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The Numerical Solution of Differential Transform Method and the Laplace Transform Method for Second Order Differential Equation

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Abstract:

DTM is one of the method which gives series solution. The approach mainly rests on the DTM which is one of the approximate methods. The method can easily be applied to many problems and is capable of reducing the size of computational work. Some examples are presented to show the efficiency and simplicity of the method. In this paper one dimensional Differential Transform Method (DTM) is applied on the second order differential equation .The numerical result obtain by DTM are compared with the solution which are obtain by Laplace transform method.

Keywords:

Differential Transform Method, Laplace transform method, system of simultaneous linear differential equation

Introduction:

A variety of methods, exact, approximate and purely numerical are available for the solution of differential equations. Most of these methods are computationally intensive because they are trial-and error in nature, or need complicated symbolic computations. The differential transformation technique is one of the numerical methods for ordinary differential equations. The concept of differential transformation was first proposed by Zhou [12] in 1986 [2-5] and (Arikhoglu[1] and Ozkol,Ayaz[3], Chen[5] and Ho, 1996, 1999; Hassan and Abdel-Halim[8] 2008, Duan[7,] Khaled Batiha[9]) it was applied to solve linear and non-linear initial value problems in electric circuit analysis. This method constructs a semi -analytical numerical technique that uses Taylor series for the solution of differential equations in the form of a polynomial. It is different from the high-order Taylor series method which requires symbolic computation of the necessary derivatives of the data functions. The Taylor series method is computationally time-consuming especially for high order equations. The differential transform is an iterative procedure for obtaining analytic Taylor series solutions of differential equations. The Differential transformation method is very effective and powerful for solving various kinds of differential equation.

Basic ideas of Differential Transform Method:

The Differential Transform Method

The transformation of the k^{th} derivative of a function with one variable is follows:

$$U(k) = \frac{1}{k!}\left(\frac{d^k u(x)}{dx^k}\right) \ at \ x = x_0, \qquad \ldots(1)$$

Where $u(x)$ is the original function and $U(k)$ is the transformed function and the differential inverse transformation $U(k)$ is defined by,

$$u(x) = \sum_{k=0}^{k=\infty} U(k)(x-x_0)^k \qquad \ldots(2)$$

When $x_0 = 0$, the function $u(x)$ defined in (2) is express as

$$u(x) = \sum_{k=0}^{k=\infty} U(k)\, x^k \qquad \ldots(3)$$

Equation (3) implies that the concept of one dimensional differential transform is almost is same as the one dimensional Taylors series expansion. We use following fundamental theorems on differential transform method

Theorem 1] If $u(x) = \alpha g(x) \pm \beta h(x) \ \ then \ \ U(k) = \alpha G(k) \pm \beta H(k)$

Theorem 2] If $u(x) = x^m \ then \ U(K) = \delta(k-m) \ where \ \delta(k-m) = \begin{cases} 1, \ if \ k = m \\ 0\,, \ if \ k \neq m \end{cases}$

Theorem 3] If $u(x) = e^x \ then \ U(k) = \frac{1}{k!}$

Theorem 4] If u (x) = g (x) h (x) then $U(k) = \sum_{l=0}^{k} G(l)H(k-l)$

Theorem 5] If $y(x) = y_1(x)\, y_2(x) \ then \ Y(k) = \sum_{k_1=0}^{k} Y_1(k_1)\, Y_2(k-k_1)$

Theorem 6] If $y(x) = \frac{d^n y_1(x)}{dx^n} \ , \ then \ Y(k) = \frac{(k+n)!}{k!} Y_1(k+n)$

Theorem 7] If $y(x) = e^{\lambda x} then \ Y(k) = \frac{\lambda^k}{k!} \ , \ \lambda \ is \ constant$

Theorem 8] If y $(x) = \sin(wx + \alpha) \ then \ Y \ (k) = \frac{w^k}{k!}\sin(\frac{k\pi}{2} + \alpha)$,where α, w are constant

Theorem 9] If $(x) = \cos(wx + \alpha) \ then \ Y \ (k) = \frac{w^k}{k!}\cos(\frac{k\pi}{2} + \alpha)$, where α, w are constant

Numerical Examples

Example1

Consider second order differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dx} + 2y = 24$ with the initial condition $y(0) = 10, y'(0) = 0$ We apply DTM ,with initial conditions $U(0) = 10,\ U(1) = 0$

$$U(k+1) = \frac{1}{(k+1)(k+2)}[-3(k+1)U(k+1) - 2U(k) + 24(\delta)]$$

Put $k = 1$, U(2) = 10

Put $k = 2, U(3) = -10$

Put $k = 3, U(4) = \frac{70}{12}$ and so on

Therefore, the closed form of the solution can be easily written as

$$y(t) = \sum_{k=0}^{k=2} U(k)t^k$$

$$= 10 + 10t^2 - 10t^3 + \frac{70}{12}t^4$$

Using the Laplace transform method, the exact solution of this example is

$$y(t) = 12 - 4e^{-t} + 2e^{-2t}$$

Example2

Consider second order differential equation

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = 0$$

with the initial condition $y(0) = -1, y'(0) = 7$

We apply DTM ,with initial conditions $U(0) = -1,\ U(1) = 7$

$$U(k+2) = \frac{1}{(k+1)(k+2)}[2(k+1)U(k+1) - 5U(k)]$$

Put k = 1, $U(2) = \frac{19}{2}$

Put k = 2, $U(3) = \frac{1}{2}$

Put k = 3, $U(4) = \frac{-89}{24}$

And so on

Therefore, the closed form of the solution can be easily written as

$$y(t) = \sum_{k=0}^{k=2} U(k)t^k$$

$$= -1 + 7t + \frac{19}{2}t^2 + \frac{1}{2}t^3 - \frac{89}{24}t^4$$

Using the Laplace transform method, the exact solution of this example is

$$y(t) = -e^t cos2t + 4e^{2t} sin2t$$

Example3

Consider second order differential equation

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 6e^{-t}$$

with the initial condition $y(0) = -2, y'(0) = 8$

We apply DTM ,with initial conditions $U(0) = -2,\ U(1) = 8$, we get

$$U(k+2) = \frac{1}{(k+1)(k+2)}\left[\frac{-6}{k!} - 4(k+1)U(k+1) - 4U(k)\right]$$

Put k = 1, $U(2) = -15$

Put k = 2, $U(3) = \frac{41}{3}$

Put k = 3, $U(4) = \frac{-107}{12}$

And so on

Therefore, the closed form of the solution can be easily written as

$$y(t) = \sum_{k=0}^{k=2} U(k)t^k$$

$$= -2 + 8t - 15t^2 + \frac{41}{3}t^3 - \frac{107}{12}t^4$$

Using the Laplace transform method, the exact solution of this example is

$$y(t) = 6e^{-t} - 8e^{-2t}$$

Conclusion:

In this paper, the Differential Transformation Method (DTM) has been successfully applied to find exact and approximate solution of the second order differential equations. The method was used in a direct way without using linearization, perturbation or restrictive assumptions. Therefore, it is not affected by computation round off errors and one is not faced with the necessities of large computer memory and time. This method unlike most numerical techniques provides a closed-form solution. A specific advantage of this method over any purely numerical method is that it offers a smooth, functional form of the solution over a time step. It may be concluded that DTM is very powerful and efficient in finding analytical as well as numerical solutions for wide classes of linear differential equations.

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