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K - Power-3 Heronian Mean Labeling of Graphs

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ABSTRACT

We describe a function f as K – Power 3. If $f: V(G) \to \{K, K+1, \ldots\ldots, K+q\}$constitute both the induced edge labelling and take $f(e = yz)$be an injective function and express it as, then a graph's Heronian Mean Labelling $G = (V, E)$ with p nodes and q lines is

$$f(e) = \left\lceil \sqrt[3]{\left[\frac{f(y)^3+(f(y)f(z))^{(\frac{3}{2})}+f(z)^3}{3}\right]} \right\rceil \text{ or } \left\lfloor \sqrt[3]{\left[\frac{f(y)^3+(f(y)f(z))^{(\frac{3}{2})}+f(z)^3}{3}\right]} \right\rfloor$$

with distinct edge labels.

In this manuscript we have proved the K– Power -3 Mean labeling behaviour of Path, Twig Graph, Triangular ladder $L_n, L_n \odot K_1$. We have also investigated K - Super power -3 Heronian Mean labelling of graghs. Also, we prove that K_n is not K– Power - 3 Heronian Mean graph and K - Super power -3 Heronian Mean labelling of Snake related graphs like triangular, alternative triangular and double triangular snake graphs.

KEYWORDS: Power - 3 Heronian Mean Graph, K - Power 3 Heronian Mean Graph, K - Super power -3 Heronian Mean graph.AMS Classification: 05C78

1. INTRODUCTION

$G = (V(G), E(G))$ is a simple, connected, undirected graph with p nodes and q lines. The contents of this paper concise, a synopsis of definitions and additional material is provided. The terms used here are those that are defined by Harary [1]. An integer label is placed to a graph's edges, vertices, or both, depending on the circumstances. J.A. Gallian [2] published an informative survey on graph labeling. The labeling is referred to as a vertex labeling (or edge labeling) if the set of vertices (or edges) represents the mapping's domain. The idea of Mean labelling of graphs was first presented by Somasundaram and Ponraj [4-5]. Labelling with a Harmonic Mean was first proposed by S.S. Sandhya and S.D. Deepa [6]. Power - 3 Mean Labelling was introduced by S.S. Sandhya and S.Sreeji [7] and their behaviour was examined. In this Mnauscript, we analyse K-Power 3 Heronian Mean labeling for numerous graphs and we establish the c oncept of K -Power 3 Heronian Mean labeling.

Explanation: 1.1

A graph $G's$ with p nodes and q lines is said to be considered to be Power -3 Heronian Mean graph if the vertices $v \in V$ can be labeled with unique labels $g(x)$ from $1, 2, \ldots\ldots, q+1$ so that $e = yz$ is labeled in each edge with

$$g(e) = \left\lceil \sqrt[3]{\left[\frac{f(y)^3+(f(y)f(z))^{(\frac{3}{2})}+f(z)^3}{3}\right]} \right\rceil \text{ or}$$

$$g(e) = \left\lfloor \sqrt[3]{\left[\frac{f(y)^3+(f(y)f(z))^{(\frac{3}{2})}+f(z)^3}{3}\right]} \right\rfloor.$$

The line labels become visible. In this instance, f is referred to as $G's$ Power - 3 Heronian Mean labelling [3]. Here, G is referred to as Power -3 Heronian Mean Graph, and f is a Power - 3 Mean labeling of G.

A graph $G = (V, E)$ with p nodes and q lines is said to be K -super power- 3 Heronian Mean graphs If it's feasible to assign unique labels to each of the nodes $x \in v$, $g(x) = K, K+1, +2, +3, ..., K+q-1$ so that each line labelled $e = yz$ is such that when

$$g(e) = \left\lceil \sqrt[3]{\frac{\left[f(y)^3 + (f(y)f(z))^{\left(\frac{3}{2}\right)} + f(z)^3 \right]}{3}} \right\rceil$$ or

$$\left\lfloor \sqrt[3]{\frac{\left[f(y)^3 + (f(y)f(z))^{\left(\frac{3}{2}\right)} + f(z)^3 \right]}{3}} \right\rfloor.$$

Consequently, the generated edge labels are unique. A graph where K -Super power is admitted K -super power - 3 heronian mean graphs is the name given to heronian mean labelling.

Explanation: 1.2

We describe a function f as K – Power - 3. If $f : V(G) \rightarrow \{K, K+1, +2, +3 ......, K+q\}$ constitute both the persuaded edge labelling and an injective function $f(e = yz)$ be defined as, then the Heronian Mean Labeling of a graph $G = (V, E)$ employing q lines and p nodes is $g(e) = \left\lceil \frac{\left[f(y)^3 + f(z)^3 \right]}{2} \right\rceil^{1/3}$ or $\left\lfloor \frac{\left[f(y)^3 + f(z)^3 \right]}{2} \right\rfloor^{1/3}$ with distinct edge labels [8-10].

Explanation: 1.3

A Twig graph, which is a tree, is created when every internal vertex on a path is connected by exactly two pendent edges.

Definition 1.4

The graph known as a Triangular ladder TL_n, $n \geq 2$. It is generated through integrating the edges $y_i \; z_{i+1}$ for $1 < i \leq n-1$, where the nodes of L_n are y_i and z_i, $1 \leq i \leq n$ the two pathways of length n in L_n are $y_1, y_2, ...\, y_n$ and $z_1, z_2, ...z_n$.

Definition 1.5

The graph that results from taking n copies of a graph H and one copy of a graph G, if G has order n connecting each vertex in the i^{th} copy of H to every other vertex in G is called the Corona of G with H, or $G \odot H$.

2. MAIN RESULTS

Statement: 2.1

Path P_n admits K - Power -3 Heronian Mean graph.

Proof:

Let the nodes and lines of P_n be $V(P_n) = (y_i ; 1 \leq i \leq n)$ and $E(P_n) = \{e_i = (y_i, y_i + 1); \; 1 \leq i \leq n-1\}$ respectively.

Define a function g from Path's vertex set to the collection $\{K, K+1, +2, +3, ..., K+q\}$ by,

$g(y_i) = K + i + 1$; i fluctuates from 1 to $n-1$

The edges are labelled with, $g(y_i, y_{i+1}) = K + i - 1$; i fluctuates from 1 to $n-1$

It is evident that f labels P_n as K -Power -3 Heronian Mean, and as a result,

P_n remains a K - Power - 3 Heronian Mean graph.

Illustration: 2.2

501- Power - 3 Heronian mean labeling of $\mathbf{P_5}$

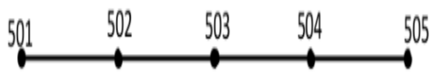

Figure: 2.1

Statement: 2.3

Let's pretend the graph is G that emerges from linking a single edge to each of the two sides of a vertex in P_n. G is therefore referred to as a Heronian Mean graph with K -Power- 3.

Proof:

Let G be the graph that results from connecting one edge to each of the two sides of a vertex in P_n.

Let $z_1, z_2, z_3, \ldots, z_n$ be a Path in P_n. Consider that the pendant vertices next to z_i are y_i and w_i.

It typically consists of 3n-1 edges and 3n vertices.

Define g in terms of $V(G)$ *to* $\{K, K+1, +2, +3, ... , K+q\}$ by,

$g(y_i)=K+3i-3$, i fluctuates from 1 to n

$g(z_i)=K+3i-2,$ i fluctuates from 1 to n

$g(w_i)=K+3i-1,$ i fluctuates from 1 to n

Then the edges are labelled v with, $g(z_i,z_{i+1})=K+3i-1,$ i fluctuates from 1 to n

$g(y_i,z_i)=K+3(i-1)$ i fluctuates from 1 to n

$g(z_i,w_i)=K+3i-2,$ i changes from 1 to n

Since an outcome, we acquire specific values for the edges. G remains a K-Power-3 Heronian mean graph as a consequence.

Illustration: 2.4

The G5 label for 401 Power-3 Heronian Mean appears below.

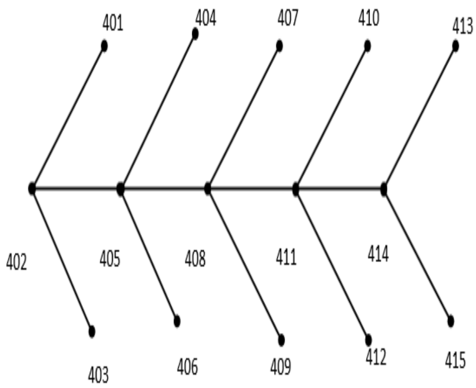

Figure: 2.2

Statement: 2.5

Triangular snake T_n stands K - Power-3 Heronian Mean graph.

Proof:

Consider Tn as triangular serpent. To acquire it, each pair of paths is attached to a new vertex, let's say z_i .

Every edge of a P_n can be swapped out for a cyclic graph c3. It typically has 3n edges and 2n+1 vertices.

Express a function g from $V(T_n)$ to $\{K+1,+2,+3,\ ......,K+q\}$ by,

$g(y_1)=K,$,

$g(y_i)=K+3i-4,$ i fluctuates from 2 to n

$g(z_1)=K+1,$

$g(z_i)=K+3(i-1)$, i fluctuates from to $n-1$

Next, the labels for the induced edges are obtained by,

$g(y_i,y_{i+1})=K+3i-2,$ i fluctuates from 1 to $n-1$

$g(y_i,z_i)=K+3(i-1)$ i fluctuates from 1 to n

$g(y_i,z_{i-1})=K+3i-4,$ i fluctuates from 1 to n.

Then we obtain a distinct value for the edges. Hence T_n remains a K - Power -3 Heronian Mean graph.

Illustration: 2.6

Following is 301 - Power-3 Heronian Mean labeled T_5.

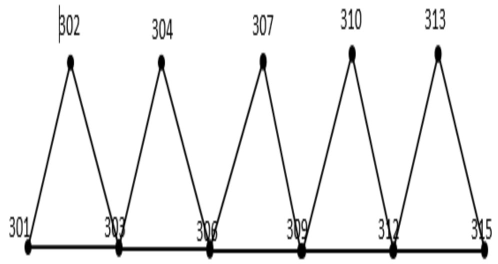

Figure: 2.3

Statement: 2.7

Quadrilateral snake Q_n stays as a K- Power-3 Heronian Mean graph.

Proof:

Let Q_n represent Quadrilateral snake.

It is obtained by joining each pair of a path's vertices to a pair of additional vertices, let's say y_i and w_i.

A cyclic structure C_5, which typically contains 3n-2 vertices and 4n-4 boundaries, is capable of changing every edge in a P_n graph.

Describe a function $g:V(Q_n)\{K,K+1,+2,+3,\ ......,K+q\}$ by,

$g(y_i)=K+i,$, i fluctuates from 1 to n

$g(y_i)=K+4(i-1),$ i fluctuates from 2 to n

$g(z_i)=K,$ i fluctuates from 1 to n

$g(z_i)=K+4i-5,$ i fluctuates from 1 to n

$g(w_i)=K+2,$, i fluctuates from 1 to n

$g(w_i)=K+4i-3,$ i fluctuates from 2 to n

The induced edge labels are subsequently given by,

$g(y_i,z_i)=K+4(i-1)$, i differs from 2 to n

$g(y_i, w_i) = K + 4i - 3$, i varies from 1 to n

$g(z_i, z_{i+1}) = K + 4i - 2$, i varies from 1 to $n-1$

$g(w_i, z_{i+1}) = K + 4i - 1$ i varies from 1 to $n - 1$

It is obvious that the label f is a K-Power-3 Heronian Mean.

Thus, Qn is a graph of K -Power-3 Heronian Mean.

Illustration: 2.8

202 - Power -3 Heronian Mean labeling Q_5 is shown below.

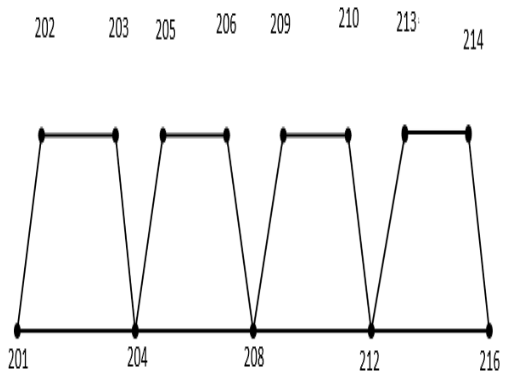

Figure: 2.4

Statement: 2.9

Alternative triangular $A(T_n)$ stays as a K- Power-3 Heronian Mean graph

Proof:

Let $A(T_n)$ be an Alternative triangular Graph.

Describe a function $g : V(A(T_n)) \{ K, K+1, +2, +3, ..., K+q \}$ by,

$g(y_i) = K + 2i - 2$, i differs from 1 to n

$g(z_i) = K + 4i - 3$, i differs from 1 to n

Edges are labeled by,

$g(y_i, z_i) = K + 4(i-1)$, i fluctuates from 1 to n

$g(z_i, y_{i+1}) = K + 4i - 2$, i fluctuates from 1 to $n-1$

$g(y_i, y_{i+1}) = K + 2i - 1$, i fluctuates from 1 to $n-1$

Clearly, f is a K - Power-3 Heronian mean labeling. Hence $A(T_n)$ remains a K - Power -3 Heronian Mean graph.

Illustration: 2.10

55 - Power-3 Heronian Mean labeling $A(T_3)$ is shown below.

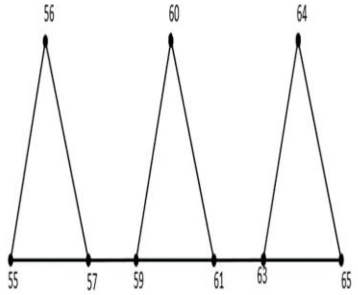

Figure: 2.5

Statement: 2.11

For every n, Comb $P_n \odot K_1$ is a K - Power-3 Heronian Meangraphs.

Proof:

Assume that $P_n \odot K_1$ is a Comb that is obtained by joining every vertex in P_n with a full graph K_1. It typically has 2n-1 edges and 2n vertices.

Describe a function $g : V(P_n \odot K_1) \rightarrow \{ K, K+1, +2, +3, ..., K+q \}$ by,

$g(y_i) = K + 2i - 1$ i fluctuates from 1 to n

$g(z_i) = K + 2(i-1)$, i fluctuates from 1 to n

Then the induced edge labels are

$g(y_i, y_{i+1}) = K + 2i - 1$, i differs from 1 to $n-1$

$g(y_i, z_i) = K + 2(i-1)$, i differs from 1 to n

Then we obtain a dissimilar value for the edges.

Hence $P_n \odot K_1$ remains a K - Power-3 Heronian Mean graph.

Example: 2.12

75 - Power-3 Heronian Mean labeling $P_5 \odot K_1$ is shown below

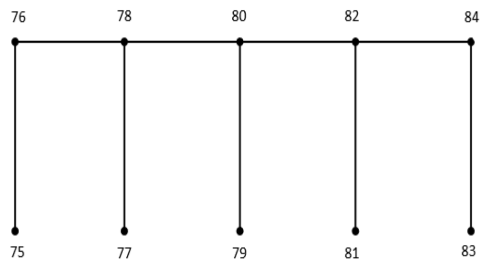

Figure: 2.6

Statement: 2.13

Ladder L_n stays as a K -Power-3 Heronian Mean graph

Proof:

Denote a ladder graph by L_n. In the graph L_n, let y_i, i fluctuates from 1 to n and z_i i differs from 1 to n be the vertices of two paths with length n. Participate in V_i and Ui.

It typically has 3n-2 edges and 2n vertices

State a function $g : V\left(L_n\right) \to \{K, K+1, +2, +3, \ldots\ldots, K+q\}$ by,

$g\left(y_i\right) = K + 3i - 3$, i fluctuates from 1 to n

$g\left(z_i\right) = K + 3i - 2$, i fluctuates from 1 to n

Next, the labels for the induced edges are obtained by,

$g\left(y_i, z_i\right) = K + 3i - 3$, i fluctuates from 1 to n

$g\left(y_i, y_{i+1}\right) = K + 3i - 2$, i fluctuates from 1 to n

$g(z_i, z_{i+1}) = K + 3i - 1$, i fluctuates from 1 to n

It is evident that f has a K-Power-3 Heronian Mean label. L_n remains a K-Power-3 Heronian Mean graph as a result.

Illustration 2.14

301 - Power-3 Heronian Mean labeling L4 is show below

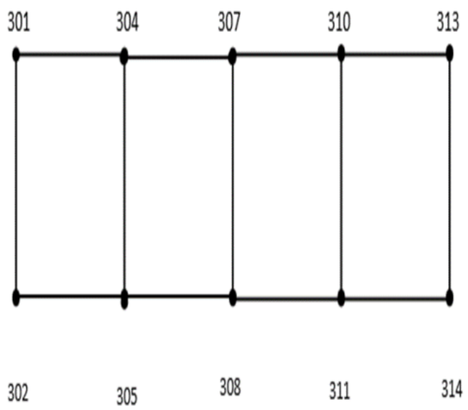

Figure: 2.7

Statement: 2.15

The triangular snake T_n $(n \geq 2)$ stays as a K-Super power-3 Heronian Mean graph for any K.

Proof

Let $\begin{cases} z_i, & i \text{ varies from } 1 \text{ to } n \\ y_i, & i \text{ varies from } 1 \text{ to } n-1 \end{cases}$ be the vertices and

$\begin{cases} e_i, & i \text{ varies from } 1 \text{ to } n \\ a_i, & i \text{ varies from } 1 \text{ to } 2(n-1) \end{cases}$ be the edges

Initially, the vertices are labelled as follows.

For

$f(z_1) = K + 4(i - 1)$ i changes from 1 to n

For $f(y_1) = K + 4i - 3$ i varies from 1 to $n-1$

Then the induced edges labels are

For $f^*(e_2) = K + 4i - 2$ i varies from 1 to $n-1$

For

$$f^*(a_2) = \begin{cases} \dfrac{2K + 4i - 3}{2} & i \text{ is odd} \\ \dfrac{2K + 4i - 2}{2} & i \text{ is even} \end{cases} \quad i \text{ varies from } 1 \text{ to } 2$$

Therefore, the edge labels are all distinct. Hence the triangular snake T_n $(n \geq 2)$ remains a K -super power -3 heronian mean graph for any K.

Statement: 2.16

The double triangular snake $D(T_n)$ $(n \geq 2)$ stays as a K - super power -3 heronian mean graph for any K

Proof:

Let $\{z_1, 1 \leq i \leq n,\ y_1, w_1 \quad 1 \leq i \leq n-1\}$ be the vertices and $\{e_1, 1 \leq i \leq n,\ a_1, b_1 \quad 1 \leq i \leq 2(n-1)\}$ be the edges.

Initially, the vertices are labelled as follows:

For $f\left(z_1\right) = K + 8\left(i - 2\right)$ i fluctuates from 1 to n

For $f\left(y_1\right) = K + 8i - 4$ i fluctuates from 1 to $n-1$

$f\left(w_1\right) = K + 8i - 4$ i fluctuates from 1 to $n-1$

Next, the generated edge labels are

For

$f^*(e_1) = K + 8i - 6$ i varies from 1 to $n-1$

$$f^*(a_1) = \begin{cases} K + 4i - 2i & \text{is odd} \\ K + 4i - 5i & \text{is even} \end{cases}$$

i varies from 1 to $2n-2$

$$f^*(b_1) = \begin{cases} K + 4i + 2i & \text{is odd} \\ K + 4i - 2i & \text{is even} \end{cases}$$

i varies from 1 to $2n-2$

Therefore, the edges labels are all distinct. Hence the double triangular snake $D(T_n)$ $(n \geq 2)$ remains a K - super power -3 heronian mean graph for any K

Statement: 2.17

The alternative triangular snake $A(T_n)$ $(n \geq 2)$ stands for K - super power -3 heronian mean graph for any K

Proof:

Let $\{ z_1, 1 \le i \le n, \; y_1, \; 1 \le i \le n/2 \}$ function as the vertices

and $\{ e_1, 1 \le i \le n-1, \; a_1, \; 1 \le i \le n \}$ be the edges

Initially, the vertices are labelled as follows.

For $f(z_1) = K + 2i - 2$ $\quad i$ fluctuates from $1 \; to \; n$

For $f(y_1) = K + 4n + 3i - 3$ $\quad i$ fluctuates from $1 \; to \; n/2$

Next, the generated edge labels are

For

$$f^*(e_1) = K + 2i - 2 \quad i \;\; varies \;\; from \;\; 1 \;\; to \;\; n-1$$

For

$$f^*(a_1) = \begin{cases} \dfrac{2K+2n+3i-4}{2} & i \; is \; odd \\ \dfrac{2K+2n+3i-6}{2} & i \; is \; even \end{cases} \qquad i \;\; changes \;\; from \;\; 1 \;\; to \;\; n$$

$$f^*(b_1) = \begin{cases} K + 4i + 2i & i \; is \; odd \\ K + 4i - 2i & i \; is \; even \end{cases} \qquad i \;\; varies \;\; from \;\; 1 \;\; to \;\; n$$

Therefore, the edges labels are all distinct. Henc the alternative triangular snake $A(T_n)(n \ge 2)$ remains a K - super power -3 heronian mean graph for any K.

CONCLUSION

The study of labeled K power- 3 heronian graphs are significant because of their wide range of uses. Not every graph has a K Power - 3 Heronian Mean. Examining the graphs that allow for Power 3 Heronian Mean Labeling is quite fascinating. Enough visuals are used to demonstrate the resulting results, improving comprehension. K- Super power -3 Heronian labeling does not satisfy for all graphs. Analyzing comparable outcomes for several additional K-Super power- 3 Heronian Mean Graphs is feasible.

DATA AVAILABILITY STATEMENT

All the data is collected from the simulation reports of the software and tools used by the authors. Authors are working on implementing the same using real world data with appropriate permissions.

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CONFLICTS OF INTEREST

The authors declare that they have no conflict of interest.

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