International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 12 Issue 11 November 2024, Page no. – 4592-4596

Index Copernicus ICV: 57.55, Impact Factor: 8.316

DOI: 10.47191/ijmcr/v12i11.07

Analysis and Solution of The Model Production System with Time Delays Using Max-Plus Algebra

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ARTICLE INFO

Published Online:
27 November 2024

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ABSTRACT

Max-Plus algebra has been used to model and analyze network problems, such as railway networks, project scheduling, queuing systems, and production systems. In this network problem, Max-Plus Algebra is used for scheduling analysis. The assumption used in this analysis is a normal situation, there are no changes in the system that affect scheduling. One factor that is important and needs to be considered is the occurrence of delays in the system which affect scheduling. The objectives of this research are 1) to analyze the effect of system delays on scheduling, and 2) to determine interventions in the form of switching on the server to overcome delay propagation due to schedule delays at certain stages in the system. The simple production system in this research involves 3 servers. The arrangement of the three servers is a combination of parallel and series arrangements. The simulation in this research uses the Scilab. The switching carried out in this research is by speeding up the process on the server by 1 unit of time. The results of this research are 1) direct delay propagation occurs if the delay occurs on the server that contains the eigenvalues. If a delay occurs on a server that does not contain eigen values then there will be no delay propagation provided that the amount of delay on that server does not exceed $(t-1)$ where t is the time required for the entire process from that server to the next server. If the delay exceeds $(t-1)$ then delay propagation will occur. 2) Based on simulations using Scilab, delay propagation can be overcome by switching one or more servers according to the delay point. Delays on the server containing the eigenvalues can only be resolved by switching the server. Meanwhile, on servers that do not contain eigenvalues, switching can be done using a combination of these servers.

KEYWORDS: Max-Plus Algebra, production system, delay, scheduling, switching

I. INTRODUCTION

Set of $\mathbb{R}_{max} = \mathbb{R} \cup \{-\infty\}$ with maximum operation $(\oplus)$ and plus $(\otimes)$ has commutative idempotent semiring structure. This structure called max-plus algebra, denoted by $(\mathbb{R}_{max}, \oplus, \otimes)$ [1], [2] Max-Plus algebra is widely used as a tool for studying dynamic linear systems. Matrix Spectral Theory over max-plus algebra is used to determine the characteristics of a dynamic linear system.

The application of Max-Plus Algebra is generally used for modeling and analyzing networks, both in the fields of health, transportation, logistics, queuing, production systems, or simulating robot movements. In the health sector, it is used to analyze queues for pharmaceutical services [3]. This relates to drug collection services, whether patented drugs or compounded drugs. Pharmacy services in Indonesia for these two types of drugs are not differentiated so scheduling is needed. In the field of public transportation, passenger waiting time is the most common issue. Passenger waiting time modeling is completed using a max-plus algebraic model with synchronization rules and a power algorithm [4]. Scheduling logistics services in ports can also be modeled using max-plus algebra [5]. Analysis of queuing theory with a one stage multi server or multi stage one server model [6], [7], Application of max-plus algebra in the production sector, for scheduling production of various types of commodities and manufacturing flow lines [8], [9], [10], [11]. Modeling robot movement with various constructions of moving legs also uses max-plus algebra [12], [13]. More complex network problems are usually assisted with Petri nets as a visualization diagram before constructing a max-plus algebraic model [9], [14].

The production system is a series of activities to process raw materials into ready-to-use materials. The process of producing ready-to-use goods can go through a series of stages. The production system consists of several machines (servers) to process materials into materials to be reprocessed by the next server or ready-to-use materials. System scheduling is important to get optimal results and the system to work efficiently. Processing time on the server and time for materials to arrive at the server for processing are two important factors that need to be considered in scheduling. Modeling a simple production system with a closed series or fork join system model can use max-plus algebra [15], [16], [17]. Network problems such as this production system require important attention in terms of periodicity. Periodicity is important so that scheduling arrangements become more controlled. Periodicity in matrix spectral theory over max-plus algebra is closely related to eigenvalues of matrix $A \in \mathbb{R}_{max}^{n\times n}$ [18], [19], [20].

The production system probably has a problem during the production process. The problem is during the delivery of materials to the server or during processing constraints on the server. Problems that occur can cause delays in production. Apart from that, the delays that occur are may to have a delay effect on subsequent processes. This is known as delay propagation. This problem is analogous to delays in train scheduling [21], [22]. This research discusses 1) analysis of the impact of a delay on the system and 2) interventions that can be provided so that there is no propagation of delays in the production system.

II. METHOD

This research is a literature study research on simple production systems. Literature studies were carried out to determine production system models, system analysis, analysis of simulation results, and determine alternative solutions from the model. The production system used in this model involves 3 servers. Analysis of the impact of delays on the system will be carried out in this model. After that, solutions were prepared as an effort to overcome the impact of delays on the system. Figure 1 below shows the flow of this research.

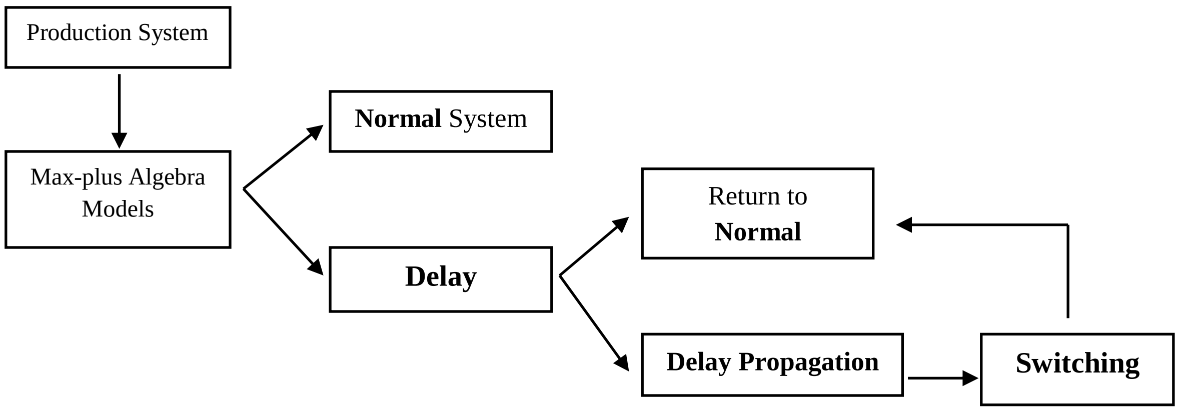

Figure 1. Step of the Research

III. RESULT AND DISCUSSION

A. *System Production Model*

The simple production system in this research uses 3 servers arranged like a Fork Model. This server arrangement combines series and parallel arrangements. Figure 2 below shows an illustration of this production system.

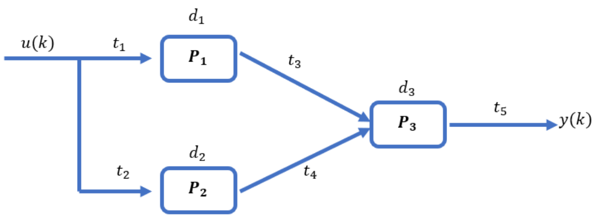

Figure 2. Production System in Fork Model

with

$u(k)$: time of raw materials are entered to system during $(k+1)^{th}$ process

$x_i(k)$: processing time of server i during k^{th} process, $i = 1, 2, 3$.

$y(k)$: time of k^{th} production has finished.

Based on the production system illustration above, the processing time for raw materials from each server is obtained as follows.

$$\begin{cases} x_1(k+1) = \max(x_1(k) + d_1,\ u(k) + t_1) \\ x_2(k+1) = \max(x_2(k) + d_2,\ u(k) + t_2) \\ x_3(k+1) = \max(x_1(k) + d_1 + t_1,\ x_2(k) + d_2 + t_4,\ x_3(k) + d_3) \end{cases} \quad (1)$$

and the time of completed production is,

$$y(k) = x_3(k) + d_3 + t_5 \quad (2)$$

Equation (1) and (2) are express in system of linear equation over max-plus algebra as follows

$$\begin{cases} x_1(k+1) = d_1 \otimes x_1(k) \oplus t_1 \otimes u(k) \\ x_2(k+1) = d_2 \otimes x_2(k) \oplus t_2 \otimes u(k) \\ x_3(k+1) = (d_1 + t_3) \otimes x_1(k) \oplus (d_2 + t_4) \otimes x_2(k) \oplus d_3 \otimes x_3(k) \end{cases} \quad (3)$$

and

$$y(k) = (d_3 + t_5) \otimes x_3(k) \quad (4)$$

A system is formed in equations (3) and (4) in the matrix equations of Max-Plus algebra, as follows.

$$\boldsymbol{x}(k+1) = A \otimes \boldsymbol{x}(k) + B \otimes \boldsymbol{u}(k) \quad (5)$$

and

$$\boldsymbol{y}(k) = C \otimes \boldsymbol{x}(k) \quad (6)$$

with,

$$A = \begin{bmatrix} d_1 & \varepsilon & \varepsilon \\ \varepsilon & d_2 & \varepsilon \\ d_1 + t_3 & d_2 + t_4 & d_3 \end{bmatrix}, B = \begin{bmatrix} t_1 \\ t_2 \\ \varepsilon \end{bmatrix}, \text{ and } C = [\varepsilon \quad \varepsilon \quad d_3]$$

The assumption in this research is $u(k) = y(k)$, that is, raw materials are immediately entered into the system once the raw materials have been produced (out of the system) [16]. Equation (5) and (7) become, as follows.

$$\boldsymbol{x}(k+1) = \bar{A} \otimes \boldsymbol{x}(k) \quad (7)$$

with $\bar{A} = A \oplus B \otimes C$. Equation (7) is a Max-Plus Algebra system model of a production system with a fork model as in figure (1) above.

Simulations are carried out to determine the performance of the production system under normal conditions. In this simulation, it takes time $d_1 = 5, d_2 = 6$ and $d_3 = 3$ respectively to process production materials on servers 1, 2 and 3. In addition, it is assumed that the time required for raw materials to arrive at the servers is $t_1 = 2$, $t_2 = 0$, $t_3 = 1$, $t_4 = 0$, and $t_5 = 0$. As a result, we obtain the matrix of the max-plus algebraic system

$$\overline{A_0} = \begin{bmatrix} 5 & \varepsilon & 5 \\ \varepsilon & 6 & 3 \\ 6 & 6 & 3 \end{bmatrix}$$

Eigen value of $\overline{A_0}$ is 6 and one of vector eigen is $[0,1,1]^T$. Simulation of production system model above used Scilab, with the initial condition $\boldsymbol{x}(0) = [0,1,1]^T$. Simulation result of scheduling time $\boldsymbol{x}(k)$ of the production system model, as follows.

0.	6.	12.	18.	24.	30.	36.	42.	48.	54.	60.	66.	72.
1.	7.	13.	19.	25.	31.	37.	43.	49.	55.	61.	67.	73.
1.	7.	13.	19.	25.	31.	37.	43.	49.	55.	61.	67.	73.

Sheduling time above show the system has time periodicity 6.

B. *Delay Affect the Performance of the System*

In this research, delays will be analyzed and simulated when k = 3 on each server.

Delay on Server 1

Based on the simulation results on server 1, the system can return to normal if the delay occurs a maximum of 4 time units. If it is more than 4 then there will be a delay propagation. Scheduling time of system after delay $\tilde{\boldsymbol{x}}(k)$ with time delay 1, 3, and 4, as follows.

0.	6.	12.	18+1.	24.	30.	36.	42.	48.	54.	60.	66.
1.	7.	13.	19	25.	31.	37.	43.	49.	55.	61.	67.
1.	7.	13.	19.	25.	31.	37.	43.	49.	55.	61.	67.

0.	6.	12.	18+3.	26.	32.	37.	43.	48.	54.	60.	66.	72.
1.	7.	13.	19	25.	31.	37.	43.	49.	55.	61.	67.	73.
1.	7.	13.	19	27.	32.	38.	43.	49.	55.	61.	67.	73.

0.	6.	12.	18+5.	28.	34.	39.	45.	50.	56.	61.	67.	73.
1.	7.	13.	19	25.	32.	38.	44.	50.	56.	62.	68.	74.
1.	7.	13.	19	29.	34.	40.	45.	51.	56.	62.	68.	74.

That implies $\boldsymbol{d}(k) = \tilde{\boldsymbol{x}}(k) - \boldsymbol{x}(k)$ with $k \geq 3$

1.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

3.	2.	2.	1.	1.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	2.	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.

5.	4.	4.	3.	3.	2.	2.	1.	1.	1.	1.	1.
0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.
0.	4.	3.	3.	2.	2.	1.	1.	1.	1.	1.	1.

Delay on Server 2

The simulation results on server 2 show that the delays that occur always cause delay propagation in the production system. The following are the simulation results $\tilde{\boldsymbol{x}}(k)$ for delays on server 2 with delay times of 1 and 2 time units.

0.	6.	12.	18	24.	31.	37.	43.	49.	55.	61.	67.	73.
1.	7.	13.	19+1	26.	32.	38.	44.	50.	56.	62.	68.	74.
1.	7.	13.	19	26.	32.	38.	44.	50.	56.	62.	68.	74.

0.	6.	12.	18	24.	32.	38.	44.	50.	56.	62.	68.	74.
1.	7.	13.	19+2	27.	33.	39.	45.	51.	57.	63.	69.	75.
1.	7.	13.	19	27.	33.	39.	45.	51.	57.	63.	69.	

We get $\boldsymbol{d}(k)$ as follows

0.	0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.
1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.
0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.

0.	0.	2.	2.	2.	2.	2.	2.	2.	2.	2.	2.
2.	2.	2.	2.	2.	2.	2.	2.	2.	2.	2.	2.
0.	2.	2.	2.	2.	2.	2.	2.	2.	2.	2.	2.

Delay on Server 3

Server 3 has similar characteristics to server 1, there is a maximum delay tolerance so that there is no delay propagation. On server 3, the system can return to normal if the processing delay does not exceed 3 time units on that server. The following are the simulation results of $\tilde{\boldsymbol{x}}(k)$ on server 3 for delays of 3 and 4 time units.

0.	6.	12.	18.	27.	32.	38.	43.	49.	54.	60.	66.
1.	7.	13.	19.	25.	31.	37.	43.	49.	55.	61.	67.
1.	7.	13.	19+3	25.	33.	38.	44.	49.	55.	61.	67.

0.	6.	12.	18.	28.	33.	39.	44.	50.	55.	61.	67.
1.	7.	13.	19.	26.	32.	38.	44.	50.	56.	62.	68.
1.	7.	13.	19+4	26.	34.	39.	45.	50.	56.	62.	68.

We get $\boldsymbol{d}(k)$ as follows

0.	3.	2.	2.	1.	1.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3.	0.	2.	1.	1.	0.	0.	0.	0.	0.	0.	0.

0.	4.	3.	3.	2.	2.	1.	1.	1.	1.	1.	1.
0.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.
4.	1.	3.	2.	2.	1.	1.	1.	1.	1.	1.	1.

Based on the simulation results of the delays of the three servers in the production system above, the following properties are obtained.

Lemma 1. *The Eigen value of matrix in the production system is the maximum processing time for materials on the server in the production system.*

Proof. Graph $\mathcal{G}(\overline{A_0})$ is a strongly connected. Based on [2], the eigen value is the maximal average of critical circuit of this graph. For all $a_{ij} \in \overline{A_0}$, $a_{ij} \leq 6$ then

$$\lambda = \frac{a_{ij}(1) + a_{ij}(2) + \cdots + a_{ij}(n)}{n} \leq 6$$

With $a_{ij}(k) \in \overline{A_0}$ for $k = 1,2,\dots,n$. We have $d_2 = 6$, then there exist a loop in this graph. Hence, $\lambda = 6$. ■

Lemma 2. *If delays occur on the server that contains the eigen values then the system will occurs delay propagation.*

Proof. In normal situations, the scheduling time for raw materials ready to be processed on each server is stated by $\boldsymbol{x}(k+1) = \overline{A_0}^{\otimes k} \otimes \boldsymbol{x}(k)$ with $\boldsymbol{x}(0) = [0,1,1]^T$ and eigen value of $\overline{A_0}$ is $\lambda = 6$. Definition eigen value of matrix $\overline{A_0}$ implies $\boldsymbol{x}(k) = [6(k-1), 6(k-1)+1, 6(k-1)+1]$. If $\delta > 0$ stated time delay at server 2 during k^{th} process, then

$$\widetilde{\boldsymbol{x}}(k) = \begin{bmatrix} 6(k-1) \\ 6(k-1)+1+\delta \\ 6(k-1)+1 \end{bmatrix}$$

It implies,

$$\widetilde{\boldsymbol{x}}(k) = \overline{A_0} \otimes \widetilde{\boldsymbol{x}}(k) = \begin{bmatrix} 6k \\ 6k+1+\delta \\ 6k+1+\delta \end{bmatrix}$$

Base on this process, it can be seen that delay propagation occurs on servers 2 and 3. If the process continues, delays will occur on all servers in the system.

C. Switching system to Solve Delay Propagation

Switching system of max-plus algebra is one of alternative to solve delay propagation problem [22].In this production system. Each server can be speed up the process production maximum in 1 time unit. The matrices of switching system, as follows.

$$\overline{A_1} = \begin{bmatrix} 4 & \varepsilon & 5 \\ \varepsilon & 6 & 3 \\ 5 & 6 & 3 \end{bmatrix}, \overline{A_2} = \begin{bmatrix} 5 & \varepsilon & 5 \\ \varepsilon & 5 & 3 \\ 6 & 5 & 3 \end{bmatrix}, \overline{A_3} = \begin{bmatrix} 5 & \varepsilon & 4 \\ \varepsilon & 6 & 2 \\ 6 & 6 & 2 \end{bmatrix}, \text{ and}$$

$$\overline{A_4} = \begin{bmatrix} 4 & \varepsilon & 4 \\ \varepsilon & 6 & 2 \\ 5 & 6 & 2 \end{bmatrix}$$

Matrices $\overline{A_1}, \overline{A_2}$, and $\overline{A_3}$ represented speed up the process in server 1, 2, and 3 respectively. Matrix $\overline{A_4}$ represented speed up both server 1 and 3 simultaneously.

Set of class switching system matrices is $\mathcal{A} = \{\overline{A_0}, \overline{A_1}, \overline{A_2}, \overline{A_3}, \overline{A_4}\}$. J is index sequence, that is sequence that contain the index of matrices in $\mathcal{A}$. The model of switching system of max-plus algebra with index sequence J of production system above is $\mathcal{M}_S = \mathcal{M}_S(\mathcal{A}, \boldsymbol{x}(0), J)$. Matrices in class of set $\mathcal{A}$ will use in switching system in order the scheduling time of production system return to normal. Switching system is more optimal if it need as few as step to return to normal scheduling time.

Definition 3. *If $(\boldsymbol{x}(k))_{k\geq 0}$ and $(\widetilde{\boldsymbol{x}}(k))_{k\geq 0}$ are normal scheduling vector and delay scheduling vector of $\mathcal{M}_S$. Vector $(\widetilde{\boldsymbol{x}}(k))_{k\geq 0}$ convergent to $(\boldsymbol{x}(k))_{k\geq 0}$ if $\exists\, N \in \mathbb{N}$ such that $\forall k \geq N$, $\widetilde{\boldsymbol{x}}(k) = \boldsymbol{x}(k)$. The smallest N is called the time resolution of delay scheduling time.*

The following is a switching system simulation to overcome delay propagation caused by delays on server 1. The switching matrix applied is $\overline{A_1}$. We have the result of $\boldsymbol{d}(k)$ is

5.	3.	3.	1.	1.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
0.	3.	1.	1.	0.	0.	0.	0.	0.	0.	0.	0.

This result shows that after delay 5 unit time in server 1, the system switch with switching matrix $\overline{A_1}$. Furhermore, for $k \geq 5$, the vector sequence $(\widetilde{\boldsymbol{x}}(k))_{k\geq 0}$ convergent to $(\boldsymbol{x}(k))_{k\geq 0}$. Index sequence of this switching system is $J = (1,1,1,1,1,0,0,0,\dots)$ with resolution time $N = 5$.

The following is a summary of solutions to delays that occur on each server.

Table 1. Alternative Solution for Delay Problem

Server	Delay(δ)	J	(N)
1	5	$(1,1,1,1,1,0,0,0,\dots)$	5
1	5	$(3,3,3,3,3,0,0,0,\dots)$	5
1	5	$(4,4,4,0,0,0,0,0,\dots)$	3
2	1	$(2,0,0,0,0,0,0,0,\dots)$	1
2	2	$(2,2,0,0,0,0,0,0,\dots)$	2
3	4	$(3,3,3,3,0,0,0,0,\dots)$	4
3	4	$(4,4,4,0,0,0,0,0,\dots)$	3

Based on table 1 above, a delay of 5 time units on server 1 can be overcome by speeding up the process on servers 1 and 3 simultaneously with a resolution time of 3. Delays on server 2 can only be overcome by speeding up the process on server 2 with a resolution time equal to delays that occur. The delay characteristics on server 3 are similar to server 1, namely that it can be resolved with a resolution time of 3.

IV. CONCLUSION

Delay problem in production system produces two possibilities: 1) affect system performance temporary, namely the system can return to normal without any intervention and 2) affects the overall performance of the system. The second condition requires intervention switching system in order to scheduling time of the system return to normal immediately.

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