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Arithmetic Sequential Graceful Labeling on Step Grid Graph

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ARTICLE INFO	ABSTRACT
Published Online: 15 July 2024 **Corresponding Author:** P. Sumathi	Let G be a simple, finite, connected, undirected, non-trivial graph with p vertices and q edges. $V(G)$ be the vertex set and $E(G)$ be the edge set of G. Let $f: V(G) \to \{a, a+d, a+2d, a+3d, \ldots, 2(a+qd)\}$ where a ≥ 0 and $d \geq 1$ is an injective function. If for each edge $uv \in E(G)$, $f^*: E(G) \to \{d, 2d, 3d, 4d, \ldots, qd\}$ defined by $f^*(uv) = \lvert f(u) - f(v) \rvert$ is a bijective function then the function f is called arithmetic sequential graceful labeling. The graph with arithmetic sequential graceful labeling is called arithmetic sequential graceful graph. In this paper, arithmetic sequential graceful labeling for some special graphs were studied.

KEYWORDS: Graceful labeling, Step grid graph, Path union of step grid graph, Cycle of step grid graph, Star of step grid graph.

1. INTRODUCTION

A fascinating area of research in graph theory is labeling. Giving values to edges or vertices is the process of labeling. It was Alexander Rosa [2] who first proposed the idea of graceful labeling. Later, a few labeling techniques were presented. See Gallian's dynamic survey [3] for further details. V J Kaneria1 , Meera Meghpara , H M Makadia Pasaribu[4] proved that open star of grid graph is graceful. V J Kaneria1 , Meera Meghpara , H M Makadia Pasaribu[5]proved that star of grid graph is graceful. V. J. Kaneria, H. M. Makadia and M. M. Jariya[6] proved that cycle of graph is Graceful labeling. V. J. Kaneria, H. M. Makadia proved that step grid graph is graceful[8]. Here are the some of the definitions which are helpful in this article.

2. DEFINITIONS

Definition 2.1:

Take $P_n, P_n, P_{n-1,\ldots,} P_2$ paths on $n, n, n-1, n-2, \ldots, 3, 2$ vertices and arrange them vertically. A graph obtained by joining horizontal vertices of given successive path is known as a step grid graph of size n, where $n \geq 3$. It is denoted by Sg_n.

Definition 2.2:

let G be a graph and $G_1, G_2, G_3, \ldots, G_n, n \geq 2$ be n copies of graph G. Then the graph obtained by adding an edge from $G_i$to $G_{i+1} (1 \leq i \leq n-1)$ is called path union of G.

Definition 2.3:

For a cycle C_n, each vertex of C_n is replaced by connected graphs $G_1, G_2, G_3, \ldots, G_n$ and is known as cycle of graphs. We shall denote it by $C(G_1, G_2, G_3, \ldots, G_n)$. If we replace each vertex by a graph G, i.e.$G_1 = G, G_2 = G, G_3 = G, \ldots, G_n = G$,such cycle of graph G is denoted by $C(n. G)$.

Definition 2.4:

Let G be a graph on n vertices. The graph obtained by replacing each vertex of the star $K_{1,n}$ by a copy of G is called a star of G and it is denoted by G^*

3. Main Results

Theorem 3.1:

A step grid graph $Sg_n, n \geq 3$ admits arithmetic sequential graceful labeling.

Proof:

Let G be a step grid graph. A graph with vertex set $V(G) = \{u_{i,j} : 1 \leq i \leq 2, 1 \leq j \leq n\} \cup \{u_{i,j} : 3 \leq i \leq n, 1 \leq j \leq n+2-i\}$ and the edge set is $E(G) = \{u_{i,j}\, u_{i,(j+1)}: 1 \leq i \leq 2, 1 \leq j \leq n\} \cup \{u_{i,j}\, u_{i,(j+1)}: 3 \leq i \leq n, 1 \leq j \leq n+2-i\} \cup \{u_{1,j}\, u_{2,j}: 1 \leq j \leq n\} \cup \{u_{i,j} u_{i+1,j}: 2 \leq i \leq n-1, 1 \leq j \leq n+1-i\}$ where $n \geq 3$ is known as a step grid graph of size n. It is denoted by Sg_n.

Here $|V| = \frac{n^2+3n-2}{2}, |E| = n^2+n-2$

We define a function $f: V(G) \to \{a, a+d, a+2d, a+3d, \ldots, 2(a+qd)\}$

The vertex labeling are as follows,

$f(u_{1,j}) = a + \left[\frac{2(n^2+n-2)-1+j^2}{4}\right] d, if\ j \equiv 1(mod\ 2), 1 \le j \le n.$

$f(u_{1,j}) = a + \left[\frac{2(n^2+n-2)-j^2}{4}\right] d, if\ j \equiv 0(mod\ 2), 1 \le j \le n.$

$f(u_{i,j}) = a + [f(u_{i-1,j+1}) + (-1)^j]d, when\ i = 2, 1 \le j \le (n-i+1)$

$f(u_{i,j}) = a + [f(u_{i-1,j+2}) + (-1)^j]d, when\ i = 3, 1 \le j \le (n-i+1)$

$f(u_{i,1}) = a + [(n-i+1)^2 - 1]d, \frac{n}{2} \le i \le n$

$f(u_{i,2}) = a + [(n^2+n-2) - (n-i+1)(n-i)]d, \frac{n}{2} \le i \le n$

$f(u_{i,j}) = a + [f(u_{i+1,j-2}) + (-1)^{j-1}]d, when\ 2 \le i \le n-1, 3 \le j \le (n+2-i)$

From the function $f^*: E(G) \to \{d, 2d, 3d, 4d, \dots, qd\}$ we get the edge labels of the step grid graph $Sg_n, n \ge 3$ as follows

Table 1. Edge labels of the graph $Sg_n, n \ge 3$

$f^*(u\,v), \forall\ uv \in E(G)$	**Edge labels**
$f^*(u_{1,j}u_{i,j})$	$= \left[\left[\frac{2(n^2+n-2)-j^2}{4}\right] - [f(u_{i-1,j+1}) + (-1)^j]\right] d,$ $if\ j \equiv 0(mod\ 2), 1 \le j \le n, when\ i = 2, 1 \le j \le (n-i+1)$ $= \left[\frac{2(n^2+n-2)-j^2}{4} - [f(u_{i-1,j+2}) + (-1)^j]\right] d,$ $if\ j \equiv 0(mod\ 2), 1 \le j \le n, when\ i = 3, 1 \le j \le (n-i+1)$
$f^*(u_{1,j}u_{i,j})$	$= \left[\frac{2(n^2+n-2)-1+j^2}{4} - [f(u_{i-1,j+1}) + (-1)^j]\right] d,$ $if\ j \equiv 1(mod\ 2), 1 \le j \le n, when\ i = 2, 1 \le j \le (n-i+1)$ $= \left[\frac{2(n^2+n-2)-1+j^2}{4} - [f(u_{i-1,j+2}) + (-1)^j]\right] d,$ $if\ j \equiv 1(mod\ 2), 1 \le j \le n, when\ i = 3, 1 \le j \le (n-i+1)$
$f^*(u_{i,1}u_{i,2})$	$= [[(n-i+1)^2 - 1] - [(n^2+n-2) - (n-i+1)(n-i)]]d, \frac{n}{2} \le i \le n$
$f^*(u_{i,2}u_{i,j})$	$= \left[[(n^2+n-2) - (n-i+1)(n-i)] - [f(u_{i+1,j-2}) + (-1)^{j-1}]\right] d,$ $\frac{n}{2} \le i \le n, when\ 2 \le i \le n-1, 3 \le j \le (n+2-i)$

It is clear that the function f is injective and also table 1 shows that $f^*: E \to \{d, 2d, 3d, 4d, \dots, qd\}$ is bijective. Hence f is arithmetic sequential graceful labeling and the graph Sg_n is arithmetic sequential graceful graph.

Example 3.1.1: Step grid graph of Sg_8 and its graceful labeling shown in Figure -1.

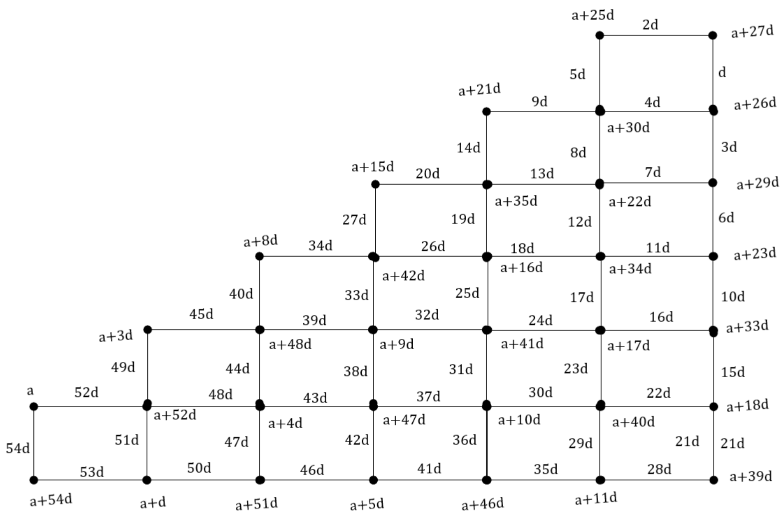

Figure -1: step grid graph with $n = 8$ and its graceful labelling.

Theorem 3.2:
A path union of r copies of step grid graph admits arithmetic sequential graceful labeling for $n \geq 3$.

Proof :
Let $V(G) = \{u_{k,i,j} : 1 \leq k \leq r, 1 \leq i \leq 2, 1 \leq j \leq n\} \cup \{u_{k,i,j} : 1 \leq k \leq r, 3 \leq i \leq n, 1 \leq j \leq n+2-i\}$ and $E(G) = \{u_{k,i,j}u_{k,i,j+1} : 1 \leq k \leq r, 1 \leq i \leq 2, 1 \leq j \leq n\} \cup \{u_{k,i,j}u_{k,i,(j+1)} : 1 \leq k \leq r, 3 \leq i \leq n, 1 \leq j \leq n+2-i\} \cup \{u_{k,1,j}\, u_{k,2,j} : 1 \leq k \leq r, 1 \leq j \leq n\} \cup \{u_{k,i,j}\, u_{k,i+1,j-1} : 1 \leq k \leq r, 2 \leq i \leq n, 1 \leq j \leq n+2-i\} \cup \{u_{k,1,1}\, u_{k+1,n,1} : 1 \leq k \leq r-1\}$

Here $|V| = \frac{r(n^2+3n-2)}{2}$, $|E| = r(n^2+n-1)-1$

Join the vertices $u_{k,1,1}$ to $u_{k+1,n,1}$ for $k = 1,2,\ldots,r-1$ by an edge to form the path union of r copies of step grid graph.

We define a function $f: V(G) \to \{a, a+d, a+2d, a+3d, \ldots, 2(a+qd)\}$

The vertex labeling are as follows

$$f(u_{1,j}) = a + \left[\frac{2(n^2+n-2)-1+j^2}{4}\right] d, \text{if } j \equiv 1 (\text{mod } 2), 1 \leq j \leq n.$$

$$f(u_{1,j}) = a + \left[\frac{2(n^2+n-2)-j^2}{4}\right] d, \text{if } j \equiv 0 (\text{mod } 2), 1 \leq j \leq n.$$

$$f(u_{i,j}) = a + [f(u_{i-1,j+1}) + (-1)^j]d, \text{when } i = 2, 1 \leq j \leq (n-i+1)$$

$$f(u_{i,j}) = a + [f(u_{i-1,j+2}) + (-1)^j]d, \text{when } i = 3, 1 \leq j \leq (n-i+1)$$

$$f(u_{i,1}) = a + [(n-i+1)^2 - 1]d, \ \forall \frac{n}{2} \leq i \leq n$$

$$f(u_{i,2}) = a + [(n^2+n-2) - (n-i+1)(n-i)]d, \ \forall \frac{n}{2} \leq i \leq n$$

$$f(u_{i,j}) = a + [f(u_{i+1,j-2}) + (-1)^{j-1}]d, \ \forall 2 \leq i \leq n-1, 3 \leq j \leq (n+2-i)$$

$$f(u_{1,i,j}) = a + [f(u_{i,j})]d, \quad \text{if } f(u_{i,j}) < \frac{(n^2+n-2)}{2}$$

$$f(u_{1,i,j}) = a + \left[f(u_{i,j}) + [r(n^2+n-1)-1] + \frac{(n^2+n-2)}{2}\right] d, \text{ if } f(u_{i,j}) > \frac{(n^2+n-2)}{2}, \ \forall 1 \leq i \leq n, 1 \leq j \leq n$$

$$f(u_{k,i,j}) = a + \left[f(u_{k-1,i,j}) + \frac{(n^2+n-2)}{2}\right] d, \ \text{if } f(u_{k-1,i,j}) < \frac{r(n^2+n-1)-1}{2}$$

$$f(u_{k,i,j}) = a + \left[f(u_{k-1,i,j}) - \frac{(n^2+n-2)}{2}\right] d, \text{if } f(u_{k-1,i,j}) > \frac{r(n^2+n-1)-1}{2}$$

$$\forall 2 \leq k \leq r, 1 \leq i \leq n, 1 \leq j \leq n$$

From the function $f^*: E(G) \to \{d, 2d, 3d, 4d, \ldots, qd\}$ we get the edge labels of the graph $P(r.Sg_n)$ as follows

Table 2. Edge labels of the graph $P(r.Sg_n), n \geq 3$

$f^*(u\,v), \forall\, uv \in E(G)$	**Edge labels**
$f^*(u_{1,i,j}u_{k,i,j})$	$= \left[f(u_{i,j}) - \left[f(u_{k-1,i,j}) + \frac{(n^2+n-2)}{2}\right]\right]d,$ $if\ f(u_{i,j}) < \frac{(n^2+n-2)}{2},$ $f(u_{k-1,i,j}) < \frac{r(n^2+n-1)-1}{2}\ when\ 1 \leq i \leq n,$ $1 \leq j \leq n, k = 2,3,\dots,r$

It is clear that the function f is injective and also table 2 shows that $f^*: E \to \{d, 2d, 3d, 4d, \dots, qd\}$ is bijective. Hence f is arithmetic sequential graceful labeling and the graph $P(r.Sg_n), n \geq 3$ is arithmetic sequential graceful graph.

Example 3.2.1: The Path union of 3 copies of Sg_4 and its graceful labelling shown in figure-2

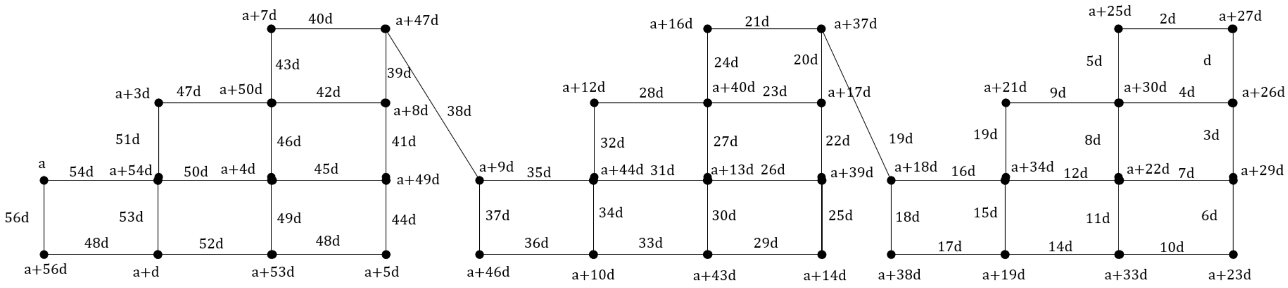

Figure -2: The Path union of 3 copies of Sg_4 and its graceful labelling.

Theorem 3.3:
The cycle of r copies of step grid graph admits arithmetic sequential graceful labeling, where $n \geq 3$ and $r \equiv 0,3(mod4)$.

Proof:

Let $V(G) = \{u_{k,i,j} : 1 \leq k \leq r, 1 \leq i \leq 2, 1 \leq j \leq n\} \cup \{u_{k,i,j} : 1 \leq k \leq r, 3 \leq i \leq n, 1 \leq j \leq n+2-i\}$ and $E(G) = \{u_{k,i,j}u_{k,i,j+1} : 1 \leq k \leq r, 1 \leq i \leq 2, 1 \leq j \leq n\} \cup \{u_{k,i,j}u_{k,i,(j+1)} : 1 \leq k \leq r, 3 \leq i \leq n, 1 \leq j \leq n+2-i\} \cup \{u_{k,1,j}\, u_{k,2,j} : 1 \leq k \leq r, 1 \leq j \leq n\} \cup \{u_{k,i,j}\, u_{k,i+1,j-1} : 1 \leq k \leq r, 2 \leq i \leq n, 1 \leq j \leq n+2-i\} \cup \{u_{k,1,1}\, u_{k+1,1,1} : 1 \leq k \leq r-1\}$

Here $|V| = \frac{r(n^2+3n-2)}{2}$, $|E| = r(n^2+n-1)$

Join the vertices $u_{k,1,1}$ with $u_{k+1,1,1}$ for $k = 1,2,\dots,r-1$ and $u_{r,1,1}$ with $u_{1,1,1}$ by an edge to form the cycle of step grid graph.

We define a function $f: V(G) \to \{a, a+d, a+2d, a+3d, \dots, 2(a+qd)\}$

The vertex labeling are as follows,

$$f(u_{1,i,j}) = a + [f(u_{i,j})]d, \quad if\ f(u_{i,j}) < \frac{(n^2+n-2)}{2}$$
$$= a + \left[f(u_{i,j}) + [r(n^2+n-1) - (n^2+n-2)]\right]d,\ if\ f(u_{i,j}) > \frac{(n^2+n-2)}{2}$$
$$\forall\, 1 \leq i \leq n, 1 \leq j \leq n.$$

$$f(u_{2,i,j}) = a + \left[f(u_{1,i,j}) + [r(n^2+n-1) - (n^2+n-2)]\right]d, if\ f(u_{1,i,j}) < \frac{r(n^2+n-1)}{2}$$

$$= a + \left[f(u_{1,i,j}) - [r(n^2+n-1) - (n^2+n-2)]\right]d, if\ f(u_{1,i,j}) > \frac{r(n^2+n-1)}{2}$$
$$\forall\, 1 \leq i \leq n, 1 \leq j \leq n.$$

$$f\left(u_{\left(\frac{r}{2}\right)+1,i,j}\right) = a + \left[f\left(u_{\left(\frac{r}{2}\right)-1,i,j}\right) + (n^2+n)\right]d\ \ if\ f\left(u_{\left(\frac{r}{2}\right)-1,i,j}\right) < \frac{r(n^2+n-1)}{2}$$
$$= a + \left[f\left(u_{\left(\frac{r}{2}\right)-1,i,j}\right) - (n^2+n-1)\right]d\ \ if\ f\left(u_{\left(\frac{r}{2}\right)-1,i,j}\right) > \frac{r(n^2+n-1)}{2}$$
$$\forall\, 1 \leq i \leq n, 1 \leq j \leq n.$$

$$f\left(u_{\left(\frac{r}{2}\right)+2,i,j}\right) = a + \left[f\left(u_{\left(\frac{r}{2}\right),i,j}\right) + (n^2+n)\right]d\ \ if\ f\left(u_{\left(\frac{r}{2}\right),i,j}\right) < \frac{r(n^2+n-1)}{2}$$
$$= a + \left[f\left(u_{\left(\frac{r}{2}\right),i,j}\right) - (n^2+n-1)\right]d\ \ if\ f\left(u_{\left(\frac{r}{2}\right),i,j}\right) > \frac{r(n^2+n-1)}{2}$$
$$\forall\, 1 \leq i \leq n, 1 \leq j \leq n.$$

$f(u_{k,i,j}) = a + [f(u_{k-2,i,j}) - (n^2 + n - 1)]d$ if $f(u_{k-2,i,j}) > \frac{r(n^2+n-1)}{2}$

$= a + [f(u_{k-2,i,j}) + (n^2 + n - 1)]d$ if $f(u_{k-2,i,j}) < \frac{r(n^2+n-1)}{2}$, $\forall k = \frac{r}{2}+3, \frac{r}{2}+4, \dots, r$, $\forall 1 \le i \le n, 1 \le j \le n$

From the function $f^*: E(G) \to \{d, 2d, 3d, 4d, \dots, qd\}$ we get the edge labels of $C(r.Sg_n)$, n ≥ 3 as follows

Table 3. Edge labels of $C(r.Sg_n)$, n ≥ 3

$f^*(u\,v), \forall\, uv \in E(G)$	**Edge labels**
$f^*(u_{1,i,j}u_{2,i,j})$	$= [f(u_{i,j}) - f(u_{1,i,j}) + 2[r(n^2+n-1) - (n^2+n-2)]]d$, if $f(u_{i,j}) > \frac{(n^2+n-2)}{2}$, $\forall 1 \le i \le n, 1 \le j \le n$ if $f(u_{1,i,j}) > \frac{r(n^2+n-1)}{2}$ $\forall 1 \le i \le n, 1 \le j \le n$
$f^*\left(u_{2,i,j}u_{\left(\frac{r}{2}\right)+1,i,j}\right)$	$= \left[f(u_{1,i,j}) - [r(n^2+n-1) - (n^2+n-2)] - f\left(u_{\left(\frac{r}{2}\right)-1,i,j}\right) + (n^2+n-1)\right]d$, if $f(u_{1,i,j}) > \frac{r(n^2+n-1)}{2}$, $\forall 1 \le i \le n, 1 \le j \le n$, if $f\left(u_{\left(\frac{r}{2}\right)-1,i,j}\right) > \frac{r(n^2+n-1)}{2}$, $\forall 1 \le i \le n, 1 \le j \le n$.
$f^*\left(u_{\left(\frac{r}{2}\right)+1,i,j}f(u_{k,i,j})\right)$	$= \left[f\left(u_{\left(\frac{r}{2}\right)-1,i,j}\right) - [n^2+n-1] - f(u_{k-2,i,j}) - [n^2+n]\right]d$, if $f\left(u_{\left(\frac{r}{2}\right)-1,i,j}\right) > \frac{r(n^2+n-1)}{2}$, $\forall 1 \le i \le n, 1 \le j \le n$, if $f(u_{k-2,i,j}) < \frac{r(n^2+n-1)}{2}$, $\forall k = \frac{r}{2}+3, \frac{r}{2}+4, \dots, r$ $\forall 1 \le i \le n, 1 \le j \le n$.

It is clear that the function f is injective and also table 3 shows that $f^*: E \to \{d, 2d, 3d, 4d, \dots, qd\}$ is bijective. Hence f is arithmetic sequential graceful labeling and the graph is arithmetic sequential graceful graph.

Example 3.3.1: The cycle of 4 copies of Sg_4 and its graceful labelling shown in figure-3

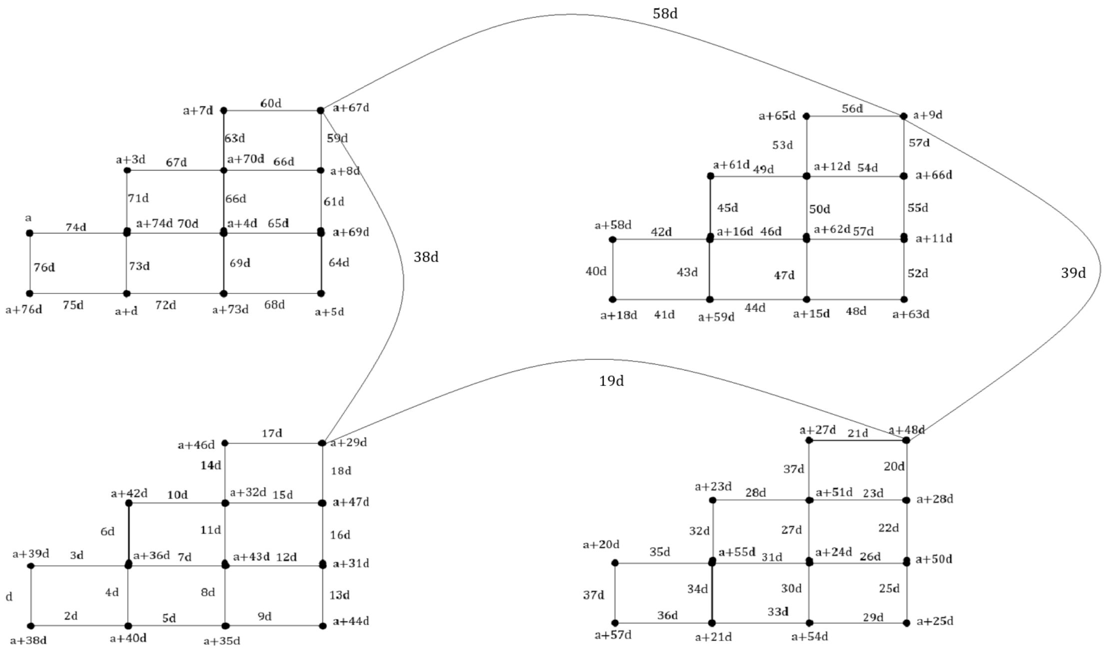

Figure-3: A cycle of 4 copies of Sg_4 and its graceful labelling.

Theorem 3.4:

Star of step grid graph Sg_n admits arithmetic sequential graceful labeling, where $n \geq 3$.

Proof:

Let $V(G) = \{u_{k,i,j} : 1 \leq k \leq r, 1 \leq i \leq 2, 1 \leq j \leq n\} \cup \{u_{k,i,j} : 1 \leq k \leq r, 3 \leq i \leq n, 1 \leq j \leq n+2-i\}$ and $E(G) = \{u_{k,i,j}u_{k,i,j+1} : 1 \leq k \leq r, 1 \leq i \leq 2, 1 \leq j \leq n\} \cup \{u_{k,i,j}u_{k,i,(j+1)} : 1 \leq k \leq r, 3 \leq i \leq n, 1 \leq j \leq n+2-i\} \cup \{u_{k,1,j}\, u_{k,2,j} : 1 \leq k \leq r, 1 \leq j \leq n\} \cup \{u_{k,i,j}\, u_{k,i+1,j-1} : 1 \leq k \leq r, 2 \leq i \leq n, 1 \leq j \leq n+2-i\} \cup \{u_{0,i,j}\, u_{k,i,j} : 1 \leq i \leq n, 1 \leq j \leq n, 1 \leq k \leq r-1\}$

Here $|V| = \frac{(n^2+3n-2)(n^2+3n)}{4}$ and $|E| = \frac{(n^2+n-1)(n^2+3n)}{2} - 1$

We define a function $f: V(G) \to \{a, a+d, a+2d, a+3d, \ldots, 2(a+qd)\}$

The vertex labeling are as follows,

$$f(u_{0,i,j}) = a + [f(u_{i,j})]d, if \ f(u_{i,j}) \geq \frac{(n^2+n-2)}{2}$$

$$= a + \left[f(u_{i,j}) + \left[\frac{(n^2+n-1)(n^2+3n)}{2} - 1 - (n^2 + n-2) \right] \right] d, \ if \ f(u_{i,j}) \leq \frac{(n^2+n-2)}{2}$$

$$\forall\, 1 \leq i \leq n, 1 \leq j \leq n.$$

$$f(u_{1,i,j}) = a + \left[f(u_{0,i,j}) + \left[\frac{(n^2+3n-2)(n^2+n-1)}{2} \right] \right] d, if \ f(u_{0,i,j}) < \frac{(n^2+3n)(n^2+n-1)-2}{4}$$

$$= a + \left[f(u_{0,i,j}) - \left[\frac{(n^2+3n-2)(n^2+n-1)}{2} \right] \right] d, if \ f(u_{0,i,j}) > \frac{(n^2+3n)(n^2+n-1)-2}{4}$$

$$\forall\, 1 \leq i \leq n, 1 \leq j \leq n.$$

$$f(u_{k,i,j}) = a + \left[f(u_{k-2,i,j}) + [n(n+1)] \right] d, if \ f(u_{k-2,i,j}) < \frac{(n^2+3n)(n^2+n-1)-2}{4}$$

$$= a + \left[f(u_{k-2,i,j}) - [n(n+1)] \right] d, if \ f(u_{k-2,i,j}) > \frac{(n^2+3n)(n^2+n-1)-2}{4}$$

$$\forall\, 1 \leq i \leq n, 1 \leq j \leq n, k = 2,3, \ldots, \frac{(n^2+3n-2)}{2}$$

From the function $f^*: E(G) \to \{d, 2d, 3d, 4d, \ldots, qd\}$ we get the edge labels of star graph of (Sg_n), n ≥ 3 as follows

Table 4. Edge labels of star graph of (Sg_n), n ≥ 3

$f^*(u\,v), \forall\, uv \in E(G)$	Edge labels
$f^*(u_{0,i,j}u_{1,i,j})$	$= \left[f(u_{i,j}) - \left[f(u_{0,i,j}) - \left[\frac{(n^2+3n-2)(n^2+n-1)}{2}\right]\right]\right] d,$ $if\ \ f(u_{i,j}) \geq \frac{(n^2+n-2)}{2},$ $f(u_{0,i,j}) > \frac{(n^2+3n)(n^2+n-1)-2}{4}\ \forall\, 1 \leq i \leq n, 1 \leq j \leq n.$
$f^*(u_{0,i,j}u_{k,i,j})$	$= \left[f(u_{i,j}) - \left[f(u_{k-2,i,j}) + [n(n+1)]\right]\right] d,$ $if\ \ f(u_{i,j}) \geq \frac{(n^2+n-2)}{2},$ $f(u_{k-2,i,j}) < \frac{(n^2+3n)(n^2+n-1)-2}{4}, \forall\, 1 \leq i \leq n, 1 \leq j \leq n,$ $k = 2,3, \dots, \frac{(n^2+3n-2)}{2}$ $= \left[f(u_{i,j}) - \left[f(u_{k-2,i,j}) - [n(n+1)]\right]\right] d,$ $if\ \ f(u_{i,j}) \geq \frac{(n^2+n-2)}{2},$ $f(u_{k-2,i,j}) > \frac{(n^2+3n)(n^2+n-1)-2}{4}, \forall\, 1 \leq i \leq n, 1 \leq j \leq n,$ $k = 2,3, \dots, \frac{(n^2+3n-2)}{2}$

It is clear that the function f is injective and also table 4 shows that $f^*: E \to \{d, 2d, 3d, 4d, \dots, qd\}$ is bijective. Hence f is arithmetic sequential graceful labeling and the graph is arithmetic sequential graceful graph.

Example 3.3.1: The star of step grid graph Sg_3 and its graceful labelling shown in figure-4

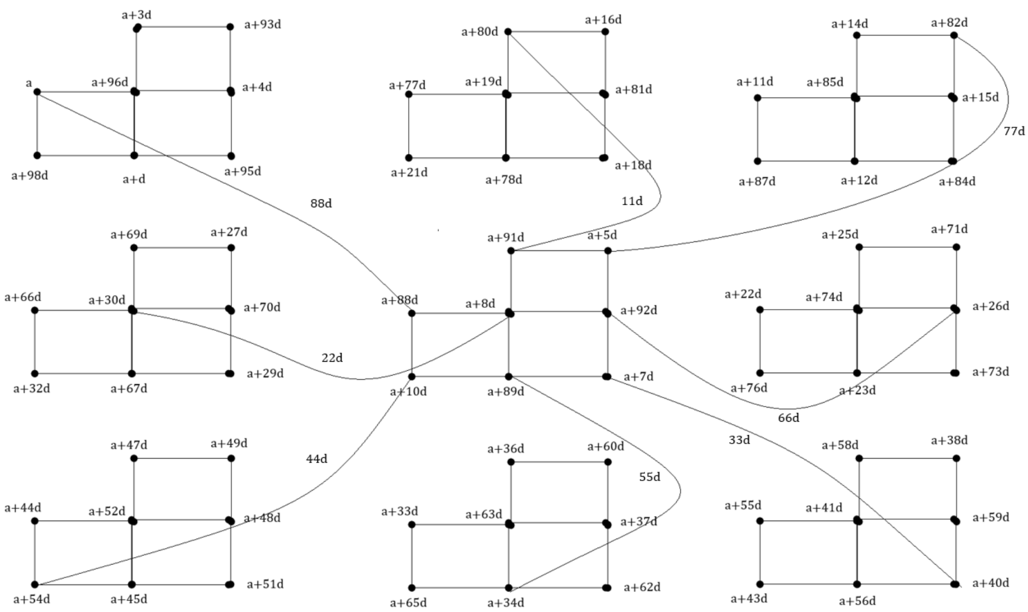

Figure-4: A star of step grid graph Sg_3 and its graceful labelling.

4. CONCLUSION

We showed here arithmetic sequential graceful labeling of some graphs obtained by step grid graph. Here we proved four new results. we discussed graceful of step grid graph, path union of step grid graph, cycle of step grid graph, star of step grid graph. Labeling pattern is demonstrated by means of illustrations, which provide better understanding of derived results. Analysing arithmetic sequential graceful on other families of graph are our future work.

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