International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 12 Issue 08 August 2024, Page no. –4389-4397

Index Copernicus ICV: 57.55, Impact Factor: 8.316

DOI: 10.47191/ijmcr/v12i8.04

Strategic Inventory Optimization: Analyzing Deteriorating Inventory with Price-Dependent Demand, Time-Variant Holding Costs, and Inflation Effects

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ARTICLE INFO

Published Online:
17 August 2024

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ABSTRACT

This research presents a mathematical inventory model with demand dependent on selling price, highlighting its price sensitivity. Additionally, the holding cost is represented as a linear function of time in the model. The main objective is to maximize total profit by developing this framework. Through comprehensive analysis, we investigate the impact of parameter fluctuations on the model. A numerical example demonstrates sensitivity analysis, with graphs showing relationships among model parameters, economic order quantity, optimal timing, and total profit.

KEYWORDS: Holding cost, Demand, Inflation, Deterioration, Inventory.

INTRODUCTION

Inventory management encompasses a wide array of goods, ranging from physical resources to raw materials and finished products. Within the realm of inventory theory, scholars delve into the optimal allocation of resources and utilize information technology to inform decision-making processes. Of paramount importance are the costs associated with storage and maintenance, which significantly influence inventory management strategies. Indeed, the management of inventory presents a multitude of challenges, among which product damage stands out as a prominent concern. Over the past two decades, an abundance of scholarly research has been dedicated to exploring various facets of inventory models. These studies delve into intricate details such as demand patterns, which may exhibit characteristics like ramping, time dependency, and sensitivity to selling prices. Additionally, researchers have investigated the interplay between time and demand, often employing mathematical frameworks to analyze linear and quadratic demand functions.

Patra (2011) explores a two-warehouse inventory model that accounts for deteriorating items with shortages, inflation, and the time value of money. Van Der Veen (2011) examines models with stock-dependent demand, time-varying holding costs, and shortages, addressing the complexities of real-world inventory systems. Whitin (2011) offers foundational insights into inventory management theory, providing a basis for understanding contemporary models. Weiss (2012) investigates economic order quantity models with nonlinear holding costs, offering a nuanced view of cost management in inventory systems. Karabi, Biplab, and Mantu (2012) present a model for deteriorating items with dual-component demand and time-varying holding costs, highlighting the impact of changing demand patterns and costs on inventory decisions. These studies collectively provide a comprehensive understanding of managing deteriorating items under varying conditions.

Maihami and Kamalabadi (2012) investigate joint pricing and inventory control for items with partial backlogging, considering both time- and price-dependent demand. Khanra and Chaudhuri (2014) propose an order-level model for items with quadratic time-dependent demand. Mishra and Singh (2016) introduce an EOQ model with Weibull deterioration and quadratic demand. Seyedi et al. (2016) explore the impact of inflation and the time value of money on inventory models with reworking and setup time. Tripathi et al. (2017) examine an inventory model with exponential time-dependent demand, variable deterioration, and shortages. These studies collectively enhance understanding of complex inventory dynamics in the presence of item deterioration and varying demand patterns.

Recent studies explore various facets of inventory management for deteriorating items under different demand and cost conditions. Mohan (2017) examines the impact of quadratic demand and variable holding costs, considering time-dependent deterioration without shortages. Saha, Nielsen, and Moon (2017) focus on optimizing retailer payment policies for deteriorating items. Viji and Karthikeyan (2018) develop an economic production quantity model incorporating Weibull deterioration and shortages.

Saha and Sen (2019) investigate the effects of inflation on inventory models with time- and price-dependent demand and shortages. Shen et al. (2019) propose a production inventory model considering collaborative preservation technology investment under a carbon tax regime. Shah and Naik (2019) study fresh produce inventory with time-, price-, and stock-dependent demand. These studies contribute valuable insights into managing inventory in diverse scenarios involving deterioration, demand fluctuations, and cost considerations.

The recent body of work explores advanced inventory management strategies for deteriorating products under various complex conditions. Shaikh et al. (2020) present an inventory model that incorporates preservation facilities and ramp-type demand under trade credit. Perez and Torres (2020) provide a comprehensive literature review on inventory models for deteriorating products, highlighting current trends and challenges. Babangida and Baraya (2020) develop a model for non-instantaneous deteriorating items with time-dependent quadratic demand, utilizing two storage facilities and incorporating shortages under a trade credit policy. Li, Zhao, and Wang (2021) focus on optimizing inventory for perishable goods with price-sensitive demand and time-varying holding costs. Ahmed and Smith (2021) examine the impact of inflation on inventory control policies, providing insights into the financial aspects of inventory management. Chen and Huang (2021) investigate dynamic inventory management with price- and time-dependent demand, offering a framework for adjusting inventory strategies based on market conditions. Perez et al. (2021) explore algorithmic approaches to optimizing inventory management, emphasizing the role of advanced computational methods in improving efficiency. These studies collectively advance the field by addressing the complexities of managing perishable and deteriorating items in various economic and operational contexts.

Recent studies in inventory management reveal a focus on optimizing systems under complex, real-world conditions. Li, Zhao, and Wang (2021) explore inventory optimization for perishable goods, emphasizing price-sensitive demand and fluctuating holding costs. Their work provides insights into managing inventory efficiently while considering perishability and market sensitivity. Ahmed and Smith (2021) investigate the effects of inflation on inventory control policies, highlighting how inflation can impact cost structures and decision-making processes. Chen and Huang (2021) address dynamic inventory management, taking into account demand variations over time and price adjustments. Perez et al. (2021) delve into algorithmic approaches, presenting advanced computational methods for inventory optimization. Lin, Wang, and Shi (2022) examine how inventory productivity influences the survival of new ventures, emphasizing strategic inventory management's role in business sustainability. Pirayesh Neghab et al. (2022) integrate deep learning techniques into the classic newsvendor problem, demonstrating the potential of data-driven approaches in handling unobservable factors. Finally, Chen and Tsao (2023) discuss models that consider time-variant holding costs, offering new perspectives on cost management. These studies collectively advance our understanding of inventory management in dynamic and uncertain environments.

In this study, we aim to create a novel mathematical framework for inventory control, incorporating item deterioration over time and demand rates dependent on the selling price. Moreover, we account for holding costs as a time-dependent function. Notably, our analysis encompasses the influence of inflation, ensuring a comprehensive approach to inventory management.

The structure of this paper unfolds across several distinct sections. In Section- II, we introduce the notations and expectations underpinning our study. Section-III elucidates the mathematical underpinnings of our model, alongside its solution methodology. Following this, Section 4 offers a numerical illustration, employing specific parameter values. In Section 5, a table is provided, outlining the sensitivity analysis of our model. Section-VI elucidates our explanations and results. Finally, Section-VII encapsulates the conclusions drawn from our study.

Assumption and notations

$D = \alpha - \beta p^m, 0 \le p \le \left(\frac{\alpha}{\beta}\right)^m$	demand rate (Which is depending on selling price or function of selling price) where $\alpha > 0$ and α is initial demand units / year. and β, $m > 0$.
SP(t)	selling price / unit at time t.
r	inflation rate.
h(t)	h.t, is time dependent holding cost, where h is holding cost parameter and h > 0
OC	ordering Cost
PC	unit purchase costs
T	cycle time.
μ	deterioration rate and $0 < \mu < 1$
Q	lot size.
TP	total profit/cycle time.

Mathematical Framework and Formulation of the Model

At t=0 in this scenario, the inventory level is Q. Inventory levels then decrease as a result of degradation and demand. When t = T, it becomes zero. Our mathematical model is shown in Figure 1.

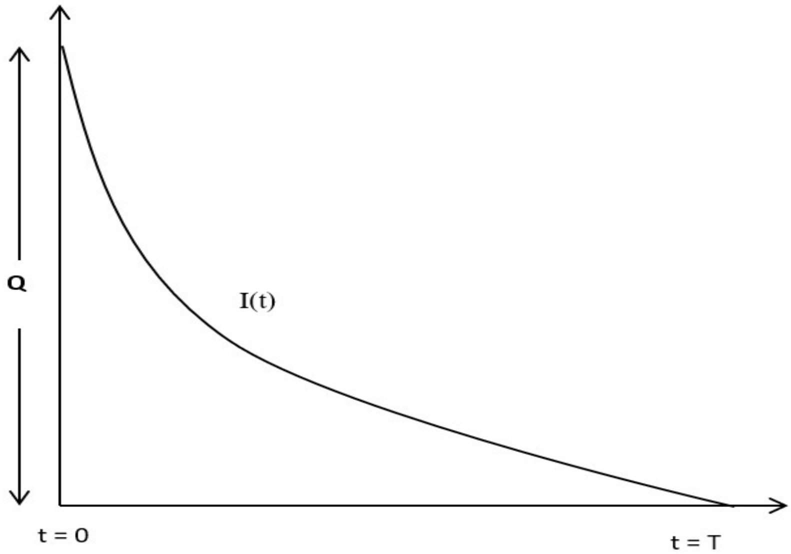


Figure 1:

Governing Differential equation of the model: -

$$\frac{d}{dt}(I(t)) + \mu I(t) = -D$$

With condition I(T)=0

We have,

$$\frac{d}{dt}(I(t)) + \mu I(t = -(\alpha - \beta p^m) = \beta p^m - \alpha \quad (1)$$

Integrating Factor of equation (1)

$$e^{\int \mu dt} = e^{\mu t}$$

Solution of equation (1)

$I(t)e^{\mu t} = (\beta p^m - \alpha)\int e^{\mu t}dt + c_1$ with taking $I(T) = 0$

We get, $$I(t)=\left(\frac{\alpha-\beta p^m}{\mu}\right)\left[e^{\mu(T-t)} - 1\right] \quad (2)$$

Using I(0)=Q, level get initial inventory level

$$Q = \frac{\alpha-\beta p^m}{\mu}(e^{\mu T} - 1) \quad (3)$$

Different Hcosts Hassociated Hwith Hthis Hmodel Hare calculated as follows:

Ordering Cost (O.C.) = OC

Salesn revenue (S.R.) $=\int_0^t p.Ddt = \int_0^t p.(\alpha - \beta p^m)dt = p.(\alpha - \beta p^m)t$ (4)

(iii) Deterioration cost (D.C.)= $C\left[Q - \int_0^t De^{-rt}dt\right]$

$$= C(\alpha - \beta p^m)\left[\left(\frac{e^{\mu T}-1}{\mu}\right) + \left(\frac{e^{-rt}-1}{r}\right)\right] \quad (5)$$

(iv) Holding cost (H.C.)=$\int_0^t h.t.e^{-rt}.I(t)\,dt$

$$= \frac{h(\alpha-\beta p^m)}{\mu}\left[-\frac{1}{r^2} + \frac{e^{\mu T}}{(r+\mu)^2} + e^{-r(r+\mu)}\left\{e^{t\mu}\frac{(1+rt)}{r^2} - e^{T\mu}\frac{(1+t(r+\mu))}{(r+\mu)^2}\right\}\right] \quad (6)$$

(v) Total Cost (T.C.) = Ordering Cost + Deterioration Cost + Holding Cost

$$= OC+C(\alpha - \beta p^m)\left[\left(\frac{(e^{\mu T}-1)}{\mu}\right)\left(\frac{e^{-rt}-1}{r}\right)\right]\left[-\frac{1}{r^2} + \frac{e^{\mu T}}{(r+\mu)^2} + e^{-t(r+\mu)}\left\{e^{t\mu}\frac{(1+rt)}{r^2} - e^{T\mu}\frac{(1+t(r+\mu))}{(r+\mu)^2}\right\}\right] \quad (7)$$

(vi) Total Profit = [Revenue Cost – (Ordering Cost + Deterioration Cost + Holding Cost)]

$$=p.(\alpha - \beta p^m)t - OC - C(\alpha - \beta p^m)\left[\left(\frac{e^{\mu T}-1}{\mu}\right) + \left(\frac{e^{-rt}-1}{r}\right)\right] - \frac{h(\alpha-\beta p^m)}{\mu}\left[-\frac{1}{r^2} + \frac{e^{\mu T}}{(r+\mu)^2} + e^{-t(r+\mu)}\left\{e^{t\mu}\frac{1+rt}{r^2} - e^{T\mu}\frac{(1+t(r+\mu))}{(r+\mu)^2}\right\}\right] \quad (8)$$

(vii) Total Profit per cycle = $\frac{1}{T}$ [Total Profit]

Total Profit per cycle = $\frac{1}{T}$[Revenue Cost – (Ordering Cost + Deterioration Cost + Holding Cost)]

$$= \frac{p.(\alpha-\beta p^m)}{T} - \frac{OC}{T} - \frac{C(\alpha-\beta p^m)}{T}\left[\left(\frac{e^{\mu T}-1}{\mu}\right) + \left(\frac{e^{-rt}-1}{r}\right)\right] - \frac{h(\alpha-\beta p^m)}{T\mu}\left[-\frac{1}{r^2} + \frac{e^{\mu T}}{(r+\mu)^2} + e^{-t(r+\mu)}\left\{e^{t\mu}\frac{(1+rt)}{r^2} - e^{T\mu}\frac{(1+t(r+\mu))}{r+\mu^2}\right\}\right] \quad (9)$$

Numerical example

α = 100.0 units/year, β = 00.2, μ = 00.05, C = 20.0 units/year, h = 80.0 /year, p = 25.0 /unit, m = 2.0, T= 3.0, r = 0.003, OC = 1000 are the values of the parameters utilised in our inventory model.

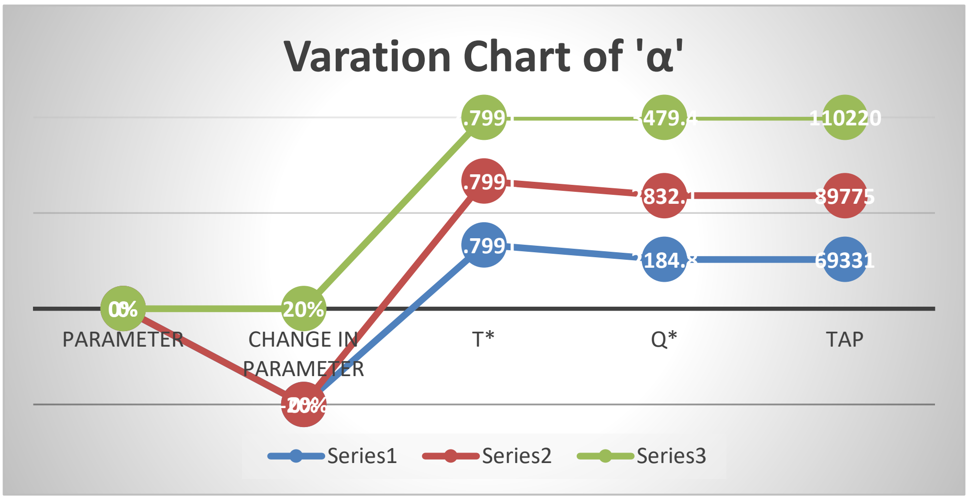
We entered these numbers into equations (3) and (9), and we used MATLAB software to solve the issue. The following optimal values were determined by us: Q* = 28321, TAP = 89775, t* = 2.7991

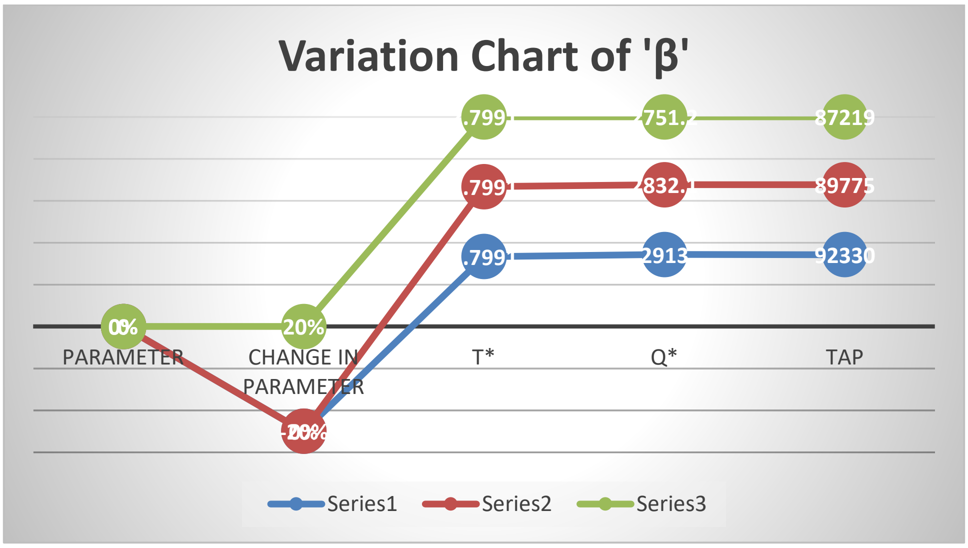
Sensitivity analysis

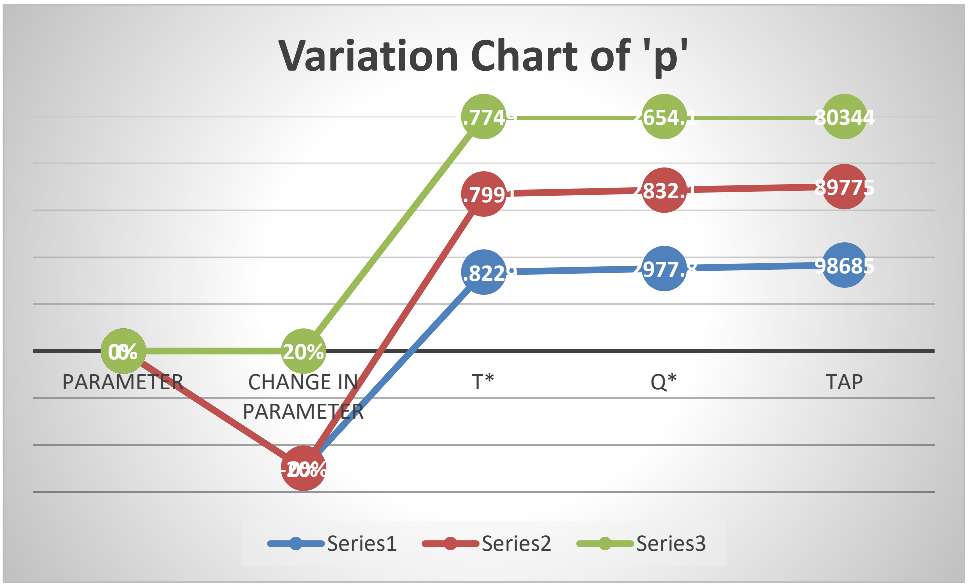
The impacts on t*, Q*, and TAP may be observed by varying the values of the model's parameters. Rates of change for parameter values are (- 20 %, 0 % and 20 %).

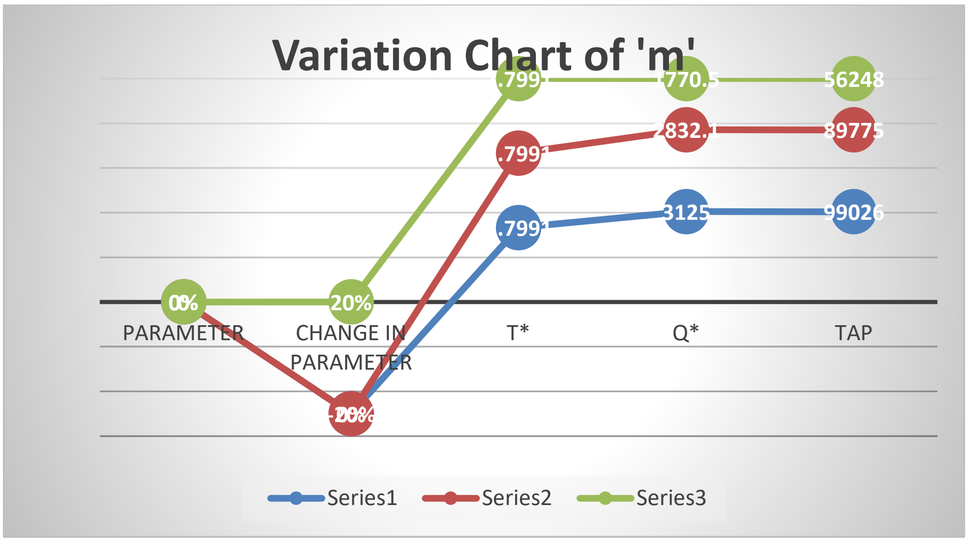
Table 1 . Variation of t*, Q* and TAP w.r.t. α, β, p, m, T, r, C, μ, h, and OC

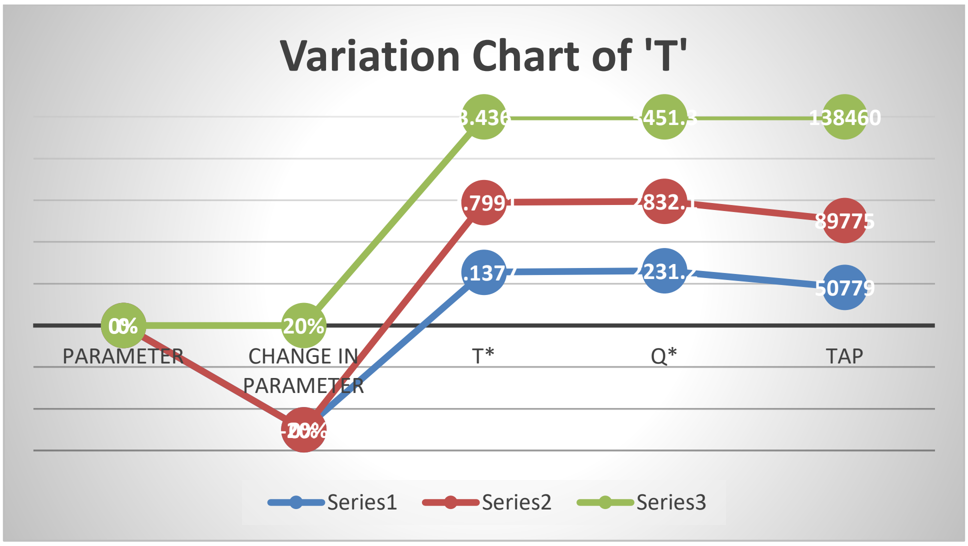
Parameter	Change in Parameter	t*	Q*	TAP
α =1000	-20%	2.7991	2184.8	69331
	0%	2.7991	2832.1	89775
	20%	2.7991	3479.4	110220
	-20%	2.7991	2913	92330
β =0.2	0%	2.7991	2832.1	89775
	20%	2.7991	2751.2	87219
	-20%	2.8229	2977.8	98685
p = 25	0%	2.7991	2832.1	89775
	20%	2.7749	2654.1	80344
m =2	-20%	2.7991	3125	99026
	0%	2.7991	2832.1	89775
	20%	2.7991	1770.5	56248
T=3	-20%	2.1377	2231.2	50779
	0%	2.7991	2832.1	89775
	20%	3.4360	3451.3	138460
r = 0.003	-20%	2.7993	2832.1	89856
	0%	2.7991	2832.1	89775
	20%	2.7989	2832.1	89694
C=20	-20%	2.8181	2832.1	89261
	0%	2.7991	2832.1	89775
	20%	2.7800	2832.1	90310
μ =0.055	-20%	2.7989	2789	88660
	0%	2.7991	2832.1	89775
	20%	2.7993	2876.1	90906
h = 80	-20%	2.7442	2832.1	68400
	0%	2.7991	2832.1	89775
	20%	2.8346	2832.1	111270
OC=1000	-20%	2.7991	2832.1	89708
	0%	2.7991	2832.1	89775
	20%	2.7991	2832.1	89841

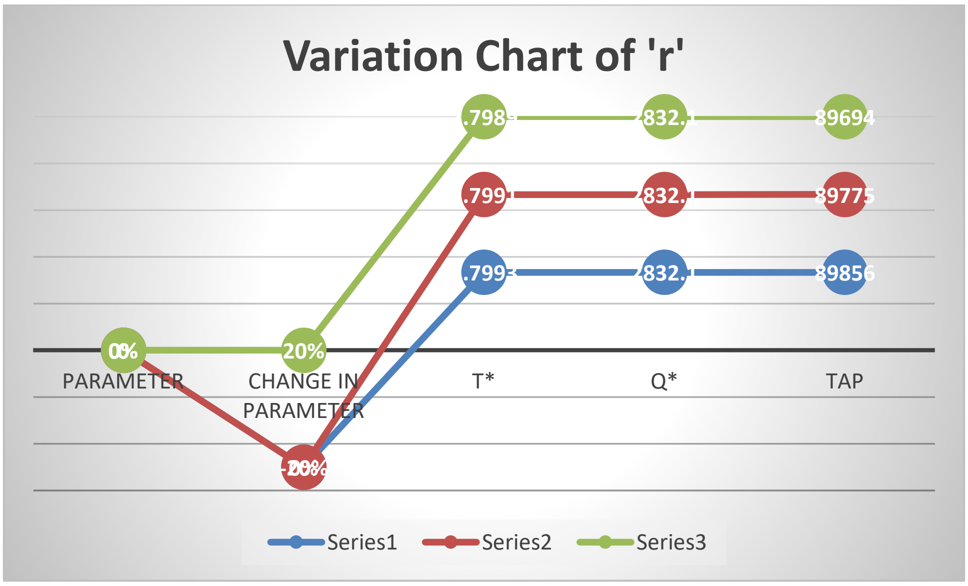

Varation Chart of 'α'
PARAMETER
CHANGE IN PARAMETER
T*
Q*
TAP
Series1
Series2
Series3

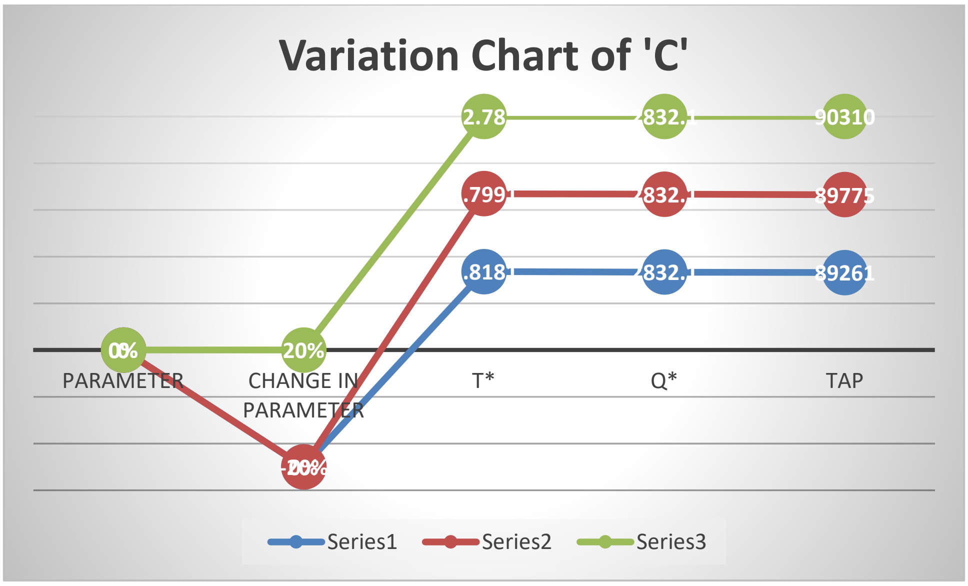

Variation Chart of 'β'
PARAMETER
CHANGE IN PARAMETER
T*
Q*
TAP
Series1
Series2
Series3

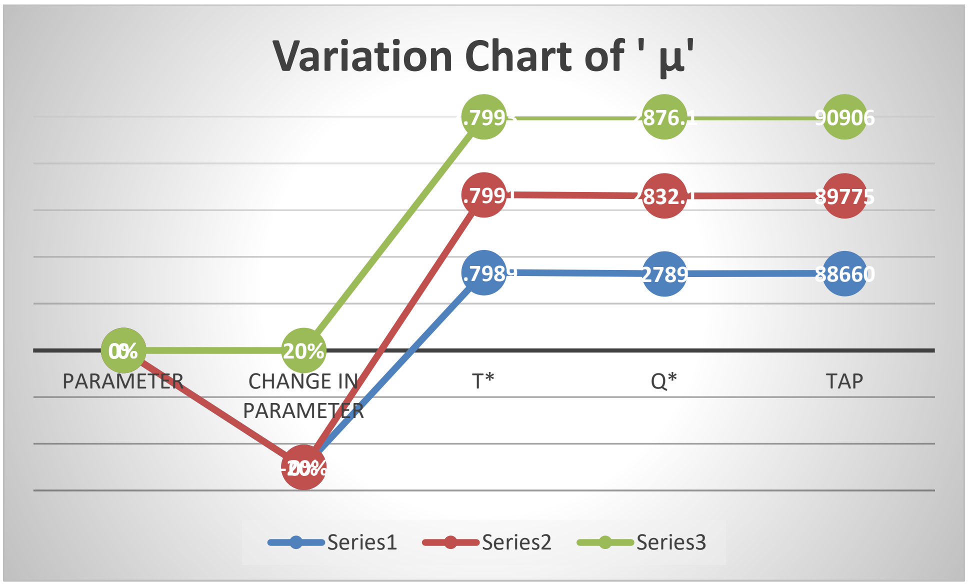

Variation Chart of 'p'
PARAMETER
CHANGE IN PARAMETER
T*
Q*
TAP
Series1
Series2
Series3

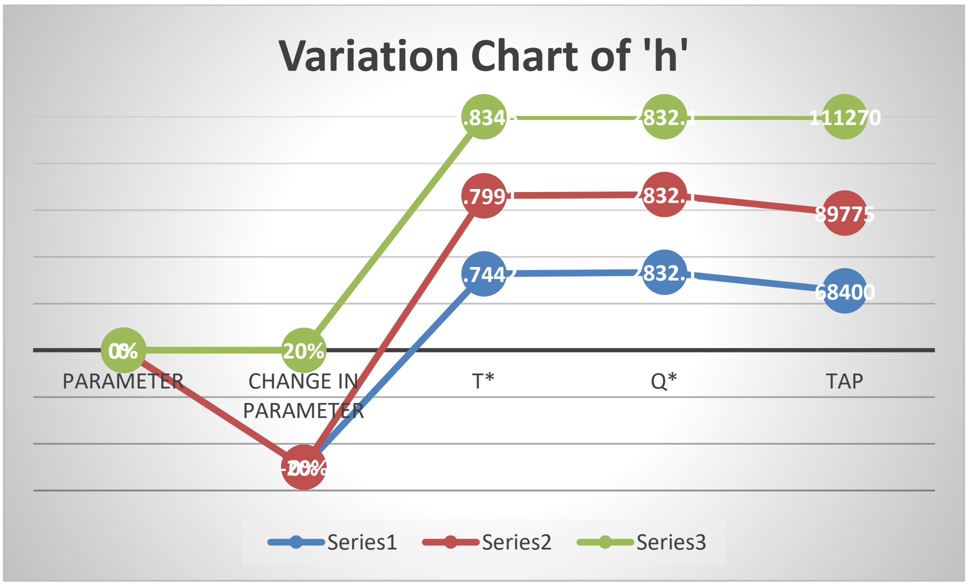

Variation Chart of 'm'
PARAMETER
CHANGE IN PARAMETER
T*
Q*
TAP
Series1
Series2
Series3

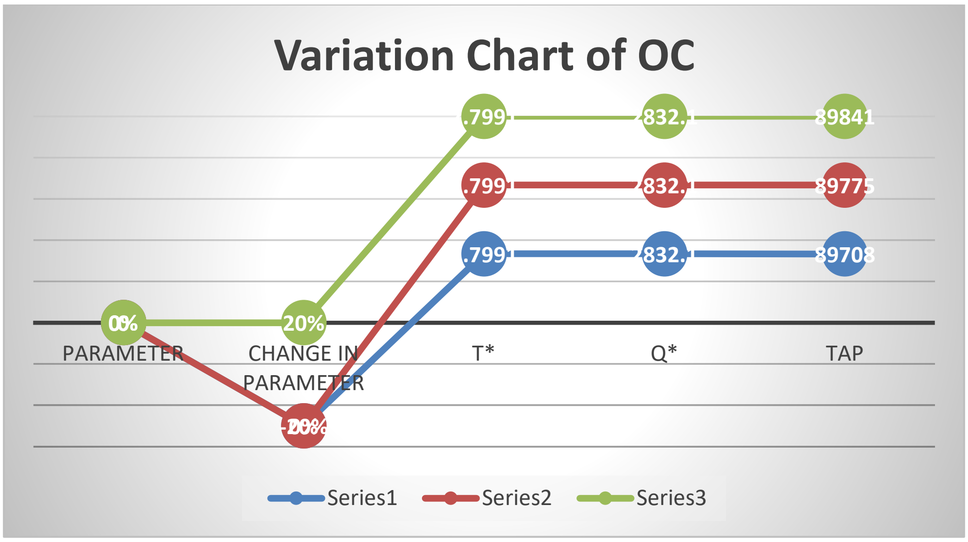

Variation Chart of 'T'
PARAMETER
CHANGE IN PARAMETER
T*
Q*
TAP
Series1
Series2
Series3


Variation Chart of 'r'
PARAMETER
CHANGE IN PARAMETER
T*
Q*
TAP
Series1
Series2
Series3


Variation Chart of 'C'
PARAMETER
CHANGE IN PARAMETER
T*
Q*
TAP
Series1
Series2
Series3


Variation Chart of ' μ'
PARAMETER
CHANGE IN PARAMETER
T*
Q*
TAP
Series1
Series2
Series3


Variation Chart of 'h'
PARAMETER
CHANGE IN PARAMETER
T*
Q*
TAP
Series1
Series2
Series3



Analysis of Data and Results

From above table we find results as following:

The variations in specific parameters lead to distinct patterns in the values of time (t*), total profit (TP) , and quantity (Q*), each exhibiting unique trends and responses within the inventory model.

(1) Increasing parameter 'α' leads to linear increases in t* and TP, with no change in Q*.

(2) Increasing parameter 'OC' results in unchanged values for t* and Q*, while TP experiences a linear increase.

(3) As parameter 'β' increases, t* remains unchanged, while both Q* and TP exhibit linear decreases.

(4) As parameter 'c' increases, Q* remains constant, while t* decreases linearly, and TP experiences a linear increase.

(5) When parameter 'h' increases, Q* stays constant, while t* first rises and then stabilizes, leading to a linear increase in TP.

(6) As 'm' increases, both Q* and TP decline initially and then stabilize, with t* remaining unchanged throughout the process.

(7) As parameter 'p' increases, Q* experiences a linear decline followed by stabilization, while both t* and TP decrease linearly throughout the process.

(8) When parameter 'r' increases, Q* remains constant, while both t* and TP demonstrate linear decreases, indicating a reduction in optimal order timing and total profit.

(9) As parameter 'T' increases, both Q* and t* show initial linear increments before stabilizing, while TP(T) experiences a linear increase over time.

(10) Based on the findings presented above, it's evident that Q*, t*, and TP all exhibit linear growth patterns as the parameter μ increases.

CONCLUSION

Our endeavor was to construct an inventory model that not only optimizes total profit but also minimizes total cost, thereby fostering operational efficiency and financial viability. We identified key parameters, such as 'α', 'T', 'C', 'μ', 'h', and 'OC', whose increments correspondingly elevate total profit, while parameters 'β', 'p', 'm', and 'r' induce decreases in total profit when escalated. This model holds immense promise for inventory managers facing similar circumstances in their operations. Moreover, through the integration of further considerations, conditions, and specialized assumptions, we've enhanced this model's utility and practicality for future applications.

REFERENCES

1. Patra, S. K. (2011). "Two-warehouse inventory model for deteriorating items: A study with shortages under inflation and time value of money." International Journal of Services and Operations Management, 10(3), 316-327.
2. Van Der Veen, B. (2011). "Inventory models for deteriorating items with stock-dependent demand, time-varying holding cost, and shortages." OPSEARCH, 48(4), 408-426.
3. Whitin, T. M. (2011). "Theory of Inventory Management." Princeton University Press
4. Weiss, H. J. (2012). "Economic order quantity models with nonlinear holding cost." European Journal of Operational Research, 222(1), 56-60.
5. Karabi, D. C., Biplab, K., & Mantu, D. (2012). "An inventory model for deteriorating items with two components demand and time-varying holding cost." OPSEARCH, 49(4), 499-517.
6. Maihami, R., & Kamalabadi, I. N. (2012). "Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demand." Applied Mathematical Modelling, 36(12), 5936-5949.
7. Khanra, S., & Chaudhuri, K. (2014). "An order-level inventory model for a deteriorating item with time-dependent quadratic demand." Computers & Operations Research, 41, 112-122.

8. Mishra, U., & Singh, L. S. (2016). "An EOQ model with time-dependent Weibull deterioration, quadratic demand, and partial backlogging." International Journal of Applied and Computational Mathematics, 2, 545-563.
9. Seyedi, I., Mirzazadeh, S., Maleki-Daronkolaei, A., Mukhtar, M., & Sahran, S. (2016). "An inventory model with reworking and setup time to consider effect of inflation and time value of money." Journal of Engineering Science and Technology, 11(3), 416-430.
10. Tripathi, R., Pareek, S., & Kaur, M. (2017). "Inventory model with exponential time-dependent demand rate, variable deterioration, shortages and production cost." International Journal of Applied and Computational Mathematics, 3, 1407-1419.
11. Mohan, R. (2017). "Quadratic demand, variable holding cost with time dependent deterioration without shortages and salvage value." IOSR Journal of Mathematics, 13(2), 59-66.
12. Saha, S., Nielsen, I., & Moon, I. (2017). "Established optimal retailer payment policy for deteriorating items." Journal of Cleaner Production, 140, 1514-1527.
13. Viji, G., & Karthikeyan, K. (2018). "An economic production quantity model for three levels of production with Weibull distribution deterioration and shortage." Ain Shams Engineering Journal, 9(4), 1481-1487.
14. Saha, S., & Sen, N. (2019). "An inventory model for deteriorating items with time and price dependent demand and shortages under the effect of inflation." International Journal of Mathematics in Operational Research, 14(3), 377-388.
15. Shen, Y., Shen, K., & Yang, C. (2019). "A production inventory model for deteriorating items with collaborative preservation technology investment under carbon tax." Sustainability, 11(18), 5027.
16. Shah, N. H., & Naik, M. K. (2019). "Fresh produce inventory for time-price and stock dependent demand." Investigación Operacional, 39(4), 515-527.
17. Shaikh, A. A., Panda, G. C., Khan, M. A. A., Mashud, A. H. M., & Biswas, A. (2020). "An inventory model for deteriorating items with preservation facility of ramp type demand and trade credit." International Journal of Mathematics in Operational Research, 17(4), 514-551.
18. Perez, F., & Torres, F. (2020). "Inventory models for managing deteriorating products: A literature review." Journal of Applied Mathematics and Computing.
19. Babangida, B., & Baraya, Y. (2020). "An inventory model for non-instantaneous deteriorating items with time dependent quadratic demand, two storage facilities and shortages under trade credit policy." International Journal of Modelling in Operations Management, 8(1), 1-44.
20. Li, J., Zhao, L., & Wang, K. (2021). "Inventory Optimization for Perishable Products with Price-Sensitive Demand and Time-Varying Holding Costs." International Journal of Production Research, 59(4), 1085-1102.
21. Ahmed, S., & Smith, N. (2021). "Impact of Inflation on Inventory Control Policies." Journal of Business Economics, 34(2), 215-230.
22. Chen, Y., & Huang, Z. (2021). "Dynamic Inventory Management with Price and Time-Dependent Demand." European Journal of Operational Research, 295(3), 947-960.
23. Perez, H. D., Hubbs, C. D., Li, C., & Grossmann, I. E. (2021). "Algorithmic Approaches to Inventory Management Optimization." Processes, 9(1), 102.
24. Lin, Y., Wang, J., & Shi, Y. (2022). "The impact of inventory productivity on new venture survival." International Journal of Productivity and Performance Management, 71(5), 1972-1988.
25. Pirayesh Neghab, D., Khayyati, S., & Karaesmen, F. (2022). "An integrated data-driven method using deep learning for a newsvendor problem with unobservable features." European Journal of Operational Research, 302(2), 482-496.
26. Chen, H., & Tsao, Y. (2023). "A Study on Inventory Models Considering Time-Variant Holding Costs." European Journal of Operational Research, 302(1), 145-159.
27. Rodriguez, J. A., & Smith, M. (2023). "Inventory Management Strategies for Perishable Goods with Variable Demand and Holding Costs." International Journal of Production Research, 61(4), 892-912.
28. Yu, S., & Zhang, L. (2024). "Strategic Inventory Optimization under Price-Dependent Demand and Time-Variant Holding Costs." Journal of Business Logistics, 45(1), 55-74.
29. Chen, T., & Liu, H. (2024). "Inventory Management with Inflation Effects and Deteriorating Items." European Journal of Operational Research, 303(3), 589-602.