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Implementation of First Passage Time Method as a Warning System to Measure the Probability of Default in Bond

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ABSTRACT

Within the rise of bond investors, an in-depth analysis of risk in bond investment will be continually needed. A lot of research mostly overviewed how a bond acts in the maturity date, while the First Passage Time is developed from the Merton Model, which helps to see credit risk from the issuing date to the maturity date. The First Passage Time Method lets us know when the model hits a certain point we call a Barrier value as the lower benchmark for the first time. It initiates a better understanding of a bond, as it does not only analyze a bond in its maturity date but also during the ongoing period. This method is applied in the Commonwealth Bank Bond I 2020, using company asset data from September 2020 to February 2023. According to R Programming output, if the barrier value is 75% of the total face value, the probability of default is 0.00003145407% with the market value of equity of IDR 21,991,492,000,000. This method will help us see a deeper view of the bond as we can set the Barrier and find whether our bond is "risky" enough, implying that investor losses will not exceed 5,06% of the initial investment within one week.

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I. INTRODUCTION

As the investment market is rapidly growing through time [1], it is much needed to do deeper research in each element. With the urgency in bond investment, it would be meaningful to project a bigger overview of the financial instrument. A larger overview is meant to be a thorough analysis throughout the on-going bond from issuing date up to maturity date, as most of the research done directly looks out for any probability of default at the maturity date. The First Passage Time Method gives a way out of this issue.

Robert C. Merton in 1974 firstly builds a model based on the capital structure of the firm, which becomes the basis of the structural approach [2]. He assumes that the firm is financed by equity and a zero coupon bond with face value K and maturity date T. In this approach, the company defaults at the bond maturity time T if its assets value falls below the face value of the bond at time T.

Black and Cox in 1976 extends the definition of default event and generalize Merton's method into the First Passage Approach [3]. In this approach, the firm defaults when the history low of the firm assets value below some barrier D. Thus the default event could take place before the maturity date T. This theory also need assumption that the corporations issues only one zero coupon bond.

The First Passage Time Method aims to give clear results of the probability of bond throughout its issuing date up to its maturity date. The method stands for the first time the company's asset value goes below the threshold value known as the Barrier which is set as the lower benchmark [4], this condition will later on "warn" the investor whether the bond has the probability of default or vice versa. In this research, The First Passage Time is directed to calculate the probability of default of Commonwealth Bank 2020 Bond I. The final data is chosen due to it being the only bond of the company as well as an analysis that includes bond ranking.

Company asset data will mainly be the data used on this research. The First Passage Time Method requires normally distributed data to calculate the volatility. Market value of equity and liability will also play an important role in this research as the calculation according to the First Passage Time will make it clearer of what the company is capable of paying and their debt.

II. THEORETICAL FRAMEWORK

2.1. Bond Valuation

Bond properties contain: issuing date, maturity date, interest rate, period, face value, and period of return. As the company asset data will be used as the main data, the normality of the company asset ln Return is crucial and will be checked through a visual test using Q-Q Plot. Return will be calculated with the following formula given that V_T is the company's asset at t, and R_t is the return on asset [5].

$$R_t = \frac{R_t}{R_{t-1}}$$

As there is another formal normality test that will be run using Jarque Bera Normality Test on the Asset's ln Return with hypothesis stated below:

H_0: data is normally distributed

H_1: data is not normally distributed

Statistics test

$$JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right)$$

Given that n = total numbers of ln return analyzed; S = ln return skewness; K= ln return kurtosis.

Rejection Region of H_0 is when $JB > \chi^2$ or $prob < sig.\alpha$

Assuming that the data is normally distributed, according to [6] the volatility will be calculated using ln return data's standard deviation with the following formula

$$S_{n-1} = \sqrt{\frac{\sum_{t=1}^{n}(R_t - \bar{R})^2}{n-1}}$$

Given that S = asset volatility; t = period of an asset data; r = asset data mean; n = total asset data period.

Volatility helps explain and picturize the asset fluctuation, the bigger it gets, the more rapid and more extreme the shift in company asset data and vice versa.

2.2. First Passage Time Method Properties

One of the crucial parts that is not common in other credit risk analysis is the usage of Barrier. Barrier is a threshold representing the lower benchmark of a "default" condition, thus whenever the company asset touches, passes, or goes below Barrier value, it will be considered as having a probability of default. Barrier value most of the time is calculated according to its face value (K). Any ratio of a (K) might represent the Barrier well.

Time Varying Default Barrier corrected the definition of bankruptcy time in Merton's Model. This model gives bondholder right to reorganize a firm if its value falls below a given barrier. To build this model, we use some assumptions for simplifying the analytic solution. Some assumptions are used, which are [7]:

1. Constant return and volatility
2. No transaction costs
3. No dividends
4. No riskless arbitrage
5. Security trading is continuous
6. Risk free rate is constant for all maturities
7. Short selling proceeds is permitted

Bond indenture provisions often include safety covenants that give bondholder right to reorganize a firm if its value falls below a given barrier. We still use geometric Brownian motion to model the total assets of the firm V_t [8]. Suppose the default barrier B is a constant valued in $(0,V_0)$, then the default time τ is modified to

$$\tau = \inf(t > 0 : V_t < B)$$

While there are few scenarios that might happen throughout a bond during its issuing period, according to [9] scenarios are stated in Table 1.

Table 1. First Passage Time Scenario

Barrier Level	Status	Assets	Liability	Equity
$B \geq K$	not default	$M_T > B, B \geq K$	K	$V_T - K$
		$B > K$	K	$B - K$
		$B = K$	K	0
$B < K$	not default	$M_T > B$	M_T	0
	default	$M_T \leq B$	B	0

Market Value of Equity in First Passage Time represent the company asset without its debt to shareholders, according to [7], it is calculated with the following formula

$$E_{FPT} = V_0\left[\Phi\left(\frac{\ln\left(\frac{V_0}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right) - \left(\frac{B}{V_0}\right)^{\frac{2r}{\sigma^2}+1}\left(1 - \Phi\left(\frac{\ln\left(\frac{KV_0}{B^2}\right) - \left(r + \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right)\right)\right]$$

$$-Ke^{-r\tau}\left[\Phi\left(\frac{\ln\left(\frac{V_0}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right) - \left(\frac{B}{V_0}\right)^{\frac{2r}{\sigma^2}+1}\left(1 - \Phi\left(\frac{\ln\left(\frac{KV_0}{B^2}\right) - \left(r - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right)\right)\right]$$

Given that $\Phi(\cdot)$ = standard normal CDF; E_{FPT} = Market value of equity in FPTM; V_0 = Company's asset value at issuing time; K = Bond face value; B = Barrier/threshold of default; r = Risk-free interest rate; τ = Time of default; σ = Asset's volatility.

Liability represents the total debt needed to be paid to shareholders of a company, it is calculated with the following formula

$$F_{FPT} = V_0 + E_{FPT}$$

Given that E_{FPT} = Market value of equity in FPTM; V_0 = Company's asset value at issuing time 0.

As the total probability of default in bond according to First Passage Time is calculated with the following formula

$$\text{PD} = \Phi\left(\frac{\ln\left(\frac{B}{V_0}\right) - \left(r - \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}\right) + \left(\frac{B}{V_0}\right)^{\frac{2r}{\sigma^2}+1}\left(\Phi\left(\frac{\ln\left(\frac{B}{V_0}\right) + \left(r - \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}\right)\right)$$

Given that V_0 = Company's asset value at issuing time; B = Barrier/threshold of default; r = Risk-free interest rate; τ = Time of default; σ = Asset's volatility.

III. RESEARCH METHODS

This research uses a bond issued by the Commonwealth Bank obtained from KSEI with the details in Table 2 [10].

Table 2. Company Asset Description

Bond Data	
Issuer	Commonwealth Bank, PT
Security Name	Obligasi I Bank Commonwealth 2020
Security Code	IDA000109607
Type	Straight Bonds
Listing Date	September 4, 2020
Maturity Date	September 3, 2023
Nominal	Rp1,000,000,000,000,-
Interest	7.50% p.a.
Interest Type	Fixed
Interest Frequency	3 months
Rating	(AAA)

While the company asset data is obtained from the company's monthly report on the Commonwealth Bank website from September 2020 - February 2023 [11]. Step by step of the data analysis are listed as

1. List out all the bonds issued by a company that currently only has one on-going bond in Indonesia, upon the list, choose the one with the best rating.
2. Determine the volatility of the company's total asset score.
3. Determine the market value of equity,
4. Determine the liability
5. Determine the probability of default using the First Passage Time model.
6. Determine the probability of default using First Passage Time.

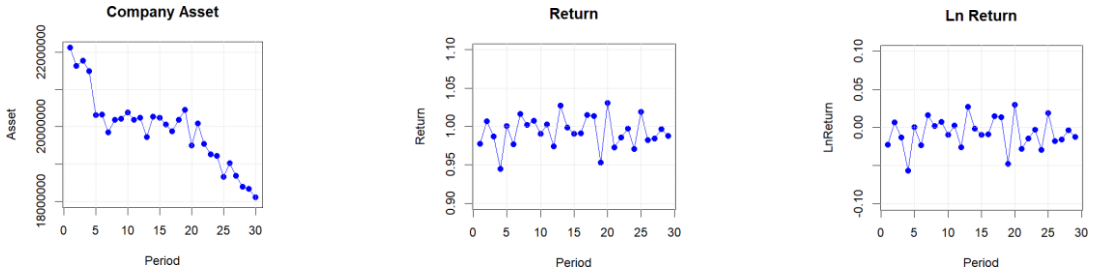
IV. RESEARCH FINDINGS AND DISCUSSION

4.1. Bond Description

Company Asset data is described as the Table 3.

Table 3. Company Asset Description

Description	Company Asset (in thousand)	Return on Asset	ln Return Asset
Number of Data	30	29	29
Mean	19,944,254,000	0,9930	-0,0070
Variance	980932133.569	0,0004	0,0004
Dev. Standard	990,420,200	0,0200	0,0200
Minimum Value	18,112,104,000	0,9450	-0,0570
Maximum Value	22,117,678,000	1,0300	0,0300



With the total numbers of asset of 30, obtained monthly from September 2020 - February 2023, it is clear that nonetheless the minimum value of the asset is relatively far from the Face Value (K) of IDR1,000,000,000,000,-. The minimum value in the return of asset was 0.945, this address that the lowest ratio of an asset from a specific period to the asset value of the period before is 94.5% with only 5.5% of downtrend, while for the maximum return was 1.030, indicating that there was a change of 3% in the asset company data. Despite the relatively small numbers in return that represent the change in the asset, it might result in a bigger number in the asset data itself. As for the Asset Ln Return, the minimum value is -0.057 and the maximum value is 0.030. Furthermore, it will be visualized in Figure 1.

With a further overview on the plotline, there was a downtrend on the Company Asset Plot within the 30 months period, it can be seen that the downtrend is constant. On the other hand, it resulted differently in the Return on Asset where there is no specific trend in the return just as the ln Return.

Two normality tests are performed to check the normality of the Ln Return. It is used because logarithmic is believed to be a more stabilized form of raw data.

The normal Q-Q plot shows that some scatter plots are far from the line in both ends, indicating that there might be some outlier in the data [12]. This will be further explained in the formal normality test using Jarque Bera [13].

Test Hypothesis

H_0: return data is normally distributed

H_1: return data is not normally distributed

Statistics test

$$JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right) = 0.6438$$

Conclusion

Based on the output of R Program, we got the value of JB = 0.64348 and p-value = 0.7249, so that we can conclude that H_0 is accepted since the P-value (0.7249) > α (0.05) means that the asset's ln Return of Commonwealth Bank is normally distributed.

4.2. First Passage Time Calculation

A volatility is estimated with the calculation of asset's return standard deviation. The Bond details used for the further calculation of First Passage Time properties are presented in Table 4 [14].

Table 4. Bond Details and First Passage Time Result

Bond Details	
Asset on September 2020	22,117,678,000,000
Asset on February 2023	18,112,104,000,000
Face Value Asset on	1,000,000,000,000
Maturity Date	36 months
BI Rate	5.75%
Asset's volatility	0.07058766
Value of Equity	21,991,492,000,000
Liability	126,185,800,000
Probability of Default	0.00003145407 %

From Table 4, there is an asset decrease along the way of on-going bond from September 2020 with company's asset IDR22,117,678,000,000 to February 2023 with company's asset IDR18,112,104,000,000 where it is still adequate to return the bond face value (K) of IDR1,000,000,000,000. On the other hand, this might not guarantee that there is no possibility for the bond to go default since the bond will have another 7 months to go for it to reach its maturity date of 36 months with the BI rate of 5.75% in 2023.

The liability of the bond contains the Commonwealth Bank responsibilities of IDR126,185,800,000. This number is adequate to be paid if we compare it to the equity in the market is IDR21,991,492,000,000,- consisting of the assets left after fulfilling the liability. With the equities and liabilities number obtained, utilizing the asset's volatility to get a better picture of the whole condition, a probability of default of 0.00003145407% is calculated, this number represents the risk of the bond.

The result is supported by the previous research. According to [15] a bond with the rating A was analyzed with First Passage Time, with the relatively small probability of default, the bond was paid without default by the company in 2017. Whereas according to [14] the bond used was released with the rating A- by the time the research was done, and with the probability of default of 0.002121936% the bond was paid well without default at maturity date

V. CONCLUSION

According to the research findings and discussion, here is the conclusion of this research.

1. The market value of equity is IDR21,991,492,000,000 implying the company's total asset without the debt.
2. The liability is IDR126,185,800,000 implying the company's total debt that needs to be paid to stakeholders, be it form of infestation such as bond, stock, etc.
3. The asset's volatility = 0.07058766 is the company's asset movement and barrier = IDR750,000,000,000 with discount rate of bond interest = 7.50% and the BI rate = 5,75% p.a. The probability of default of "Obligasi I Bank Commonwealth 2020" = 0.00003145407%

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