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# Chickenpox Childhood Disease: An Insight from Mathematical Modelling

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**ABSTRACT**

Chickenpox is an infectious disease that causes an itchy, blister-like rash on the skin and can spread through bodily fluids and body contact. This study presents a mathematical model for the transmission dynamics of chickenpox among children by considering the impact of vaccination and treatment. The qualitative analysis of the model reveals that the model has two equilibrium points, namely: the chickenpox-free and endemic equilibrium points. The disease-free equilibrium point is globally asymptotically stable whenever the basic reproduction number is less than unity ( $R_0 < 1$ ) and the endemic equilibrium point is globally stable whenever the reproduction number is greater than unity ( $R_0 > 1$ ). The normalized forward sensitivity index is also used to obtain the critical factors responsible for the transmission of chickenpox in the population. Furthermore, it reveals that parameters with negative indices will reduce the transmission of chickenpox when increased. The qualitative analysis of the model is supported by numerical simulation

**KEYWORDS:** Chickenpox, Mathematical model, Stability, Equilibrium points, Lyapunov function

## 1. INTRODUCTION

Chickenpox known as varicella is a highly contagious vaccine-preventable disease caused by the initial infection with Varicella Zoster Virus (VZV), a member of the herpes virus family. The disease results in a characteristic skin rash that forms small itchy blisters, which eventually scab over (CDC 2011a). It usually starts on the chest, back and face. It then spreads to the rest of the body; the rash and other symptoms such as fever, tiredness and headaches usually last five to seven days. Complication may occasionally include pneumonia, inflammation of the brain and bacterial skin infections. The disease is usually more serious in adults than in children.

Chickenpox is an airborne disease which easily spread via human-to-human transmission typically through coughs and sneezes of an infected person. The incubation period is 10 – 21 days, after which the characteristic rash appears (CDC 2011b). Chickenpox occurs in all parts of the world. In 2013, there were 140 million cases of chickenpox and shingles worldwide (Wang 2005). Before routine immunization, the number of cases occurring each year was so far to the number of people born. Since immunization, the number of infections in the United States has decreased nearly 90%. In 2015, chickenpox results in 6400 death globally down from 8900 in 1990. Death occurs in about 1 per 60,000 cases. Chickenpox was not separated from smallpox until the late 19th Century (Atkinson and Williams 2011). In 1888, its connection to shingles was determined. The first document use of the term chickenpox was in 1658. Various explanations have been suggested for the use of 'chicken' in the name, one being the relative mildness of the disease. Since its introduction in 1995 in the United States, the varicella vaccine has resulted in a decrease in the number of cases and complication from the disease. It protects about 70-90 percent of people from the disease with a greater benefit for severe disease (Wang et al., 2005).

Routine immunization of children is recommended in many countries (Leeuwen 2015) and immunization within three days of exposure may improve outcome in children. Treatment of those infected may include calamine lotion to help with itching keeping the finger nails short to decrease injury from scratching and the use of paracetamol (acetaminophen) to help with fevers. For those at increase rate of complication, antiviral medication such as acyclovir is recommended.

Mathematical modeling is an important tool used by researchers in studying the transmission dynamics of infectious disease in the human population (Adepoju and Ibrahim 2024a; Abimbade *et al.*, 2024; Adepoju and Olaniyi 2021; Olaniyi 2022) and quite a few studies have investigated

the dynamical spread of chickenpox in the population (Ali *et al*., 2009; Ofori *et al*., 2011; Stephen *et al*., 2014). Ali *et al* (2009) described the clinical manifestation of laboratory findings and frequency of complications in adult chickenpox patients admitted in the hospital. Their result showed that chickenpox in adults' cause's severe systemic manifestations leading to high frequency of complications with increased mortality rate, particularly in older age group and in smokers who develops varicella pneumonia and require mechanical ventilation. Ofori *et al* (2011) develop an SIR mathematical model on the spread of varicella in Ghana. Their model focused on the spread of chickenpox at the initial stages of the infection when the infected persons are absent and when they are present, taking into consideration birth and natural mortality rates. Carrington *et al* (2011) concluded that congenital varicella syndrome, maternal varicella-zoster virus pneumonia and non-neonatal varicella infections are associated with serious fetomaternal morbidity. Shrim *et al* (2012) worked on the management of varicella infection in pregnancy. Recommendations were made toward immunizing all non-immune women as part of pre-pregnancy and postpartum care and vaccination should not be administered in pregnancy. Stephen *et al* (2014) developed a deterministic mathematical model for transmission of chickenpox incorporating vaccination. The effect of some sociological factors on the outbreak of chickenpox disease was investigated by Rahman and Kuddus (2014). Their results indicated that infectious disease such as chickenpox outbreak is associated with the sociological factors, particularly education levels and religious affiliation had independent influence on parent's infection to vaccinate children. Corberans *et al* (2018) developed a deterministic mathematical model for the transmission of chickenpox incorporating vaccination for the human population.

In another development, Katherine *et al* (2018) examined a mathematical model on the role of vaccination and treatment on chickenpox transmission dynamics while Steiner *et al* (2018) proposed an epidemiological model for the dynamics of chickenpox transmission with periodic infection rate. Their results indicated that vaccines are excellent way to preventing many people from contracting chickenpox. Okolo and Abu (2019) stressed on the mathematical model of chickenpox transmission dynamic of using both effect of vaccination and human contact interaction on the transmission of chickenpox virus. Furthermore, Agbata *et al* (2019) developed and analyzed a mathematical model for the control of chickenpox with vaccination and treatment strategy and Bright *et al* (2019) investigated a deterministic mathematical model for transmission dynamic of chickenpox incorporating vaccination. Umar *et al* (2020) presented a deterministic epidemiological model incorporating the method of control adopted by national chickenpox and leprosy programme in Damaturu-Yobe and Nigeria at large while Zhang *et al* (2020) studied the mathematical model for the dynamic of chickenpox virus transmission with non-UK individuals. The effect of routine vaccination on the dynamical spread of chickenpox in Hungary was carried out by Karsai *et al* (2020). Anebi *et al* (2021) investigated the effect of vaccination strategy using Adomian decomposition method and Sanyoos *et al* (2023) stressed on mathematical model depicting the dynamics of chickenpox transmission by incorporating precautionary measure.

It is pertinent to state that this study considers the effects of vaccination and treatment of children affected with chickenpox in the human population. The organization of the work is as follows: Section 2 presents the full description of model. The analysis of the model is carried out in Section 3, while in Section 4, the numerical simulations of the system are performed. Section 5 wraps up the work with concluding remarks.

## 2. MODEL FORMULATION

The total population at time $(t)$, donated by $N(t)$, is subdivided into six mutually exclusive compartments of susceptible individual $S(t)$, vaccinated individual $V(t)$, exposed individual $E(t)$, infected individual $I(t)$, treated individual $T(t)$, recovered individual $R(t)$ respectively. The total human population is obtained as

$$N(t) = S(t) + \mathrm{V}(t) + E(t) + I(t) + T(t) + R(t) \quad (1)$$

Susceptible individuals are recruited into the population at a rate $\pi$. The effective contact rate with the probability of human being infected by the chickenpox virus per population is donated by $\frac{\beta SI}{N}$. The susceptible population increases as the vaccine's effectiveness wanes at a rate $\sigma V$. The population is further reduced by vaccination at birth and natural mortality at the rates $\theta$ and $\mu$. Then, the rate of change of susceptible individuals is given by

$$\frac{dS}{dt} = (1-\theta)\pi - \frac{\beta SI}{N} - \mu S + \sigma \mathrm{V} \quad (2)$$

A fraction of the susceptible human got vaccinated at a rate $\theta\pi$. The population in this compartment is further reduced when the natural death occurs at a rate $\mu$ and vaccine wanes at a rate $\sigma$. Then, the rate of change of vaccinated individuals is given by

$$\frac{d\mathrm{V}}{dt} = \theta\pi - (\sigma + \mu)\mathrm{V} \quad (3)$$

As more members of the susceptible class become infected, the number of exposed people rises proportionately at a rate $\frac{\beta SI}{N}$. The class reduces due to progression of exposed individual to active chickenpox infection at a rate $\omega$. The population is further reduced by natural mortality at a rate $\mu$. Then, the rate of change of exposed individuals is given by

$$\frac{dE}{dt} = \frac{\beta SI}{N} - (\omega + \mu)E \qquad (4)$$

The population of infected individual increases based on the progression of exposed infected individual to active infection at a rate $\omega E$. The population reduces as a result of treatment at a rate $\tau$. The population in this class is further decreased as a result of disease induced death and natural death at the rates $\delta$ and $\mu$. Then, the rate of change of infectious individual is given as

$$\frac{dI}{dt} = \omega E - (\mu + \tau + \delta)I \qquad (5)$$

The treatment compartment increases as a result of progression from the infected compartment at a rate $\tau I$. The population reduces due to natural death at a rate $\mu$ and recovery at a rate $\gamma$. Then, the rate of change of treatment individual is given as

$$\frac{dT}{dt} = \tau I - (\mu + \gamma)T \qquad (6)$$

The population of recovered individuals increases as a result of progression from the treatment class at a rate $\gamma T$. The population is further decreased by natural death at a rate $\mu$. Then, the rate of change of recovered individuals is given as

$$\frac{dR}{dt} = \gamma T - \mu R \qquad (7)$$

Thus, the mathematical model describing the dynamical spread of chickenpox among children is given as follows

$$\left.\begin{aligned}
\frac{dS}{dt} &= (1-\theta)\pi - \frac{\beta SI}{N} - \mu S + \sigma V \\
\frac{dV}{dt} &= \theta\pi - (\sigma + \mu)V \\
\frac{dE}{dt} &= \frac{\beta SI}{N} - (\omega + \mu)E \\
\frac{dI}{dt} &= \omega E - (\mu + \tau + \delta)I \\
\frac{dT}{dt} &= \tau I (\mu + \gamma)T \\
\frac{dR}{dt} &= \gamma T - \mu R
\end{aligned}\right\} \qquad (8)$$

The state variables (8) are subject to the initial conditions:

$S(t) > 0, V(t) > 0, E(t) > 0, I(t) > 0, T(t) > 0$ and $R(t) > 0$

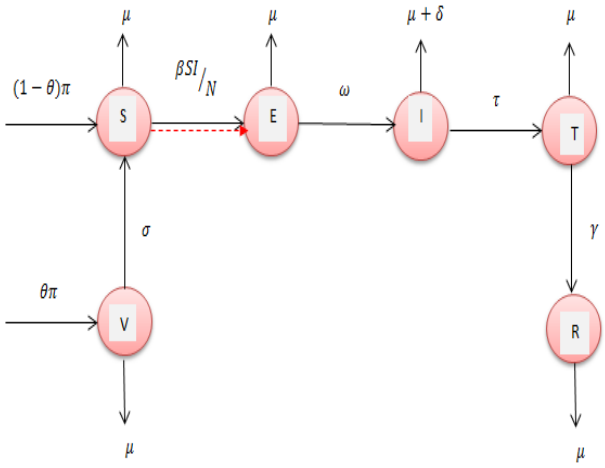


**Figure 1. Schematic diagram of the Chickenpox model**

**Table 1. Description of Variables**

| Variable | Description |
|---|---|
| $S(t)$ | Susceptible Individual |
| $V(t)$ | Vaccinated Individual |
| $E(t)$ | Exposed Individual |
| $I(t)$ | Infected Individual |
| $T(t)$ | Treated Individual |
| $R(t)$ | Recovered Individual |

**Table 2. Description of Parameters of the Chickenpox Model**

| Parameter | Description |
|---|---|
| $\pi$ | Recruitment rate |
| $\theta$ | Fraction of recruitment who are vaccinated |
| $\beta$ | Contact Rate |
| $\mu$ | Natural death rate |
| $\sigma$ | Rate at which vaccine wanes |
| $\delta$ | Progression rate from vaccinated to infected class |
| $\omega$ | Progression rate from exposed to infected |
| $\tau$ | Treatment rate |
| $\gamma$ | Recovery rate |

## 3. ANALYSIS OF THE CHICKENPOX MODEL

### 3.1 The Invariant region

***Theorem 1***: *Let* $(S,V,E,I,T,R)$ *be the solution of (8) with initial condition in a feasible region* $\Omega$, *then* $\Omega \subset R_+^6$ *is positively invariant.*

Proof: It is clear from the first six equations of the system (8) that

$$\frac{dN}{dt} = \pi - \mu N - \delta I \qquad (9)$$

so that,

$$\frac{dN}{dt} = \pi - \mu N \qquad (10)$$

Then, by standard technique it follows that

$$N(t) \le N(0)e^{-\mu t} + \frac{\pi}{\mu}(1 - e^{-\mu t}) \qquad (11)$$

If $N(0) \le \frac{\pi}{\mu}$, then $N(t) \le \frac{\pi}{\mu}$. Thus the region $\Omega$ is positively invariant. Furthermore, if $N(0) \ge \frac{\pi}{\mu}$, then the solution enters $\Omega$ in finite time. Hence the feasible region $\Omega$ attracts all the solutions in $R_+^6$.

Since the region is positively-invariant and attracting, then it is enough to investigate the dynamics of the mathematical model (8) in the feasible region. Hence, the chickenpox model (8) is mathematically and epidemiologically meaningful (Hethcote 2000).

### 3.2 Positivity of solutions

**Theorem 2:** *The solutions of the system (8) with non-negative initial conditions,* $S(t) > 0; V(t) > 0; E(t) > 0; I(t) > 0$ $T(t) > 0$; $R(t) > 0$ *remain non-negative for all time* $t > 0$.

***Proof:*** The first compartment of the system (8) gives rise to

$$\frac{dS}{dt} + \frac{\beta SI}{N} + \mu S \ge 0 \qquad (12)$$

which when solved using standard technique yields

$$\frac{d}{dt}\left[ S(t) \exp\left\{ \int_0^t \frac{\beta I(w)}{N(w)} dw + \mu t \right\} \right] \ge 0 \qquad (13)$$

This implies that

$$S(t) \ge S(0) \exp\left\{ -\int_0^t \frac{\beta I(w)}{N(w)} dw + \mu t \right\} > 0, \quad \forall t > 0$$

Using the same approach, the remaining state variables, $V(t) > 0; E(t) > 0; I(t) > 0$ $T(t) > 0$; $R(t) > 0$ are non-negative for all time $t > 0$.

### 3.3 Existence of equilibria and stability analysis

The existence of the equilibrium points (steady state solutions) of the autonomous system (8) is determined analytically and the stability analysis is also investigated in this section

#### 3.3.1 Chickenpox-free equilibrium

The chickenpox-free equilibrium of the model (8), denoted by $\varepsilon_0$, is given by

$$\varepsilon_0 = (S_0, V_0, E_0, I_0, T_0, R_0) = \left( \frac{\pi(\sigma + \mu - \theta\mu)}{(\sigma + \mu)\mu}, \frac{\theta\pi}{\sigma + \mu}, 0, 0, 0, 0 \right) \qquad (14)$$

The basic reproduction number is the average number of new cases of secondary infection caused by a single infectious individual in the population of susceptible. It is an important threshold under which the incidence of chickenpox will persist or die out in the population. The threshold parameter $R_0$ is calculated using the approach of (Driessche and Watmough 2002; Diekmann *et al*., 1990; Adepoju and Ibrahim 2024a) where the infected compartments are considered at disease-free equilibrium. It can be deduced from the model (8) that

$$\frac{d}{dt}\begin{pmatrix} E(t) \\ I(t) \end{pmatrix} = \begin{pmatrix} \frac{\beta SI}{N} \\ 0 \end{pmatrix} - \begin{pmatrix} (\omega + \mu)E \\ (\mu + \tau + \delta)I - \omega E \end{pmatrix} \qquad (15)$$

from which the transmission matrix $F$ and transition matrix $V$ are obtained at $\varepsilon_0$ respectively, by,

$$F = \begin{pmatrix} 0 & \frac{\beta\{\sigma + \mu(1-\theta)\}}{\sigma + \mu} \\ 0 & 0 \end{pmatrix}$$

and

$$V = \begin{pmatrix} \omega + \mu & 0 \\ -\omega & \mu + \tau + \delta \end{pmatrix}$$

Therefore, the basic reproduction number of the system (8), denoted by $R_0 = \rho(FV^{-1})$, where $\rho$ is the spectral radius of the product of $FV^{-1}$ is obtained as

$$R_0 = \frac{\beta\omega\{\mu(1-\theta) + \sigma\}}{\sigma + \mu} \qquad (16)$$

***Lemma 1:*** *The chickenpox-free equilibrium point,* $\varepsilon_0$, *of the model (8) is locally asymptotically if* $R_0 < 1$ *and unstable if* $R_0 > 1$

The implication of Lemma 1 from the epidemiological point of view, is that the dynamical spread of chickenpox can be effectively controlled in the population whenever $R_0 < 1$, if the initial size of the sub-population of the system (8) are within the point of attraction of $\varepsilon_0$.

#### 3.3.2 Global Stability of Chickenpox-free equilibrium

***Theorem 3:*** *The chickenpox-free equilibrium denoted* $\varepsilon_0$ *is globally asymptotically stable whenever* $R_0 < 1$.

Proof: Considering the infectious compartments, the proof is based on comparison theorem as applied in (Augusto and Gumel 2010; Erinle-Ibrahim *et al*., 2022, Adepoju *et al*., 2024c). The infected compartments of the model (8) is written in the form

$$\begin{pmatrix} \frac{dE}{dt} \\ \frac{dI}{dt} \end{pmatrix} = \begin{pmatrix} E \\ I \end{pmatrix} - \left(1 - \frac{S}{N}\right) F \begin{pmatrix} E \\ I \end{pmatrix} \quad (17)$$

where $F$ and $V$ are transmission and transition matrices respectively obtained in section (3.3.1). Since $S(t) \le N(t)$ for all $t > 0$, then it follows from (17) that

$$\begin{pmatrix} \frac{dE}{dt} \\ \frac{dI}{dt} \end{pmatrix} \le (F - V) \begin{pmatrix} E \\ I \end{pmatrix} \quad (18)$$

Now, knowing that all the eigenvalues of $|(F-V) - \lambda I|$ all have negative real parts, it follows that the differential inequality (18) is stable whenever $R_0 < 1$. Consequently, $(E(t), I(t)) \to (0, 0)$ as $t \to \infty$. Thus by comparison theorem (Huo and Feng 2013), it follows that $(E(t), I(t)) \to (0, 0)$. Then $(S(t), V(t), E(t), I(t), T(t), R(t)) \to (0, 0, 0, 0, 0, 0)$ as $t \to \infty$. Hence the chickenpox-free equilibrium, $\varepsilon_0$, is globally asymptotically stable when $R_0 < 1$

### 3.3.3 Endemic equilibrium

Let $\varepsilon^{**} = (S^*, V^*, E^*, I^*, T^*, R^*)$ be the endemic equilibrium of the system (8). At steady states, let the force of infection be $\lambda^* = \frac{\beta I}{N}$. Then solving the system (8) simultaneously at steady state yields

$$\left.\begin{aligned} S^* &= \frac{k_1\pi(1-\theta) - \sigma\theta\pi}{k_1(\lambda^* + \mu)}, \\ V^* &= \frac{\theta\pi}{k_1}, \\ E^* &= \frac{\lambda\pi(k_1(1-\theta) + \theta\sigma)}{k_1k_2(\lambda^* + \mu)} \\ I^* &= \frac{\pi\omega\lambda^*(k_1(1-\theta) + \theta\sigma)}{k_1k_2k_3(\lambda^* + \mu)} \\ T^* &= \frac{\tau\pi\omega\lambda^*(k_1(1-\theta) + \theta\sigma)}{k_1k_2k_3k_4(\lambda^* + \mu)} \\ R^* &= \frac{\gamma\tau\pi\omega\lambda^*(k_1(1-\theta) + \theta\sigma)}{k_1k_2k_3k_4\mu(\lambda^* + \mu)} \end{aligned}\right\} \quad (19)$$

Substituting $I^*$ from (19) into $\lambda^*$ and simplifying gives

$$\lambda^* = \frac{\mu}{k_3}\left(\frac{\omega}{k_2}R_0 - 1\right) \quad (20)$$

It can deduced from (20), one possible condition for which positive $\lambda^*$ can be obtained: if $R_0 > 1; k_2k_3 > 0$. Hence chickenpox will persist in population provided $R_0 > 1$.

### 3.3.4 Global stability of endemic equilibrium

***Theorem 4:*** *The endemic equilibrium point of the system (8) is globally asymptotically stable whenever the threshold parameter, $R_0$, is greater than unity.*

***Proof:*** The global asymptotic stability of the endemic equilibrium is proved following the approach of (Adepoju and Olaniyi 2021; Abimbade *et al*., 2024; Adepoju *et al*., 2024b).

Consider the Lyapunov function $L : \Omega \in R_+^6 \to R_+$ defined by

$$L = \frac{1}{2}\{(S - S^*) + (V - V^*) + (E - E^*) + (I - I^*) + (T - T^*) + (R - R^*)\}^2 \quad (21)$$

The time derivative of (21) gives

$$\begin{aligned} \dot{L} &= \{(S - S^*) + (V - V^*) + (E - E^*) + (I - I^*) \\ &+ (T - T^*) + (R - R^*)\} \\ &\frac{d}{dt}(S + V + E + I + T + R) \end{aligned} \quad (22)$$

$$\begin{aligned} &= \{(S - S^*) + (V - V^*) + (E - E^*) \\ &+ (I - I^*) + (T - T^*) + (R - R^*)\} \\ &\times \{\pi - (S + V + E + I + T + R)\mu - \delta I\} \end{aligned} \quad (23)$$

$$\begin{aligned} \dot{L} &\le \{(S - S^*) + (V - V^*) + (E - E^*) \\ &+ (I - I^*) + (T - T^*) + (R - R^*)\} \\ &\times \left\{(S + V + E + I + T + R) - \frac{\pi}{\mu}\right\} \end{aligned} \quad (24)$$

Since $N^* = \frac{\pi}{\mu}$, then simplifying (24) gives

$$\begin{aligned} \dot{L} &= \left\{\begin{matrix}(S - S^*) + (V - V^*) + (E - E^*) + (I - I^*) \\ + (T - T^*) + (R - R^*)\end{matrix}\right\} \\ &\times \left\{\begin{matrix}(S + V + E + I + T + A) \\ - (S^* + V^* + E^* + I^* + T^* + A^*)\end{matrix}\right\} \end{aligned} \quad (25)$$

$$\begin{aligned} &= -\mu\left\{\begin{matrix}(S - S^*) + (V - V^*) + (E - E^*) + (I - I^*) \\ + (T - T^*) + (R - R^*)\end{matrix}\right\} \\ &\times \left\{\begin{matrix}(S - S^*) + (V - V^*) + (E - E^*) \\ + (I - I^*) + (T - T^*) + (R - R^*)\end{matrix}\right\} \\ &= -\mu\left\{\begin{matrix}(S - S^*) + (V - V^*) + (E - E^*) \\ + (I - I^*) + (T - T^*) + (R - R^*)\end{matrix}\right\}^2 \end{aligned} \quad (26)$$

Consequently, the derivative of the continuously differentiable function $L$ is negative semi definite, that is, $\dot{L} \leq 0$, then the function $L$ is a Lyapunov function. Therefore, $\frac{dL}{dt} = 0$, provided $S = S^*, V = V^*, E = E^*, I = I^*, T = T^*, R = R^*$. Then by Lasalle's invariance principle (Lasalle 1976), the largest invariance set for which, $\frac{dL}{dt} = 0$ is the singleton set $\{\varepsilon^*\}$ which implies that the endemic equilibrium point of the chickenpox model is globally asymptotically stable.

### 3.3.5 Sensitivity Analysis

The influence of the parameters of the chickenpox model related to the basic reproduction number is examined through sensitivity analysis using the normalized forward sensitivity index. The sensitivity analysis is very important do discovering how best the dynamical spread of chickenpox can be curbed by studying the critical factors responsible for its transmission (Adepoju *et al.*, 2024). By definition, the normalized forward sensitivity index of a variable is the ratio of the relative change in the variable to the relative change in the parameter (Chitnis *et al.*, 2008; Rois *et al.*, 2021). The normalized forward sensitive indices of the basic reproduction number $R_0$, relative to its parameter $P$, is given by,

$$\Gamma_P^{R_0} = \frac{\partial R_0}{\partial P} \times \frac{P}{R_0} \tag{27}$$

The sensitivity indices of the parameters that are associated with the basic reproduction number is calculated using (27) and the results are presented in Table 3

**Table 3. Sensitivity indices of the basic reproduction number relative to its parameters**

| Parameter | Value | Sensitivity indices | Source |
|---|---|---|---|
| $\beta$ | 0.2 | 1.000 | Agbata *et al.*,(2019) |
| $\mu$ | 0.03 | -0.002 | Ofori (2011) Agbata *et al.*, (2019) |
| $\theta$ | 0.05 | 0.656 | Assumed |
| $\omega$ | 0.01 | 1.000 | CDC (2004), Agbata *et al.*, (2019) |
| $\sigma$ | 0.5 | -0.838 | Ogabi (2009) |

## 4. NUMERICAL SIMULATION

Numerical simulations of the chickenpox model were carried out using MATLAB computing software in order to corroborate the analytical solutions established in section 3 and the results are presented graphically.

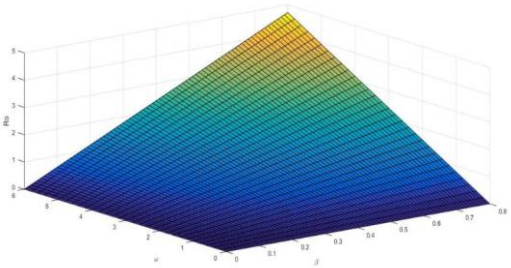

**Figure 2. Effects of $\omega$ and $\beta$ on the threshold parameter $R_0$**

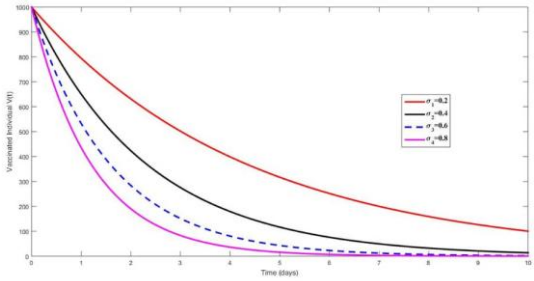

**Figure 3. Effect of varying values of vaccination waning rate on vaccinated individuals**

The influences of the transmission rate, $\beta$ and progression rate $\omega$ on the basic reproduction number are depicted in Figure 2. It is observed that both $\beta$ and $\omega$ has corresponding increase on the threshold parameter $R_0$. This implies from epidemiological viewpoint that there would be persistence of chickenpox in the population if appropriate measures to curbing the disease are not strictly adhered to. Figure 3 presents the effect of varying values of the rate at which vaccine wanes, $\sigma$ on the vaccinated individuals. It is observed that as the value of $\sigma$ increases, the population of vaccinated individuals' decreases and this might be as a result of weak immune system. The dynamics of the effect of the transmission rate, $\beta$ on the susceptible population is presented in Figure 4. It can be deduced that as the value of the transmission rate, $\beta$ increases, there is a proportionate decrease in the population of the susceptible. Ultimately, this means that adequate policy needs to be put in place so as to reduce the surge in the transmission of the disease. Figure 5 presents the global asymptotic stability of exposed individuals around the disease-free equilibrium. It is observed that regardless of the initial size of the population of the exposed individuals', the population will converge to the chickenpox-free equilibrium. This implies that chickenpox can be reduced in the population. Figure 6 shows impact of the global stability of exposed individual in the basin of endemic equilibrium. It can be deduced that regardless of the initial size of the population, the exposed individuals' will converge to a unique endemic equilibrium.

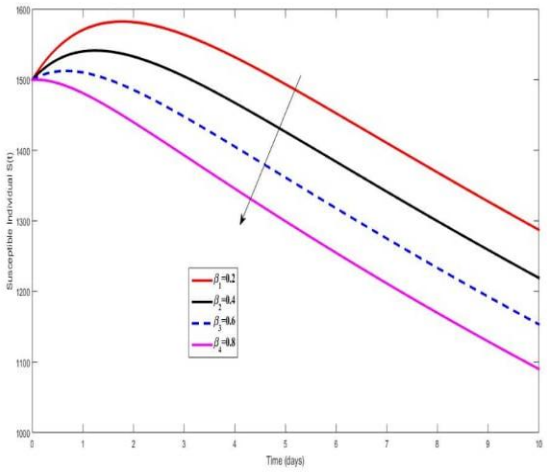


**Figure 4. The impact of varying values of transmission rate, $\beta$ on the susceptible population**

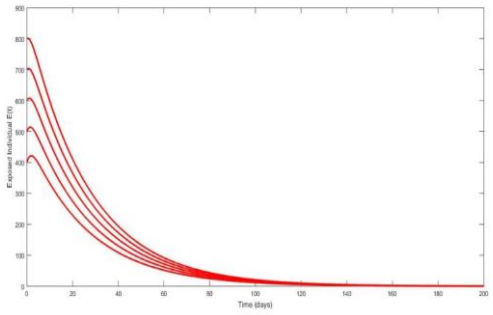


**Figure 5. Global stability of exposed individuals around chickenpox-free equilibrium**

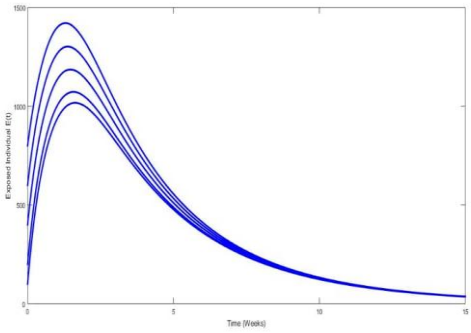


**Figure 6. Global stability of exposed individuals around chickenpox-present equilibrium**

## 5. CONCLUSION

This study is concerned with the mathematical assessment of the dynamical spread of chickenpox with vaccination and treatment among children using a system of ordinary differential equation. The model is divided into six mutually exclusive compartments of Susceptible, Vaccinated, Exposed, Infected, Treated and Recovered individuals respectively. A qualitative analysis of the model is performed to show that solution of the model is positive and bounded, implying that the model formulated is mathematically and biologically meaningful. An analytical solution of the model revealed that the model possesses two equilibrium points; the disease-free and endemic equilibrium points. The basic reproduction number was obtained using the next generation matrix method. By applying the comparison test approach, the disease-free equilibrium was shown to be globally asymptotically stable and the global asymptotic stability of the endemic equilibrium point was established using quadratic Lyapunov function. It was observed that chickenpox will persist in the children population when $R_0 > 1$. The influence of the critical parameters influencing the transmission of chickenpox was obtained using the normalized forward sensitivity index and it was deduced that parameters with positive indices will have increase on the basic reproduction number while those with negative indices will have significant reduction on the basic reproduction number when increased. Efforts should be intensified by healthcare practitioners and policy makers to reducing the value of parameters with positive indices so as to reduce the transmission of chickenpox in the population. Additionally, it is crucial to state that the values of the parameters used in this study are obtained from existing literature while the value of $\theta$ is hypothetically chosen. Real time data of chickenpox can be used to fit the model for a realistic study.

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