d_u+d_v\right)+\sqrt{d_u^2+d_v^2}]\geq\left(d_u+d_v\right)+\frac{1}{\sqrt{2}}\left(d_u+d_v\right).$$

Hence

$$\sum_{uv\in E(G)}[\left(d_u+d_v\right)+\sqrt{d_u^2+d_v^2}].$$
$$\geq\left(1+\frac{1}{\sqrt{2}}\right)\sum_{uv\in E(G)}\left(d_u+d_v\right).$$

Thus

$$KB(G)\geq\left(1+\frac{1}{\sqrt{2}}\right)M_1(G)$$

with equality if G is regular.

Theorem 2. Let G be a simple connected graph. Then

$$KB(G)\leq\left(1+\sqrt{2}\right)M_1(G)-\sqrt{2}RR(G).$$

Proof: It is known that for $1\leq x\leq y$,

$$f(x,y)=\left(x+y-\sqrt{xy}\right)-\sqrt{\frac{x^2+y^2}{2}}$$

is decreasing for each y. Thus $f(x,y)\geq f(y,y)=0.$ Hence

$$x+y-\sqrt{xy}\geq\sqrt{\frac{x^2+y^2}{2}}$$

or
$$\sqrt{\frac{x^2+y^2}{2}}\leq x+y-\sqrt{xy}.$$

Put $x= d_u$ and $y= d_v$, we get

$$\sqrt{\frac{d_u^2+d_v^2}{2}}\leq\left(d_u+d_v\right)-\sqrt{d_ud_v}$$

or
$$\sqrt{d_u^2+d_v^2}\leq\sqrt{2}[\left(d_u+d_v\right)-\sqrt{d_ud_v}].$$

Thus
$$\left(d_u+d_v\right)+\sqrt{d_u^2+d_v^2}$$
$$\leq\left(d_u+d_v\right)+\sqrt{2}[\left(d_u+d_v\right)-\sqrt{d_ud_v}]$$

which implies

$$\sum_{uv\in E(G)}\left(d_u+d_v\right)+\sqrt{d_u^2+d_v^2}$$
$$\leq\left(1+\sqrt{2}\right)\sum_{uv\in E(G)}\left(d_u+d_v\right)-\sqrt{2}\sum_{uv\in E(G)}\sqrt{d_ud_v}.$$

Thus

$$KB(G) \le (1+\sqrt{2})M_1(G) - \sqrt{2}RR(G).$$

Theorem 3. Let G be a simple connected graph. Then

$$KB(G) < 2M_1(G).$$

Proof: It is known that for $1 \le x \le y$,

$$\sqrt{x^2+y^2} < x+y$$

$$(x+y)+\sqrt{x^2+y^2} < 2(x+y)$$

Setting $x = d_u$ and $y = d_v$, we get

$$(d_u+d_v)+\sqrt{d_u^2+d_v^2} < 2(d_u+d_v).$$

Thus

$$\sum_{uv\in E(G)} [(d_u+d_v)+\sqrt{d_u^2+d_v^2}] < 2\sum_{uv\in E(G)} (d_u+d_v).$$

Hence $\quad KB(G) < 2M_1(G).$

3. RESULTS FOR DENDIMER NANOSTARS $D_1[n]$

In this section, we consider a family of dendrimer nanostars with n growth stages, denoted by $D_1[n]$, where $n \ge 0$. The molecular graph of $D_1[4]$ with 4 growth stages is depicted in Figure 1.

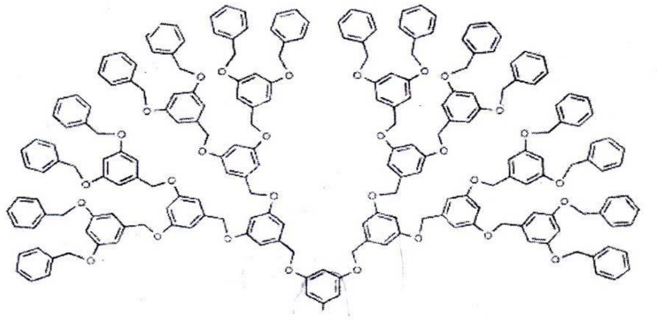

Figure 1. The molecular graph of $D_1[4]$.

Let G be the molecular graph of dendrimer nanostar $D_1[n]$. From Figure 1, it is easy to see that the vertices of dendrimter nanostar $D_1[n]$ are either of degree 1, 2 or 3. We obtain that G has $2^{n+4} - 9$ vertices and $18 \times 2^n - 11$ edges. Also by calculation, we partition the edge set $E(D_1[n])$ into three sets as follows:

$E_1 = \{uv \in E(G) \mid d_u = 1, d_v = 3\}$, $\quad |E_1| = 1$.

$E_2 = \{uv \in E(G) \mid d_u = d_v = 2\}$, $\quad |E_2| = 6 \times 2^n - 2$.

$E_3 = \{uv \in E(G) \mid d_u = 2, d_v = 3\}$, $|E_3| = 12 \times 2^n - 10$.

Theorem 4. The Kepler Banhatti index of a dendrimer nanostar $D_1[n]$ is given by

$$KB(G) = (84+12\sqrt{2}+12\sqrt{13})2^n$$
$$-54+\sqrt{10}+4\sqrt{2}-10\sqrt{13}.$$

Proof: We have

$$KB(G) = \sum_{uv\in E(G)} [(d_u+d_v)+\sqrt{d_u^2+d_v^2}]$$

$$=1\left[(1+3)+\sqrt{1^2+3^2}\right]$$
$$+(6\times 2^n-2)\left[(2+2)+\sqrt{2^2+2^2}\right]$$
$$+(12\times 2^n-10)\left[(2+3)+\sqrt{2^2+3^2}\right]$$

$$=(84+12\sqrt{2}+12\sqrt{13})2^n - 54+\sqrt{10}+4\sqrt{2}-10\sqrt{13}.$$

Theorem 5. The Kepler Banhatti exponential of a dendrimer nanostar $D_1[n]$ is given by

$$KB(G,x) = 1x^{4+\sqrt{10}} + (6\times 2^n-2)x^{4+2\sqrt{2}}$$
$$+(12\times 2^n-10)x^{5+\sqrt{13}}.$$

Proof: We have

$$KB(G,x) = \sum_{uv\in E(G)} x^{(d_u+d_v)+\sqrt{d_u^2+d_v^2}}$$

$$=1x^{(1+3)+\sqrt{1^2+3^2}} + (6\times 2^n-2)x^{(2+2)+\sqrt{2^2+2^2}}$$
$$+(12\times 2^n-10)x^{(2+3)+\sqrt{2^2+3^2}}$$
$$=1x^{4+\sqrt{10}} + (6\times 2^n-2)x^{4+2\sqrt{2}} + (12\times 2^n-10)x^{5+\sqrt{13}}.$$

Theorem 6. The modified Kepler Banhatti index of a dendrimer nanostar $D_1[n]$ is

$$^{m}KB(G) = \frac{1}{4+\sqrt{10}} + \frac{6\times 2^n-2}{4+2\sqrt{2}} + \frac{12\times 2^n-10}{5+\sqrt{13}}.$$

Proof: We have

$$^{m}KB(G) = \sum_{uv\in E(G)} \frac{1}{(d_u+d_v)+\sqrt{d_u^2+d_v^2}}$$

$$=\frac{1}{(1+3)+\sqrt{1^2+3^2}} + \frac{6\times 2^n-2}{(2+2)+\sqrt{2^2+2^2}} + \frac{12\times 2^n-10}{(2+3)+\sqrt{2^2+3^2}}$$

$$=\frac{1}{4+\sqrt{10}} + \frac{6\times 2^n-2}{4+2\sqrt{2}} + \frac{12\times 2^n-10}{5+\sqrt{13}}.$$

Theorem 7. The modified Kepler Banhatti exponential of a dendrimer nanostar $D_1[n]$ is given by

$$^{m}KB(G,x) = 1x^{\frac{1}{4+\sqrt{10}}} + (6\times 2^n-2)x^{\frac{1}{4+2\sqrt{2}}}$$
$$+(12\times 2^n-10)x^{\frac{1}{5+\sqrt{13}}}.$$

Proof: We have

$$ {}^{m}KB(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{(d_u+d_v)+\sqrt{d_u^2+d_v^2}}} $$

$$ =1x^{\frac{1}{(1+3)+\sqrt{1^2+3^2}}}+\left(6\times 2^n-2\right)x^{\frac{1}{(2+2)+\sqrt{2^2+2^2}}} $$

$$ +\left(12\times 2^n-10\right)x^{\frac{1}{(2+3)+\sqrt{2^2+3^2}}} $$

$$ =1x^{\frac{1}{4+\sqrt{10}}}+\left(6\times 2^n-2\right)x^{\frac{1}{4+2\sqrt{2}}}+\left(12\times 2^n-10\right)x^{\frac{1}{5+\sqrt{13}}}. $$

4. RESULTS FOR DENDIMER NANOSTARS $D_3[n]$

In this section, we consider of dendrimer nanostars with n growth stages, denoted by $D_3[n]$, where $n\geq 0$. The molecular structure of $D_3[n]$ with 3 growth stages is shown in Figure 2.

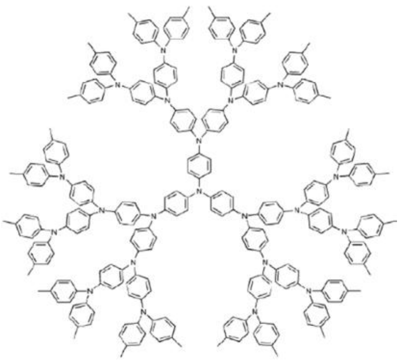

Figure 2. The molecular structure of $D_3[3]$

Let G be the graph of a dendrimer nanostar $D_3[n]$. From Figure 2, it is easy to see that the vertices of dendrimter nanostar $D_3[n]$ are either of degree 1, 2 or 3. By algebraic method, we obtain that G has $24\times 2^n-20$ vertices and $24\times 2^{n+1}-24$ edges. Also by algebraic method, we obtain that the edge set $E(D_3[n])$ can be divided into four partitions:

$E_1=\{uv\in E(G)\mid d_G(u)=1,\ d_G(v)=3\}$ $|E_1|=3\times 2^n$.

$E_2=\{uv\in E(G)\mid d_G(u)=d_G(v)=2\}$ $|E_2| = 12\times 2^n-6$.

$E_3=\{uv\in E(G)\mid d_G(u)=2,\ d_G(v)=3\}$ $|E_3|=24\times 2^n-12$.

$E_4=\{uv\in E(G)\mid d_G(u)=d_G(v)=3\}$ $|E_4| = 9\times 2^n-6$.

Theorem 8. The Kepler Banhatti index of a dendrimer nanostar $D_3[n]$ is given by

$$ KB(G)=\left(12\sqrt{10}+114\sqrt{2}+72\sqrt{5}\right)2^n-60\sqrt{2}-36\sqrt{5}. $$

Proof: We have

$$ KB(G)=\sum_{uv\in E(G)}\left(d_u+d_v\right)+\sqrt{d_u^2+d_v^2} $$

$$ =3\times 2^n\left[(1+3)+\sqrt{1^2+3^2}\right] $$

$$ +\left(12\times 2^n-6\right)\left[(2+2)+\sqrt{2^2+2^2}\right] $$

$$ +\left(24\times 2^n-12\right)\left[(2+3)+\sqrt{2^2+3^2}\right] $$

$$ +\left(9\times 2^n-6\right)\left[(3+3)+\sqrt{3^2+3^2}\right] $$

$$ =\left(12\sqrt{10}+114\sqrt{2}+72\sqrt{5}\right)2^n-60\sqrt{2}-36\sqrt{5}. $$

Theorem 9. The Kepler Banhatti exponential of a dendrimer nanostar $D_3[n]$ is given by

$$ KB(G,x)=3\times 2^n x^{4+\sqrt{10}}+\left(12\times 2^n-6\right)x^{4+2\sqrt{2}} +\left(24\times 2^n-12\right)x^{5+\sqrt{13}}+\left(9\times 2^n-6\right)x^{6+3\sqrt{2}}. $$

Proof: We have

$$ KB(G,x)=\sum_{uv\in E(G)} x^{(d_u+d_v)+\sqrt{d_u^2+d_v^2}} $$

$$ =3\times 2^n x^{(1+3)+\sqrt{1^2+3^2}}+\left(12\times 2^n-6\right)x^{(2+2)+\sqrt{2^2+2^2}} $$

$$ +\left(24\times 2^n-12\right)x^{(2+3)+\sqrt{2^2+3^2}}+\left(9\times 2^n-6\right)x^{(3+3)+\sqrt{3^2+3^2}} $$

$$ =3\times 2^n x^{4+\sqrt{10}}+\left(12\times 2^n-6\right)x^{4+2\sqrt{2}} $$

$$ +\left(24\times 2^n-12\right)x^{5+\sqrt{13}}+\left(9\times 2^n-6\right)x^{6+3\sqrt{2}}. $$

Theorem 10. The modified Kepler Banhatti index of a dendrimer nanostar $D_3[n]$ is

$$ {}^{m}KB(G)=\frac{3\times 2^n}{4+\sqrt{10}}+\frac{12\times 2^n-6}{4+2\sqrt{2}} +\frac{24\times 2^n-12}{5+\sqrt{13}}+\frac{9\times 2^n-6}{6+3\sqrt{2}}. $$

Proof: We have

$$ {}^{m}KB(G)=\sum_{uv\in E(G)}\frac{1}{\left(d_u+d_v\right)+\sqrt{d_u^2+d_v^2}} $$

$$ =\frac{3\times 2^n}{(1+3)+\sqrt{1^2+3^2}}+\frac{12\times 2^n-6}{(2+2)+\sqrt{2^2+2^2}} $$

$$ +\frac{24\times 2^n-12}{(2+3)+\sqrt{2^2+3^2}}+\frac{9\times 2^n-6}{(3+3)+\sqrt{3^2+3^2}} $$

$$ =\frac{3\times 2^n}{4+\sqrt{10}}+\frac{12\times 2^n-6}{4+2\sqrt{2}}+\frac{24\times 2^n-12}{5+\sqrt{13}}+\frac{9\times 2^n-6}{6+3\sqrt{2}}. $$

Theorem 11. The modified Kepler Banhatti exponential of a dendrimer nanostar $D_3[n]$ is given by

$$ {}^{m}KB(G,x)=3\times 2^n x^{\frac{1}{4+\sqrt{10}}}+\left(12\times 2^n-6\right)x^{\frac{1}{4+2\sqrt{2}}} +\left(24\times 2^n-12\right)x^{\frac{1}{5+\sqrt{13}}}+\left(9\times 2^n-6\right)x^{\frac{1}{6+3\sqrt{2}}}. $$

Proof: We have

$$ {}^{m}KB(G,x) = \sum_{uv \in E(G)} x^{\frac{1}{(d_u+d_v)+\sqrt{d_u^2+d_v^2}}} $$

$$ = 3\times 2^n x^{\frac{1}{(1+3)+\sqrt{1^2+3^2}}} + (12\times 2^n - 6)\, x^{\frac{1}{(2+2)+\sqrt{2^2+2^2}}} $$

$$ +(24\times 2^n - 12)\, x^{\frac{1}{(2+3)+\sqrt{2^2+3^2}}} + (9\times 2^n - 6)\, x^{\frac{1}{(3+3)+\sqrt{3^2+3^2}}} $$

$$ = 3\times 2^n x^{\frac{1}{4+\sqrt{10}}} + (12\times 2^n - 6)\, x^{\frac{1}{4+2\sqrt{2}}} $$

$$ +(24\times 2^n - 12)\, x^{\frac{1}{5+\sqrt{13}}} + (9\times 2^n - 6)\, x^{\frac{1}{6+3\sqrt{2}}}. $$

5. CONCLUSION

We have introduced the Kepler Banhatti and modified Kepler Banhatti indices and their exponentials of a graph. Furthermore the Kepler Banhatti and modified Kepler Banhatti indices and their exponentials for two families of dendrimer nanostars are determined. Also some mathematical properties of Kepler Banhatti index are obtained.

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