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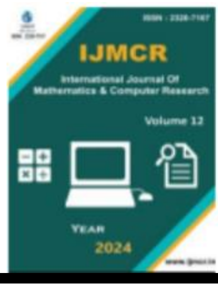

Weaker Forms on Separation Axioms

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ARTICLE INFO	ABSTRACT
Published Online: **21 June 2024** Corresponding Author: **T.D. Rayanagoudar**	In general topology, the concepts of separation axioms play an important role. The main goal of this paper is to present the study of weaker forms of separation axioms called spgωα-T_1, spgωα-T_2, spgωα-regular and spgωα-normal spaces using spgωα-open sets in topological spaces. Further, the properties spgωα-compact, spgωα-connected and spgωα-Lindelof spaces have been defined and studied their basic characterizations in topological spaces.

KEYWORDS: spgωα-open sets, spgωα-closed sets, spgωα-continuous functions, spgωα-Irresolute functions, spgωα-compact spaces, spgωα-connected spaces.

1. INTRODUCTION AND PRELIMINARIES:

Njastad [8], in 1984 introduced and defined α-open sets. Following the work on α-open sets, many topologists focused on generalization of topological concepts using semi-open and α-open sets. These sets play an important role in the generalization of continuity in topological spaces. Maheshwari and Prasad [6] introduced s-normal spaces using semi-open sets. Nori and Popa [9], Dorsett [2], Arya [1] and Munshi [7] studied g-regular and g-normal spaces using g-closed sets in topological spaces.

In this paper, author establish the properties of weaker forms of separation axioms called spgωα-T_0, spgωα-T_1 and spgωα-T_2 spaces, spgωα-regular and spgωα-normal spaces using spgωα-closed sets in topology. Also, we defined several properties related to them. Further, authors explained the properties of spgωα-compact spaces, spgωα-connected spaces, spgωα-Lindelöf spaces in topological spaces and several properties related to them.

Definition 2.1. [4] A subset A of a TS R is said to be a semi generalized ωα -closed (briefly spgωα-closed) if cl(A) $\subset$ U whenever A $\subset$ U and U is ωα-open in R.

The family of all spgωα-closed subsets of a space R is denoted by spgωα-C(R).

Definition 2.2. [4] The intersection of all spgωα-closed sets containing a subset A of R is called spgωα-closure of A and is denoted by spgωα-cl(A).

A set A is spgωα-closed if and only if spgωα-cl(A) = A.

Definition 2.3. [4] The union of all spgωα-open sets containing a subset A of R is called spgωα-interior of A and it is denoted by spgωα-int(A).

A set A is called spgωα-open if and only if spgωα-int(A) = A.

Definition 2.4. A function f: R $\rightarrow$ S is called a

(i) spgωα-continuous [12] if f^{-1}(V) is spgωα-closed in R for every closed set V in S.

(ii) spgωα-irresolute [12] if f^{-1}(V) is spgωα-closed in R for every spgωα-closed set V in S.

(iii) spgωα-open [12] if f(V) is spgωα-open in S for every open set V in R.

2. SPGωA-SEPARATION AXIOMS

The weaker forms of separation axioms are found in this section, such as spgωα-T_0, spgωα-T_1 and spgωα-T_2 spaces and their related concepts.

Definition 2.1: Let (R, τ) be a TS. Then R is said to be a spgωα-T_0 if for each r_1, $r_2 \in R^*$ with $r_1 \neq r_2$, there exists a spgωα-open set containing one but not the other.

Example 2.2: Consider R ={r_1, r_2, r_3} and τ = {R, φ, {r_3}}. The R is spgωα-T_0.

Theorem 2.3: A space R is spgωα-T_0iffspgωα-closures of distinct points are distinct.

Proof: Let $r_1, r_2 \in R$, where R is spgωα-T_0. Then there exists U $\in$spgωα-O(R) with $r_1 \in$ U, $r_2 \notin$ U, and so $r_1 \notin R-U$ and $r_2 \in R-$ U, where R$-$ U$\in$spgωα-C(R). We have, spgωα$-$cl({r_2}) is intersection of the spgωα-closed sets, which contains r_2.

Thus, $r_2 \in$spgωα−cl({r_2}). As, $r_1 \notin$R−U and so $r_1 \notin$spgωα−cl({r_1}). Thus, spgωα−cl({r_1}) ≠ spgωα−cl({r_2}).
On the other hand, let $r_1, r_2 \in$R holds for any pair of different points spgωα−cl({r_1}) ≠ spgωα−cl({r_2}). So, there exists at least one point r∈R with r ∈ spgωα−cl({r_1}) and r∉spgωα−cl({r_2}). Further, we have to prove that $r_1 \notin$spgωα−cl({r_2}). Let r_1 ∈spgωα−cl({r_2}). Then, spgωα−cl({r_1}) ⊆spgωα-cl({r_2}). So, r∈spgωα−cl({r_2}) which is a wrong. Thus, $r_1 \notin$spgωα−cl({r_2}) shows that $r_1 \in$R −spgωα-cl({r_2}) with R is spgωα-cl({r_2}) is spgωα-open set containing r_1 but not r_2. Hence R is spgωα-T_0.

Theorem 2.4: The property of spg-T_0 space is a hereditary.

Definition 2.5: A function Ψ: R → S is pre-spgωα-open, if Ψ(V) is spgωα-open in R, for every V ∈spgωα-O(R).

Theorem 2.6: Let Ψ: R → S be bijective, pre spgωα-open. If R is spgωα-T_0, then S is spgωα-T_0.

Proof: Let Ψ be bijective, pre-spgωα-open with R is spgωα-T_0. Let $s_1, s_2 \in$ S with $s_1 \neq s_2$. Since Ψ is bijective, with $r_1, r_2 \in$ R with Ψ(r_1) = s_1, Ψ(r_2) = s_2. So, there exists spgωα-open set U such that $r_1 \in$ U, $r_2 \notin$ U. Thus, Ψ(U*) is spgω-open set containing Ψ(r_1) but not Ψ(r_2). So, there exists spgωα-open set Ψ(U) ∈ S such that $s_1 \in$Ψ(U), $s_2 \notin$Ψ(U). Thus, S is spgωα -T_0.

Theorem 2.7: The properties listed below are equivalent for a space R, where spgωα-O(R) is open under arbitrary union:

(i) R is spgωα-T_0,
(ii) Each singleton set is spgωα-closed,
(iii) Each subset of R is the intersection of all spgωα-open set containing it,
(iv) The set {s} is the intersection of all spgωα-open set containing the point $r_1 \in$ R.

Proof: (i) → (ii): Let r∈ R with R is spgωα-T_0. For each s ∈ R with s ≠ r, there exists U ∈spg-O(R, s) but not r. Thus, s ∈ U⊆ $\{r\}^c$. So, $\{r\}^c$= ∪{U : s∈$\{r\}^c$}, that is $\{r\}^c$ is the union of spgωα-open sets and so {r} is spgωα-closed.
(ii) → (iii): Let (ii) holds and A⊆ R. Then for each s ∉A, there exists $\{s\}^c$ with A⊆$\{s\}^c$ where $\{s\}^c$ is spgωα-open in R. We get, A = ∩{$\{s\}^c$: s ∈ A^c} and so, the set A is the intersection of all spgωα-open sets containing A.
(iii) → (iv): Proof is obvious.
(iv) → (i): Let (4) holds and r, s ∈ R with r≠s. According to the assumption, there is a spgωα-open set U_r, with r∈ U_r, s ∉U_r. Hence the condition of spgωα-T_0 space satisfied. Hence, R is spgωα-T_0.

Definition 2.8: A space R is said to be spgωα-T_1 if for each pair of points r, s in R with r ≠ s, there exists a spgωα-open sets U and V with the condition r ∈ U, s ∉ U and s ∈ V, r ∉ V.

Example 2.9: Let R = {r_1, r_2, r_3} and τ = {R, φ, {r_1}, {r_2, r_3}}. The space R is spgωα-T_1.

Theorem 2.10: Every spgωα-T_1 space is spgωα-T_0 space.

Remark 2.11: The converse of the above examples does not hold from the following example.

Example 2.12: Let R={r_1, r_2, r_3} and τ = { R, φ, {r_1}}. The space R is spgωα-T_0 but not spgωα-T_1. There is no spgωα-open set G with $r_1 \in${r_1} and $r_2 \notin${r_1} holds for $r_1 \neq r_2$.

Theorem 2.13: A space R is spgωα-T_1iff R has a singleton subset {r} which is spgωα-closed in R.

Proof: Let r ∈ R and let s ∈ $\{r\}^c$. Then s ≠ r, by R is spgωα-T_1. So, there is a spgωα-open set G such that s ∈ G but r ∉G, that is for each s∈ $\{r\}^c$, there exists G∈spgωα-O(R) with s∈G⊆ $\{r\}^c$. Thus, ∪{s: s≠ r} ⊆∪{G: s ≠ r} ⊆ $\{r\}^c$. Thus, $\{r\}^c$⊆∪{G: s≠ r} ⊆ $\{r\}^c$, that is $\{r\}^c$ = ∪{G: s≠r}. As, G is spgωα-open, then $\{r\}^c$ is spgωα-open in R and so, {r} is spgωα-closed in R.
On the other hand, let r, s ∈ R be spgωα-closed with r≠ s. Then $\{r\}^c$,$\{s\}^c$ are spgωα-open with s ∈ $\{r\}^c$ but r ∉$\{r\}^c$ and r∈ {s} but s∈ $\{s\}^c$. Then there are spgωα-open sets $\{r\}^c$ and $\{s\}^c$ under r∈ $\{s\}^c$, s ∈ $\{s\}^c$ and s∈ $\{r\}^c$, r∉ $\{r\}^c$. So, R is spgωα-T_1.

Theorem 2.14: Let Ψ: R → S be spgωα.C, injective and S is T_1. Then R is spgωα-T_1.

Proof: Let $r_1, r_2 \in$ R with $r_1 \neq r_2$. There exists $s_1, s_2 \in$ S with $s_1 \neq s_2$ such that Ψ(r_1) = s_1 and Ψ(r_2) = s_2. As S is T_1, U, V ∈spgωα-O(S), so that $s_1 \in$ U, $s_2 \notin$U and $s_1 \notin$V, $s_2 \in$ V. That is Ψ(r_1) ∈ U, Ψ(r_2) ∉U and Ψ(r_1) ∉ V, Ψ(r_2) ∈ V. So, $r_1 \in \Psi^{-1}$(U), $r_2 \notin \Psi^{-1}$ (U), $r_1 \notin \Psi^{-1}$ (V), $r_2 \in \Psi^{-1}$ (V), where Ψ^{-1} (U), Ψ^{-1} (V) ∈spgωα-O(R) follows from spgωα.C. Thus, if $r_1, r_2 \in$R with $r_1 \neq r_2$, there exist Ψ^{-1} (U), Ψ^{-1} (V) ∈spgωα-O(R) such that $r_1 \in \Psi^{-1}$(U), $r_2 \notin \Psi^{-1}$ (U) and $r_1 \notin \Psi^{-1}$ (V), $r_2 \in \Psi^{-1}$ (V). So, R is spgωα-T_1.

Definition 2.15: Let R be TS. Then R is said to be spgωα-T_2 if there are disjoint spgωα-open sets U, V with the condition r ∈ U and s ∈ V holds for each r, s∈ R, where r≠ s.

Example 2.16: Let R = {r_1, r_2, r_3} and τ={R, φ, {r_1}, {r_2}, {r_1, r_2}}. The space R is spgωα-T_2.

Remark 2.17: Every spgωα-T_2 space is spgωα-T_1. The converse of the implication is not true follows from the example.

Example 2.18: From Example 2.16, it is clear that the space R is spgωα-T_2 but not spgωα-T_1.

Theorem 2.19: The intersection of all spgωα-closed neighborhoods of each point of R is a singleton set if and only if the space R is spgωα-T_2.

Proof: Let r, s∈ R where r ≠ s. Then U, V ∈spgωα-O(R) such that r∈ U, s∈ V and U ∩ V = φ, so r∈ U ⊆ R−V follows from the definition. Thus, R − V is spgωα-closed neighborhood of r excluding s. Hence, s not in the form part of intersection of all spgωα-closed neighborhoods of r. As s is an arbitrary, the singleton set {r} is the intersection of all spgωα-closed neighborhoods of r.

On the other hand, let us consider r is the intersection of all spgωα-closed neighborhoods of any point r∈ R and y any arbitrary point of R with r ≠ s. As s does not belong to the intersection, there exists a spgωα-closed neighborhood S of r with s∉S. So, there exists U ∈spgωα-O(S) such that s ∈ U ⊆S. Consequently, U, R − S ∈spgωα-O(R) under the condition r ∈ U and s ∈ R− S with U ∩ (R−S) = φ. Hence, R is spgωα-T_2.

Theorem 2.20: If Ψ: R → S is spgωα.C, injective with S is T_2, then R is spgωα-T_2.

Proof: Consider two distinct points r_1, r_2∈ R. As Ψ is injective, there exists s_1, s_2∈ S with s_1 = Ψ(r_1) and s_2 = Ψ(r_2). There exist U, V ∈ O(S) with U ∩V = ϕ such that s_1∈ U, s_2∈V as S is T_2. So, r_1∈Ψ^{-1}(U) and r_2∈Ψ^{-1}(V)∈spgωα-O(R). Consider Ψ^{-1}(U)∩Ψ^{-1}(V) = Ψ^{-1}(U ∩V) = Ψ^{-1}(φ) = φ. Thus, for each r_1, r_2∈R with r_1≠ r_2, there are Ψ^{-1}(U), Ψ^{-1}(V) ∈spgωα-O(R) with Ψ^{-1}(U) ∩ Ψ^{-1}(V) = φ with r_1∈Ψ^{-1}(U), r_2 ∈Ψ^{-1}(V). Thus, R is spgωα-T_2.

3. SPGωA-REGULAR SPACES:

In this section we introduce the spgωα-regular spaces in topological spaces and obtained some of their properties.

Definition 3.1: A space R is spgωα-regular if there are open sets U, V such that F ⊆ U, r∈ V with U ∩V = ϕ holds for every F ∈spgωα-C(R) and r∈ F.

Remark 3.2: Every spgωα-regular space is regular but not conversely.

Example 3.3: Consider R= {r_1, r_2, r_3} and τ = {R, φ, {r_1}, {r_2, r_3}}. Then R is regular but not spgωα-regular.

Theorem 3.4: Let R be spgωα-regular and S is a spgωα-closed and open subset of R. The subspace S is spgωα-regular.

Proof: Let F be spg-closed subset of S with s ∈ S− F and so F ∈spgωα-C(R). Since R is spgωα-regular, there exist U, V∈ O(R) with s∈ U and F⊆V and U ∩V = ϕ. Thus U ∩ Sand V ∩ S are disjoint open sets in S with s∈ U ∩ S, F⊆ V ∩ S. Thus, S is spgωα-regular.

Theorem 3.5: Let Ψ: R → S is spgωα.I, bijective, open with R is spgωα-regular. Then S is spgωα-regular.

Proof: Let F ∈spgωα-C(S) with s∈F. For some point r∈ R, let s = Ψ(r). Then Ψ^{-1}(s) ∈ R and Ψ^{-1}(F) ∈spgωα-C(R) as Ψ is spgωα.I where r∈Ψ^{-1}(F). There U, V∈ O(R) with r∈ U, Ψ^{-1}(F) ⊆ V with U∩ V = ϕ. Then, s = Ψ(r) ∈ Ψ(U) and F ⊆ Ψ(V). Thus, Ψ(U), Ψ(V) ∈ O(R) such that s ∈ Ψ(U), F ⊆Ψ(V) where Ψ(U) ∩ Ψ(V) = φ holds for all s ∈ S. Hence S is spgωα-regular.

Theorem 3.6: Let Ψ is pre spgωα-open, closed, injective and S is spgωα-regular. Then R is spgωα-regular.

Proof: Let r∈ R and F ∈spgωα-C(R) with r∈ F. As Ψ is pre spgωα-open, Ψ(F) ∈spgωα-C(S) such that Ψ(r) ∈Ψ(F). As S is spgωα-regular, U, V ∈ O(R) such that Ψ(r) ∈ U and Ψ(F) ⊆ V, that is r∈Ψ^{-1}(U), F ⊆Ψ^{-1}(V) with Ψ^{-1}(U) ∩ Ψ^{-1}(V) = φ. Thus, for each r∈ R and for every F∈spgωα-C(R) with r∈ F, there exist Ψ^{-1}(U), Ψ^{-1}(V) ∈ O(R) such that r∈Ψ^{-1}(U) and F ⊆Ψ^{-1} (V) with Ψ^{-1}(U) ∩ Ψ^{-1}(V) = φ. Thus, R is spgωα-regular.

4. SPGωA-NORMAL SPACES

In this section, we introduce the spgωα-normal spaces in topological spaces and obtained some of their properties

Definition 4.1: A space R is said to be spgωα-normal if for any disjoint A, B ∈spgωα-C(R), there exist U, V∈O(R) with A⊆U, B ⊆V and U ∩ V = ϕ.

Example 4.2: Consider R = {r_1, r_2, r_3, r_4} and τ = {R, φ, {r_1}, {r_2}, {r_1, r_2}, {r_2, r_4}, {r_1, r_2, r_4}, {r_2, r_3, r_4}}. Then R is spgωα-normal.

Example 4.3: Let R= {r_1, r_2, r_3} and τ = {R, φ, {r_1}, {r_2, r_3}}. Then R is normal but not spgωα-normal.

Theorem 4.4: A space R is normal if and only if there exists spg-open set A ⊆ U ⊆ cl(U) ⊆ V holds for each closed set A and an open set V containing A.

Theorem 4.5: Let Ψ: R →S be spgωα.I, bijective, open mapping and R is spgωα-normal. Then S is spgωα-normal

Proof: Let A, B ∈spgωα-C(S) with A ∩B = ϕ. As Ψ is spgωα.I, Ψ^{-1}(A), Ψ^{-1}(B) ∈spgωα-C(R). Then there exist U, V ∈ O(R) with Ψ^{-1}(A) ⊆ U and Ψ^{-1}(B) ⊆ V, as Ψ is open. As Ψ is bijective, Ψ(U), Ψ(V) ∈ O(S) such that A ⊆ Ψ(U) and B ⊆ Ψ(V) with Ψ(U) ∩ Ψ(V) = φ. Hence S is spgωα-normal.

Theorem 4.6: Let R is spgωα-normal and S is spgωα-closed subset of R. Then the subspace S is spgωα-normal.

Proof: Let A, B ∈spgωα-C(S) with A∩B = ϕ. Then A, B ∈ spgωα-C(R). As R is spgωα-normal, there are U, V∈ O(R) such that A⊆ U and B ⊆ V. As a result, U ∩ S, V ∩ S are disjoint open subsets in the subspace S with A ⊆ U ∩S, B ⊆ V ∩ S. Thus, S is spgωα-normal.

5. SPGωA-COMPACTNESS

In this section, we introduce spgωα-compact, countably spgωα-compact and spgωα-Lindelöf using spgωα-open sets in TS and studied their properties.

Definition 5.1: A collection {B^*_i: i∈I} of spgωα-open sets in a **TS R** is called spgωα-open cover if $B^* \subseteq \bigcup_{i\in I} B^*_i$.

Definition 5.2: A TS R is called spgωα-compact if every spgωα-open cover of R has a finite subcover.

Definition 5.3: A subset B* of a TS Ris spgωα-compact relative to R if for every collection {B^*_i: i∈I} of spgωα-open sets of R with $B^* \subseteq \bigcup_{i\in I} B_i^*$ there exists a finite subset I_0 of I such that $B^* \subseteq \bigcup_{i\in I_0} B^*_i$.

Definition 5.4: A subset B* of a TS R is spgωα-compact if B* is spgωα-compact of the subspace of R.

Theorem 5.5: A spgωα-closed subset of spgωα-compact space is spgωα-compact relative to R.

Proof: Let A* ∈spgωα-C(R*). Then (R- A*) ∈spgωα-O(R*).

Let S = {A^*_i: i∈I} be a spgωα-open cover of A* by spgωα-open subsets. Then S* = S ∪(R - A*) is a spgωα-open cover, that is R = [∪ {A^*_i: i∈I}] ∪(R - A*).

But, from hypothesis R is spgωα-compact and hence S* is reducible to a finite subcover, say S* = R ∪ A^*_{i1}∪ A^*_{i2}∪ ...∪ A^*_{in}∪ (R - A*), A^*_{ik}∈ S*. Since A* ∩ (R - A*) = ∅, so A* ⊆ A^*_{i1}∪ A^*_{i2}∪…...∪ A^*_{in} ∈ S. Thus, a spgωα-open cover S contains a finite subcover and so A* is spgωα-compact relative to R.

Theorem 5.6: Let Ψ be surjective, spgωα.C. If R is spgωα-compact, then S is compact.

Proof: Let {A^*_i: i∈I} be an open cover. Since Ψ is spgωα.C, then {$\Psi^{-1}(A^*_i)$: i∈I} is spgωα-open cover of R which has a finite subcover say {$\Psi^{-1}(A^*_i)$: i=1…...n}.

Thus R = $\bigcup_{i=1}^{n}\psi^{-1}(A^*_i)$, that is Ψ(R) = $\bigcup_{i=1}^{n}A^*_i$. As Ψis surjective, that is S* = $\bigcup_{i=1}^{n}A^*_i$.

Thus {A^*_1, A^*_2, ... A^*_n} is a finite subcover of {A^*_i: i∈I}. So, S is compact.

Theorem 5.7: If a function Ψis spgωα.I and B be a subset of R which is spgωα-compact relative to R, then Ψ(B) is spgωα-compact relative to S.

Proof: Let {A^*_i: i∈I} be any collection of spgωα-open sets in with Ψ(B) = $\bigcup_{i\in I}A^*_i$.

Then B ⊆ $\bigcup_{i\in I}\psi^{-1}(A^*_i)$, where {$\Psi^{-1}(A^*_i$: i∈ I} is spgωα-open in R. As B is spgωα-compact, there exists finite subcollection {A^*_1, A^*_2, ..., A^*_n} such that B ⊆ $\bigcup_{i\in I_0}\Xi^{-1}(A^*_i)$.

Thus Ψ(B) ⊆ $\bigcup_{i\in I_0}A_i$ and so Ψ(B) is spgωα-compact relative to S.

Theorem 5.8: Every spgωα-compact space is compact.

Theorem 5.9: A TS R* is spgωα-compact if and only if every family of spgωα-closed sets of R having finite intersection property has a non-empty intersection.

Proof: Suppose R is spgωα-compact and {A^*_i: i∈I} be a family of spgωα-closed sets with finite intersection property. We need to prove that $\bigcap_{i\in I}A_i \neq \phi$.

On the contrary $\bigcap_{i\in I}A_i^* = \phi$. Then $R - \bigcup_{i\in I}A^*_i = R.$, that is $\bigcup_{i\in I}(R - A^*_i) = R.$

The cover {R – A^*_i; i∈I} is a spgωα-open cover of R. As R is spgωα-compact, spgωα-open cover {R – A^*_i; i∈I} has a finite subcover, say {R – A^*_i; i=1…...n}, that is R = $\bigcup_{i=1}^{n}(R^* - A^*_i)$, which implies that R = R – $\bigcap_{i=1}^{n}A^*_i$, that is R – R = R - $\left[R - \bigcap_{i=1}^{n}A_i\right]$ and so $\phi = \bigcap_{i=1}^{n}A^*_i$. This contradicts the assumption. Hence $\bigcap_{i=1}^{n}A^*_i \neq \phi$.

Other part, suppose every family of spgωα-closed sets of R with finite intersection property has a non-empty intersection. To prove that R is spgωα-compact. Suppose R is not a spgωα-compact. Then there exists a spgωα-open cover say {G_i: i∈I} having no finite subcover. That is for any finite sub family {G_i: i =1...n} of {G_i: i∈I}, we have $\bigcup_{i=1}^{n}G_i \neq R$ which implies that $R - \bigcup_{i=1}^{n}G_i \neq$ R – R, and so $\bigcap_{i=1}^{n}(R - G_i) \neq \phi$.

Then the family {R – G_i: i∈I} of spgωα-closed sets has a finite intersection property. By assumption $\bigcap_{i=1}^{n}(R - G_i) \neq \phi$, that is R – $\bigcup_{i=1}^{n}G^*_i \neq \phi$ and so $\bigcup_{i=1}^{n}G_i \neq R$. Hence {$G_i$: i∈ I} is not a cover of R, which contradicts the fact that {G_i: i∈I} is a cover for R. So, a spgωα-open cover {G_i: i∈I} has a finite subcover {G_i: i =1…...n} and so R is spgωα-compact.

Definition 5.10: A TS R is said to be countably spgωα-compact (C.spgωα-compact) if every countable spgωα-open cover of R has a finite subcover.

Theorem 5.11: If R is a C.spgωα-compact space, then R is countably compact.

Proof: Let {A_i: i∈I} be a countable open cover of R by open sets in R. Then {A_i: i∈I} is C.spgωα-open cover of R.As R is C.spgωα-compact, the countable spgωα-open cover of R has a finite subcover, say S = {A^*_i: i=1....n}.Hence R is countably compact.

Theorem 5.12: Every spgωα-compact space is C.spgωα-compact.

Theorem 5.13: If Ψis spgωα.C from a C.spgωα-compact space R onto S, then S is countably compact.

Proof: Let {A^*_i :i∈I} be a countable open cover of S. As Ψ is spgωα.C, then {$\Psi^{-1}(A^*_i)$: i∈I} is countable spgωα-open cover of R. Since R is C.spgωα-compact, the countable spgωα-open cover {$\Psi^{-1}(A^*_i)$: i∈I} of R has a finite subcover say {$\Psi^{-1}(A^*_i)$: i=1...n}.

Thus R = $\bigcup_{i=1}^{n}\psi^{-1}(A*_i)$ implies Ψ(R) = $\bigcup_{i=1}^{n}A*_i$ and so S = $\bigcup_{i=1}^{n}A*_i$. Hence {$A*_1$, $A*_2$, ..., $A*_n$} is a finite subcover for S, so S is countably compact.

Theorem 5.14: The image of a countably spgωα-compact space under spgωα.I is C.spgωα-compact.

Proof: Let Ψbe a spgωα.I from a C.spgωα-compact space R onto S and {$A*_i$: i∈I} be a countable spgωα-open cover of S. Then {$\Psi^{-1}(A*_i)$: i∈I} is a countable spgωα-open cover of R. Since R is C.spgωα-compact, the countable spgωα-open cover {$\Psi^{-1}(A*_i)$: i∈I} of R has a finite subcover say {$\Psi^{-1}(A*_i)$: i =1...n}. Thus R = $\bigcup_{i=1}^{n}\psi^{-1}(A*_i)$, that isΨ(R) = $\bigcup_{i=1}^{n}A*_i$ and so S = $\bigcup_{i=1}^{n}A_i*$. So, S is C.spgωα-compact.

Definition 5.15: A TS R is spgωα-Lindelöf (spgωα.L) if every spgωα-open cover of R has a countable subcover.

Theorem 5.16: Every spgωα.L space is Lindelöf.

Proof: Let R be spgωα.L. Let {$A*_i$: i∈I} be an open cover of R and so {$A*_i$: i∈I} is spgωα-open cover of R is spgωα-open in R. As R is spgωα.L, the spgωα-open cover {$A*_i$: i∈I} of R has countable subcover. Hence R is Lindelöf.

Theorem 5.17: Every spgωα-compact space is spgωα.L.

Proof: Let R be spgωα-compact and {$A*_i$: i∈I} be spgωα-open cover of R. Then {$A*_i$: i∈I} has a finite subcover say {$A*_i$: i=1...n}. Since every finite subcover is always a countable subcover and so {$A*_i$: i=1......n} is a countable subcover for R. Hence R is spgωα.L.

Theorem 5.18: If Ψ is spgωα.C from a spgωα.L space R onto S, then S is Lindelöf.

Proof: Let {$A*_i$: i∈I} be an open cover of S. AsΨ is spgωα.C, {$\Psi^{-1}(A*_i)$: i∈I} is spgωα-open cover of R. Since R is spgωα.L, the spgωα-open cover {$\Psi^{-1}(A*_i)$:i∈I} has a countable subcover say S = {$\Psi^{-1}(A_{i_n})$: n∈N}. Therefore R = $\bigcup_{n\in N}\psi^{-1}(A*_{i_n})$, that isΨ (R) = S = $\bigcup_{n\in N}A*_{i_n}$, where { $A*_{i_n}$: n∈N} is a countable subcover for S and so S is Lindelöf.

Theorem 5.19: The image of spgωα.L under spgωα.I is spgωα.L.

Proof: Let Ψbe a spgωα.I form spgωα.L space R onto S. Let {A*: i∈I} be a spgωα-open cover of S. Then {Ψ^{-1}(A*): i∈I} is spgωα-open cover of R as Ψ is spgωα.I. Since R is spgωα.L, the spgωα-open cover {Ψ^{-1}(A*): i∈I} of R has a countable subcover say {$\Psi^{-1}(A_{i_n})$: n∈N}. Thus R = $\bigcup_{n\in N}\psi^{-1}(A*_{i_n})$ which implies Ψ(R) = S = $\bigcup_{n\in N}A_{i_n}$, that is { A_{i_n} : n∈N} is a countable subfamily of {A_i: i∈I}. Hence S is Lindelöf.

Theorem 5.20: If R is spgωα.L and countable spgωα-compact, then R is spgωα-compact.

Proof: Suppose R is countable spgωα-compact and spgωα.L and {A_i: i∈I} be a spgωα-open cover of R. As R is spgωα.L, {A_i: i∈I} has a countable subcover say { A_{i_n} : n∈N}. Therefore { A_{i_n} : n∈N} is a countable subcover of R and { A_{i_n} : n∈N} is a subfamily of {A_i: i∈I} and so { A_{i_n} : n∈N} is a countably spgωα-open cover of R. Since R is C.spgωα.-compact, { A_{i_n} : n∈N} has a finite subcover say { A_{i_n} : n∈N} ⊆ {A_i :i∈I} and so { A_{i_n} : n∈N} is a finite subcover of {A_i :i∈I} for R. Hence R is spgωα-compact.

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