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Nano $(1,2)^{\star}$-π-Closed Sets and its Generalizations

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ABSTRACT

In this paper, we introduce the notions of nano $(1,2)^{\star}$-π-closed sets and nano $(1,2)^{\star}$-πg-closed sets, nano $(1,2)^{\star}$-$w\pi g$-closed sets and nano $(1,2)^{\star}$-rwg-closed sets use it to obtain a characterization and preservation theorems of quasi-normal spaces.

KEYWORDS: nano $(1,2)^{\star}$-π-closed sets and nano $(1,2)^{\star}$-πg-closed sets, nano $(1,2)^{\star}$-$w\pi g$-closed sets and nano $(1,2)^{\star}$-rwg-closed sets

I. INTRODUCTION AND PRELIMINARIES

All of these theories were superseded by Lellis Thivagar et al,[7], who introduced a new topology to investigate the imperfect data in mathematics. Because of its small size, the new topology is called Nano Topology. Determining the lower and upper approximations and boundary regions of a subset of a universe reduces it to a maximum of five open sets, regardless of the universe's size. The nano open sets are the components of the nanotopology. Nanotopology in $\check{C}$ech rough closure space, weak forms of nanotopology, and other topics were further established by Lellis Thivagar et al, [8]. In [2], Buvaneshwari et al examined the idea of nano$(1,2)^{\star}$-g-closed sets. The unique notions of generalized closed sets and maps were recently developed and researched by a number of nanobiologists (res. [4, 3, 5, 9]). In this paper, we introduce the notions of nano $(1,2)^{\star}$-π-closed sets and nano $(1,2)^{\star}$-πg-closed sets, nano $(1,2)^{\star}$-$w\pi g$-closed sets , nano $(1,2)^{\star}$-rwg-closed sets and submaximal space use it to obtain a characterization and preservation theorems of quasi-normal spaces.

Definition 1.1 [10] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \cup_{x\in U} \{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \cup_{x\in U} \{R(x): R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 1.2 [7] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,
3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

Definition 1.3 [3] Let U be the universe, R be an equivalence relation on U and $\tau_{R_{1,2}}(X) = \cup \{\tau_{R_1}(X), \tau_{R_2}(X)\}$ where $X \subseteq U$. Then it is satisfies the following axioms:

1. U and $\phi \in \tau_{R_{1,2}}(X)$.
2. The union of the elements of any sub collection of $\tau_{R_{1,2}}(X)$ is in $\tau_{R_{1,2}}(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_{R_{1,2}}(X)$ is in $\tau_{R_{1,2}}(X)$.

Then $\tau_{R_{1,2}}(X)$ is a topology on U called the Nano bitopology on U with respect to X. $(U, \tau_{R_{1,2}}(X))$ is called the Nano bitopological space. Elements of the Nano bitopology are known as Nano $\tau_{1,2}$-open sets in U. Elements of $(\tau_{R_{1,2}}(X))^c$ are called Nano $\tau_{1,2}$-closed sets in $\tau_{R_{1,2}}(X)$.

Definition 1.4 [3] If $(U, \tau_{R_{1,2}}(X))$ is a Nano bitopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

1. The Nano $(1,2)^{\star}$ closure of A is defined as the intersection of all Nano $(1,2)^{\star}$ closed sets containing A and it is denoted by $N_{\tau_{1,2}}$-$cl(A)$.
2. The Nano $(1,2)^{\star}$ interior of A is defined as the union of all Nano $(1,2)^{\star}$ open subsets of A contained in A and it is denoted by $N_{\tau_{1,2}}$-$int(A)$.

Definition 1.5 Let $(U, \tau_{R_{1,2}}(X))$ be a Nano bitopological space and $A \subseteq U$. Then A is said to be

1. Nano $(1,2)^{\star}$-α-open [5] if $A \subseteq N_{\tau_{1,2}}$-$int(N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(A)))$.
2. Nano $(1,2)^{\star}$-semi open [3] if $A \subseteq N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(A))$.
3. Nano $(1,2)^{\star}$-pre open [6] if $A \subseteq N_{\tau_{1,2}}$-$int(N_{\tau_{1,2}}$-$cl(A))$.
4. Nano $(1,2)^{\star}$-regular open [4] if $A \subseteq N_{\tau_{1,2}}$-$int(N_{\tau_{1,2}}$-$cl(A))$.

The complements of the above mentioned open sets are called their respective closed sets.

Definition 1.6 Let $(U, \tau_{R_{1,2}}(X))$ be a Nano bitopological space and $A \subseteq U$. Then A is said to be

1. Nano $(1,2)^{\star}$-g-closed set [2] if $N_{\tau_{1,2}}$-$cl(G) \subseteq V$ where $G \subseteq V$ and V is nano $\tau_{1,2}$-open.
2. Nano $(1,2)^{\star}$-gp-closed set [6] if $N_{(1,2)^{\star}}$-$pcl(A) \subseteq H$ whenever $A \subseteq H$ and H is Nano $\tau_{1,2}$-open in U.
3. Nano $(1,2)^{\star}$-αg-closed set [5] if $N_{(1,2)^{\star}}$-$\alpha cl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano $\tau_{1,2}$-open in V.
4. Nano $(1,2)^{\star}$-rg-closed set [4] if $N_{\tau_{1,2}}$-$cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano $(1,2)^{\star}$-regular open.

The collection of all Nano $(1,2)^{\star}$-g-closed set (resp. Nano $(1,2)^{\star}$-gp-closed set, Nano $(1,2)^{\star}$-αg-closed set and Nano $(1,2)^{\star}$-rg-closed set) in U is denoted by $N_{(1,2)^{\star}}$-g-closed set, (resp. $N_{(1,2)^{\star}}$-gp-closed set, $N_{(1,2)^{\star}}$-αg-closed set and $N_{(1,2)^{\star}}$-rg-closed set). The complements of the above mentioned closed sets are called their respective open sets.

Definition 1.7 [11] A subset S of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is said to be a nano $(1,2)^{\star}$-locally closed set (short in $N_{(1,2)^{\star}}$-LC set) if $S = F \cap G$ where F is $N_{\tau_{1,2}}$-open and G is $N_{\tau_{1,2}}$-closed in $(U, \tau_{R_{1,2}}(X))$.

II. NANO $(1,2)^{\star}$-π-OPEN SETS AND QUASI NANO $(1,2)^{\star}$-NORMAL

Definition 2.1 A subset A of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is called a nano $(1,2)^{\star}$-π-open (short in $N_{(1,2)^{\star}}$-π-open) if the finite union of $N_{(1,2)^{\star}}$-regular open sets. The complement of $N_{(1,2)^{\star}}$-π-open if $A^c = U - A$ is $N_{(1,2)^{\star}}$-π-closed.

Definition 2.2 A nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is said to be a nano $(1,2)^{\star}$-pre-normal if for any pair of disjoint $N_{\tau_{1,2}}$-closed sets A and B of U, there exist disjoint $N_{(1,2)^{\star}}$-preopen sets E and F of U such that $A \subseteq E$ and $B \subseteq F$.

Definition 2.3 A nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is said to be quasi nano $(1,2)^{\star}$-pre normal if for every pair of disjoint $N_{(1,2)^{\star}}$-π-closed subsets A and B of U, there exist disjoint nano $(1,2)^{\star}$-preopen subsets E and F of U such that $A \subseteq E$ and $B \subseteq F$.

Theorem 2.4 For a nano bitopological space $(U, \tau_{R_{1,2}}(X))$, the following are equivalent.

1. U is quasi nano $(1,2)^{\star}$-pre normal space.
2. for every pair of $N_{(1,2)^{\star}}$-π-open subsets E and F of U whose union is U, there exist $N_{(1,2)^{\star}}$-preclosed

subsets G and H of U such that $G \subseteq E$ and $H \subseteq F$ and $G \cup H = U$.

3. For any $N_{(1,2)^{\star}}$-π-closed set A and each $N_{(1,2)^{\star}}$-π-open set B such that $A \subseteq B$, there exists $N_{(1,2)^{\star}}$-preopen set E such that $A \subseteq E \subseteq N_{\tau_{1,2}}$-$pcl(E) \subseteq B$.

4. For every pair of disjoint $N_{(1,2)^{\star}}$-π-closed subsets A and B of U there exist $N_{(1,2)^{\star}}$-preopen subsets E and F of U such that $A \subseteq E$, $B \subseteq F$ and $N_{\tau_{1,2}}$-$pcl(E) \cap N_{\tau_{1,2}}$-$pcl(F) = \phi$.

Proof : $(1) \Rightarrow (2)$: Let E and $F)$ be any $N_{(1,2)^{\star}}$-π-open subsets of a quasi nano $(1,2)^{\star}$-pre normal space U such that $E \cup F = U$. Then, $U \backslash E$ and $U \backslash F$ are $N_{(1,2)^{\star}}$-π-closed subsets of U. By quasi nano $(1,2)^{\star}$-π-normality of U, there exist disjoint $N_{(1,2)^{\star}}$-π-open subsets E_1 and F_1 of U such that $U \backslash E \subseteq E_1$ and $U \backslash F \subseteq F_1$. Let $G = U \backslash E_1$ and $H = U \backslash F_1$. Then G and H are $N_{(1,2)^{\star}}$-preclosed subsets of U such that $G \subseteq E$ and $H \subseteq F$ and $G \cup H = U$.

$(2) \Rightarrow (3)$: Let A be a $N_{(1,2)^{\star}}$-π-closed set and B be a $N_{(1,2)^{\star}}$-π-open subset of U such that $A \subseteq B$. Then, $U \backslash A$ and B are $N_{(1,2)^{\star}}$-π-open subsets of U whose union is U. Then by (2), there exist $N_{(1,2)^{\star}}$-preclosed sets G and H of U such that $G \subseteq U \backslash A$ and $H \subseteq B$ and $G \cup H = U$. Then $A \subseteq U \backslash G$, $B \subseteq U \backslash H$ and $(U \backslash G) \cap (U \backslash H) = \phi$. Let $E = U \backslash G$ and $F = U \backslash H$. Then E and F are disjoint $N_{(1,2)^{\star}}$-preopen sets such that $A \subseteq E \subseteq U \backslash F \subseteq B$. Since $U \backslash F$ is $N_{(1,2)^{\star}}$-preclosed, then we have $N_{\tau_{1,2}}$-$pcl(E) \subseteq U \backslash F$. Thus, $A \subseteq E \subseteq N_{\tau_{1,2}}$-$pcl(E) \subseteq B$.

$(3) \Rightarrow (4)$: Let A and B are any two disjoint $N_{(1,2)^{\star}}$-π-closed sets of U. Then $A \subseteq U \backslash B$ where $U \backslash B$ is $N_{(1,2)^{\star}}$-π-open. Then, by (3), there exists a $N_{(1,2)^{\star}}$-preopen subset E of U such that $A \subseteq E \subseteq N_{\tau_{1,2}}$-$pcl(E) \subseteq U \backslash B$. Let $F = U \backslash N_{\tau_{1,2}}$-$pcl(E)$. Then F is $N_{(1,2)^{\star}}$-preopen subset of U. Thus, we obtain, $A \subseteq E$, $B \subseteq F$ and $N_{\tau_{1,2}}$-$cl(E) \cap N_{\tau_{1,2}}$-$pcl(F) = \phi$.

$(4) \Rightarrow (1)$: Obvious.

Theorem 2.5 For of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$, if U is quasi nano $(1,2)^{\star}$-pre-normal, then for any disjoint $N_{(1,2)^{\star}}$-π-closed subsets A and B of U there exist $N_{(1,2)^{\star}}$-gp-open subsets E and F of U such that $A \subseteq E$ and $B \subseteq F$.

Proof. Let U be a quasi nano $(1,2)^{\star}$-pre-normal space. Let A and B be any disjoint $N_{(1,2)^{\star}}$-π-closed subsets of U. By quasi nano $(1,2)^{\star}$-pre-normality of U, there exist disjoint $N_{(1,2)^{\star}}$-pre open subsets E and F of U such that $A \subseteq E$ and $B \subseteq F$. Thus, E and F are disjoint $N_{(1,2)^{\star}}$-gp-open subsets U such that $A \subseteq E$ and $B \subseteq F$.

Remark 2.6 For a subset of a nano bitopological space, we have following implications:

$N_{(1,2)^{\star}}$-regular open $\Rightarrow N_{(1,2)^{\star}}$-$\pi$-open $\Rightarrow N_{\tau_{1,2}}$-open.

In this diagram, none of the above implications is reversible, as shown in the following examples.

Example 2.7 Let $U = \{y_1, y_2, y_3\}$ with $U/R_1 = \{\{y_1\}, \{y_2\}, \{y_3\}\}$ and $X = \{y_1\}$ then $\tau_{R_1}(X) = \{\phi, \{y_1\}, U\}$ and let $U/R_2 = \{\{y_2\}, \{y_1, y_3\}\}$ and $X = \{y_2\}$ then $\tau_{R_2}(X) = \{\phi, \{y_2\}, U\}$. Then the sets in $\{\phi, \{y_1\}, \{y_2\}, \{y_1, y_2\}, U\}$ are called $N_{\tau_{1,2}}$-open and the sets in $\{\phi, \{y_3, y_4\}, \{y_1, y_3, y_4\}, \{y_2, y_3, y_4\}, U\}$ are called $N_{\tau_{1,2}}$-closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then the subset $\{y_1, y_2\}$ is $N_{(1,2)^{\star}}$-π-open set but not $N_{(1,2)^{\star}}$-regular open.

Example 2.8 Let $U = \{y_1, y_2, y_3, y_4\}$ with $U/R_1 = \{\{y_1\}, \{y_2\}, \{y_3\}, \{y_4\}\}$ and $X = \{y_1\}$ then $\tau_{R_1}(X) = \{\phi, \{y_1\}, U\}$ and let $U/R_2 = \{\{y_1\}, \{y_4\}, \{y_2, y_3\}\}$ and $X = \{y_2, y_3\}$ then $\tau_{R_2}(X) = \{\phi, \{y_2, y_3\}, U\}$. Then the sets in $\{\phi, \{y_1\}, \{y_2, y_3\}, \{y_1, y_2, y_3\}, U\}$ are called $N_{\tau_{1,2}}$-open and the sets in $\{\phi, \{y_4\}, \{y_1, y_4\}, \{y_2, y_3, y_4\}, U\}$ are called $N_{\tau_{1,2}}$-closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then the set subset $\{y_1\}$ is $N_{\tau_{1,2}}$-open set but not $N_{(1,2)^{\star}}$-π-open.

Note 2.9 Given a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ and a subset $A \subseteq U$, we denote by $N_{\tau_{1,2}}(A)$ the simple extension of $N_{\tau_{1,2}}$ over A, i.e., the collection of sets $P \cup (Q \cap A)$, where $P \in N_{\tau_{1,2}}$ and $Q \in N_{\tau_{1,2}}$. Note that $N_{\tau_{1,2}}(A)$ is a nano bitopology on U finer than $N_{\tau_{1,2}}$.

Definition 2.10 A subset A of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is called

1. nano $(1,2)^{\star}$-dense if $N_{\tau_{1,2}}$-$cl(A) = U$,

2. nano $(1,2)^{\star}$-nowhere dense if $N_{\tau_{1,2}}$-$int(N_{\tau_{1,2}}$-$cl(A)) = \phi$.

Lemma 2.11 If A is a nano $(1,2)^{\star}$-dense subset of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then A is also nano $(1,2)^{\star}$-dense in $(U, \tau_{R_{1,2}}(X))$.

Definition 2.12 A nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is called

1. nano $(1,2)^{\star}$-submaximal if each nano $(1,2)^{\star}$-dense subset of U is $N_{\tau_{1,2}}$-open.
2. nano $(1,2)^{\star}$-g-submaximal if every nano $(1,2)^{\star}$-dense subset is $N_{(1,2)^{\star}}$-g-open.
3. nano $(1,2)^{\star}$-rg-submaximal if every nano $(1,2)^{\star}$-dense subset is $N_{(1,2)^{\star}}$-rg-open.

Theorem 2.13 Every subspace S of a nano $(1,2)^{\star}$-submaximal space U is nano $(1,2)^{\star}$-submaximal.

Proof.Let A be a nano $(1,2)^{\star}$-dense subset of S. Then $N_{\tau_{1,2}}$-$cl(A) \cap S = S$ and so $S \subseteq N_{\tau_{1,2}}$-$cl(A)$. Since $A \cup (U \backslash N_{\tau_{1,2}}$-$cl(A))$ is nano $(1,2)^{\star}$-dense in U, then it is an $N_{\tau_{1,2}}$-open subset of U. Hence $S \cap (A \cup (X \backslash N_{\tau_{1,2}}$-$cl(A))) = A$ is $N_{\tau_{1,2}}$-open in S or equivalently S is nano $(1,2)^{\star}$-submaximal.

Theorem 2.14 For a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ the following conditions are equivalent:

1. U is nano $(1,2)^{\star}$-submaximal.
2. For any $A \subseteq U$, the subspace $N_F r(A) = N_{\tau_{1,2}}$-$cl(A) \backslash N_{\tau_{1,2}}$-$int(A) = N_{\tau_{1,2}}$-$cl(A) \cap N_{\tau_{1,2}}$-$cl(X \backslash A)$ is discrete.

Proof. (1) $\Rightarrow$ (2): Let $x \in N_F r(A)$. Since A is nano $(1,2)^{\star}$-dense in A, then so is $A \cup \{x\}$. Since A is submaximal according to Theorem 2.13, then $A \cup \{x\} = N_{\tau_{1,2}}$-$cl(A) \cap P$, where P is $N_{\tau_{1,2}}$-open in U. In the same way it can be seen that $(X \backslash A) \cup \{x\} = N_{\tau_{1,2}}$-$cl(X \backslash A) \cap Q$, where Q is $N_{\tau_{1,2}}$-open in U. Thus $\{x\} = (A \cup \{x\}) \cap ((X \backslash A) \cup \{x\}) = N_{\tau_{1,2}}$-$cl(A) \cap N_{\tau_{1,2}}$-$cl(X \backslash A) \cap P \cap Q$. Hence $\{x\}$ is $N_{\tau_{1,2}}$-open in $N_F r(A)$ and so $N_F r(A)$ is discrete.

(2) $\Rightarrow$ (1): Let A be nano $(1,2)^{\star}$-dense in X. By (4) $N_{\tau_{1,2}}$-$cl(A) \backslash N_{\tau_{1,2}}$-$int(A) = U \backslash N_{\tau_{1,2}}$-$int(A)$ is discrete and thus $A \backslash N_{\tau_{1,2}}$-$int(A)$ is its $N_{\tau_{1,2}}$-open subset. Hence $A \backslash N_{\tau_{1,2}}$-$int(A) = (X \backslash N_{\tau_{1,2}}$-$int(A)) \cup P$, where P is $N_{\tau_{1,2}}$-open in U. Thus $A \backslash N_{\tau_{1,2}}$-$int(A) \subseteq P$ and so $A \backslash N_{\tau_{1,2}}$-$int(A) \subseteq P \backslash N_{\tau_{1,2}}$-$int(A)$. For the reverse inclusion if $x \in U \backslash N_{\tau_{1,2}}$-$int(A)$, then $x \in (X \backslash N_{\tau_{1,2}}$-$int(A)) \cap U = A \backslash N_{\tau_{1,2}}$-$int(A)$. This shows that $A \backslash N_{\tau_{1,2}}$-$int(A) = U \backslash N_{\tau_{1,2}}$-$int(A)$ and hence $A = U \cup N_{\tau_{1,2}}$-$int(A)$. Thus A is $N_{\tau_{1,2}}$-open in U.

Recall that a space U is called a $N_{(1,2)^{\star}}$-$T_{1/2}$-space if it satisfies the following statements:

1. Every $N_{(1,2)^{\star}}$-g-closed set is $N_{\tau_{1,2}}$-closed.
2. Every singleton is either $N_{\tau_{1,2}}$-open or $N_{\tau_{1,2}}$-closed.
3. The complement of a finite set is $N_{(1,2)^{\star}}$-LC set.

Theorem 2.15 Every subset I of a nano $(1,2)^{\star}$-submaximal space U is nano $(1,2)^{\star}$-locally closed and hence U is a $N_{(1,2)^{\star}}$-$T_{1/2}$-space.

Proof. Since by Theorem 2.13. $N_{\tau_{1,2}}$-$cl(I)$ is nano $(1,2)^{\star}$-submaximal and since I is its nano $(1,2)^{\star}$-dense subset, then I is $N_{\tau_{1,2}}$-open in I. Hence $I = X \cap N_{\tau_{1,2}}$-$cl(I)$, where X is $N_{\tau_{1,2}}$-open and I is $N_{\tau_{1,2}}$-closed. Thus I is $N_{(1,2)^{\star}}$-LC set.

The more general result is as follows

Theorem 2.16 For a nano bitopological space U the following are equivalent:

1. U is nano $(1,2)^{\star}$-submaximal.
2. Every subset of U is nano $(1,2)^{\star}$-locally closed.

Proof. $(1) \Rightarrow (2)$: Since by Theorem 2.15.

(2) $\Rightarrow$ (1): If I is nano $(1,2)^{\star}$-dense in U, then by (2) $I = A \cap B$, where A is $N_{\tau_{1,2}}$-open and B is $N_{\tau_{1,2}}$-closed. Since $I \subseteq B$, then $U = I \subseteq B$. So $B = U$ and hence $I = X$. Hence I is $N_{\tau_{1,2}}$-open in U.

Note: Nano $(1,2)^{\star}$-$T_{1/2}$-spaces (briefly, $N_{(1,2)^{\star}}$-$T_{1/2}$-spaces) need not be nano $(1,2)^{\star}$-submaximal; not even metric spaces are always nano $(1,2)^{\star}$-submaximal.

Theorem 2.17 Let $(U, \tau_{R_{1,2}}(X))$ be a nano bitopological space. For any subset I of U, the following statements are equivalent:

1. U is nano $(1,2)^{\star}$-rg-submaximal ,
2. Every nano $(1,2)^{\star}$-codense subset I of U is nano $(1,2)^{\star}$-regular closed.

Proof. (1)$\Rightarrow$(2) Let I be a nano $(1,2)^{\star}$-codense subset of U. Since $U \backslash I$ is nano $(1,2)^{\star}$- dense set, by (1), $U \backslash I$ is $N_{(1,2)^{\star}}$-regular open. Hence I is $N_{(1,2)^{\star}}$-regular closed.

(2)⇒(1) Let I be a nano $(1,2)^{\star}$-dense subset of U. Since $U \backslash I$ is nano $(1,2)^{\star}$-codense set, $U \backslash I$ is $N_{(1,2)^{\star}}$-regular closed. Hence I is $N_{(1,2)^{\star}}$-regular open.

Theorem 2.18 Every nano $(1,2)^{\star}$-rg-submaximal space is a nano $(1,2)^{\star}$-g-submaximal space.

Proof. Let $(U, \tau_{R_{1,2}}(X))$ be a nano $(1,2)^{\star}$-rg-submaximal space and $I \subseteq U$ be a nano $(1,2)^{\star}$-dense set. Since $(U, \tau_{R_{1,2}}(X))$ is nano $(1,2)^{\star}$-rg-submaximal, I is nano $(1,2)^{\star}$-regular open in $(U, \tau_{R_{1,2}}(X))$. Since every nano $(1,2)^{\star}$-regular open is $N_{\tau_{1,2}}$-open, I is $N_{\tau_{1,2}}$-open. Hence $(U, \tau_{R_{1,2}}(X))$ is a nano $(1,2)^{\star}$-g-submaximal space as every nano $(1,2)^{\star}$-dense set is a $N_{\tau_{1,2}}$-open set in $(U, \tau_{R_{1,2}}(X))$.

III. NANO $(1,2)^{\star}$-πg-CLOSED SETS

Definition 3.1 A subset A of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is said to be nano $(1,2)^{\star}$-π- generalized closed (in short $N_{(1,2)^{\star}}$-πg-closed) if $N_{\tau_{1,2}}$-$cl(A) \subseteq G$ whenever $A \subseteq G$ and G is $N_{(1,2)^{\star}}$-π-open in U.

The complement of $N_{(1,2)^{\star}}$-πg-closed set is called $N_{(1,2)^{\star}}$-πg-open.

Remark 3.2 For a subset of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$, we have the following implications.

$N_{\tau_{1,2}}$-closed

↓

$N_{(1,2)^{\star}}$-g-closed→$N_{(1,2)^{\star}}$-πg-closed→ $N_{(1,2)^{\star}}$-rg-closed

In this diagram, none of the above implications is reversible, as shown in the following examples.

Example 3.3 In example 2.8, then

1. the subset $\{y_1\}$ is $N_{(1,2)^{\star}}$-πg-closed set but not $N_{\tau_{1,2}}$-closed.
2. the subset $\{y_1\}$ is $N_{(1,2)^{\star}}$-πg-closed set but not $N_{(1,2)^{\star}}$-g-closed.

Example 3.4 Let $U = \{y_1, y_2, y_3, y_4\}$ with $U/R_1 = \{\{y_1\}, \{y_2\}, \{y_3\}, \{y_4\}\}$ and $X = \{y_1\}$ then $\tau_{R_1}(X) = \{\phi, \{y_1\}, U\}$ and let $U/R_2 = \{\{y_1\}, \{y_2\}, \{y_3, y_4\}\}$ and $X = \{y_2\}$ then $\tau_{R_2}(X) = \{\phi, \{y_2\}, U\}$. Then the sets in $\{\phi, \{y_1\}, \{y_2\}, \{y_1, y_2\}, U\}$ are called $N_{\tau_{1,2}}$-open and the sets in $\{\phi, \{y_3, y_4\}, \{y_1, y_3, y_4\}, \{y_2, y_3, y_4\}, U\}$ are called $N_{\tau_{1,2}}$-closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then the subset $\{y_1, y_2\}$ is $N_{(1,2)^{\star}}$-rg-closed set but not $N_{(1,2)^{\star}}$-πg-closed.

Theorem 3.5 For a nano bitopological space $(U, \tau_{R_{1,2}}(X))$, the following are equivalent:

1. U is extremely disconnected.
2. Every subset of U is $N_{(1,2)^{\star}}$-πg-closed.
3. The nano bitopology on U generated by the $N_{(1,2)^{\star}}$-πg-closed sets is the discrete one.

Proof. $(1) \Rightarrow (2)$ Let $A \subseteq G$, where A is an arbitrary subset of U and G is $N_{(1,2)^{\star}}$-π-open. Since G is the finite union of $N_{(1,2)^{\star}}$-regular open sets and U is extremely disconnected, G is the finite union of $N_{\tau_{1,2}}$-clopen sets and hence $N_{\tau_{1,2}}$-clopen. Thus $N_{\tau_{1,2}}$-$cl(A) \subseteq N_{\tau_{1,2}}$-$cl(G) = G$, which shows that A is $N_{(1,2)^{\star}}$-πg-closed.

$(2) \Rightarrow (1)$ Let F be a $N_{(1,2)^{\star}}$-regular open set of U. Since by (2) F is also $N_{(1,2)^{\star}}$-πg-closed, we have $N_{\tau_{1,2}}$-$cl(F) \subseteq F$, which shows that F is $N_{\tau_{1,2}}$-closed and hence $N_{\tau_{1,2}}$-clopen. Thus, U is extremally disconnected.

$(2) \Rightarrow (3)$ This is obvious.

Theorem 3.6 The following are equivalent for a nano bitopological space $(U, \tau_{R_{1,2}}(X))$:

1. U is quasi nano $(1,2)^{\star}$-normal.
2. for any disjoint $N_{(1,2)^{\star}}$-π-closed sets P and F, there exist disjoint $N_{(1,2)^{\star}}$-g-open sets G and E such that $P \subseteq G$ and $F \subseteq E$.
3. for any disjoint $N_{(1,2)^{\star}}$-π-closed sets P and F, there exist disjoint $N_{(1,2)^{\star}}$-πg-open sets G and E such that $P \subseteq G$ and $F \subseteq E$.
4. for any $N_{(1,2)^{\star}}$-π-closed set P and any $N_{(1,2)^{\star}}$-π-open set E containing P, there exists a $N_{(1,2)^{\star}}$-g-open set G of U such that $P \subseteq G \subseteq N_{\tau_{1,2}}$-$cl(G) \subseteq E$.
5. for any $N_{(1,2)^{\star}}$-π-closed set P and any $N_{(1,2)^{\star}}$-π-open set E containing P, there exists a $N_{(1,2)^{\star}}$-πg-open set G of U such that $P \subseteq G \subseteq N_{\tau_{1,2}}$-$cl(G) \subseteq E$.

Proof. It is obvious that $(1) \Rightarrow (2)$, $(2) \Rightarrow (3)$ and $(4) \Rightarrow (5)$.

$(3) \Rightarrow (4)$ Let P be any $N_{(1,2)^{\star}}$-π-closed set of U and E any $N_{(1,2)^{\star}}$-π-open set containing P. There exist disjoint $N_{(1,2)^{\star}}$-πg-open sets G, K such that $P \subseteq G$ and $U - E \subseteq K$. We know that a subset A is $N_{(1,2)^{\star}}$-πg-

open if and only if $B \subseteq N_{\tau_{1,2}}$-$int(A)$ whenever B is nano $N_{(1,2)^{\star}}$-π-closed and $B \subseteq A$, since we have $U - E \subseteq N_{\tau_{1,2}}$-$int(F)$ and $G \cap N_{\tau_{1,2}}$-$int(F) = \phi$. Therefore, we obtain $N_{\tau_{1,2}}$-$cl(G) \cap N_{\tau_{1,2}}$-$int(F) = \phi$ and hence $P \subseteq G \subseteq N_{\tau_{1,2}}$-$cl(G) \subseteq U - N_{\tau_{1,2}}$-$int(F) \subseteq E$.

$(5) \Rightarrow (1)$ Let P, F be any disjoint $N_{(1,2)^{\star}}$-π-closed sets of U. Then $P \subseteq U - F$ and $U - F$ is $N_{(1,2)^{\star}}$-π-open and hence there exists a $N_{(1,2)^{\star}}$-πg-open set H of U such that $P \subseteq H \subseteq N_{\tau_{1,2}}$-$cl(H) \subseteq U - F$. Put $G = N_{\tau_{1,2}}$-$int(H)$ and $E = U - N_{\tau_{1,2}}$-$cl(H)$. Then G and E are disjoint $N_{\tau_{1,2}}$-open sets of U such that $P \subseteq G$ and $F \subseteq E$. Therefore, U is quasi nano $(1,2)^{\star}$-normal.

IV. NANO WEAKLY $(1,2)^{\star}$-πg-CLOSED SETS AND NANO $(1,2)^{\star}$-REGULAR WEAKLY GENERALIZED CLOSED SETS

Definition 4.1 A subset F of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is called a nano $(1,2)^{\star}$-weakly π-generalized closed set(in short $N_{(1,2)^{\star}}$-$w\pi g$-closed) if$F \subseteq K, K \in N_{(1,2)^{\star}}$-open $\Rightarrow N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) \subseteq K$.

The complement of $N_{(1,2)^{\star}}$-$w\pi g$-open if $F^c = U - F$ is $N_{(1,2)^{\star}}$-$w\pi g$-closed.

Lemma 4.2 In a nano bitopological space $(U, \tau_{R_{1,2}}(X))$, every $N_{(1,2)^{\star}}$-πg-closed set is $N_{(1,2)^{\star}}$-$w\pi g$-closed.

Remark 4.3 The converse of Lemma 4.2 is need not be true in general as shown in the following example.

Example 4.4 Let $U = \{y_1, y_2, y_3, y_4\}$ with $U/R_1 = \{\{y_1\}, \{y_2, y_3, y_4\}\}$ and $X = \{y_1\}$ then $\tau_{R_1}(X) = \{\phi, \{y_1\}, U\}$ and let $U/R_2 = \{\{y_1, y_4\}, \{y_2, y_3\}\}$ and $X = \{y_2, y_3\}$ then $\tau_{R_2}(X) = \{\phi, \{y_2, y_3\}, U\}$. Then the sets in $\{\phi, \{y_1\}, \{y_2, y_3\}, \{y_1, y_2, y_3\}, U\}$ are called $N_{\tau_{1,2}}$-open and the sets in $\{\phi, \{y_4\}, \{y_1, y_4\}, \{y_2, y_3, y_4\}, U\}$ are called $N_{\tau_{1,2}}$-closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then the subset $\{y_3\}$ is $N_{(1,2)^{\star}}$-$w\pi g$-closed but not $N_{(1,2)^{\star}}$-πg-closed.

Definition 4.5 A subset F of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is called a nano $(1,2)^{\star}$-regular weakly generalized closed set(in short $N_{(1,2)^{\star}}$-rwg-closed) if $F \subseteq K, K \in N_{(1,2)^{\star}}$-regular open $\Rightarrow N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) \subseteq K$.

The complement of $N_{(1,2)^{\star}}$-rwg-open if $F^c = U - F$ is $N_{(1,2)^{\star}}$-rwg-closed.

Proposition 4.6 In a nano bitopological space $(U, \tau_{R_{1,2}}(X))$, every $N_{(1,2)^{\star}}$-$w\pi g$-closed set is $N_{(1,2)^{\star}}$-rwg-closed.

Proof. Let F be any $N_{(1,2)^{\star}}$-$w\pi g$-closed set and I be $N_{(1,2)^{\star}}$-regular open set containing F. Then I is $N_{(1,2)^{\star}}$-π-open $\subseteq F$. Comprise $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) \subseteq I$. Hence F is $N_{(1,2)^{\star}}$-rwg-closed.

Remark 4.7 The converse of Proposition 4.6 is need not be true in general as shown in the following example.

Example 4.8 Let $U = \{y_1, y_2, y_3, y_4\}$ with $U/R_1 = \{\{y_4\}, \{y_1, y_2, y_3\}\}$ and $X = \{y_4\}$ then $\tau_{R_1}(X) = \{\phi, \{y_4\}, U\}$ and let $U/R_2 = \{\{y_1, y_2\}, \{y_3, y_4\}\}$ and $X = \{y_3, y_4\}$ then $\tau_{R_2}(X) = \{\phi, \{y_3, y_4\}, U\}$. Then the sets in $\{\phi, \{y_4\}, \{y_1, y_3\}, \{y_1, y_3, y_4\}, U\}$are called $N_{\tau_{1,2}}$-open and the sets in $\{\phi, \{y_2\}, \{y_2, y_4\}, \{y_1, y_2, y_3\}, U\}$ are called $N_{\tau_{1,2}}$-closed. In the nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then the subset $\{y_1, y_3, y_4\}$ is a $N_{(1,2)^{\star}}$-rwg-closed set but not $N_{(1,2)^{\star}}$-$w\pi g$-closed.

Definition 4.9 A subset F of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is called a nano $(1,2)^{\star}$-weakly generalized closed set(in short $N_{(1,2)^{\star}}$-wg-closed) if $F \subseteq K, K \in N_{\tau_{1,2}}$-open $\Rightarrow N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) \subseteq K$.

The complement of $N_{(1,2)^{\star}}$-wg-open if $F^c = U - F$ is $N_{(1,2)^{\star}}$-wg-closed.

Theorem 4.10 In a nano bitopological space $(U, \tau_{R_{1,2}}(X))$, every $N_{(1,2)^{\star}}$-wg-closed set is $N_{(1,2)^{\star}}$-$w\pi g$-closed.

Proof. Let F be any $N_{(1,2)^{\star}}$-wg-closed set and I be a $N_{(1,2)^{\star}}$-π-open set containing F. Then I is a $N_{\tau_{1,2}}$-open set containing F. We have $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) \subseteq I$. Hence F is $N_{(1,2)^{\star}}$-$w\pi g$-closed.

Theorem 4.11 For a nano bitopological space $(U, \tau_{R_{1,2}}(X))$, if a subset F is both $N_{\tau_{1,2}}$-closed and $N_{(1,2)^{\star}}$-αg-closed then $N_{(1,2)^{\star}}$-$w\pi g$-closed.

Proof. Let F be $N_{(1,2)^{\star}}$-αg-closed and I be $N_{(1,2)^{\star}}$-π-open $\subseteq F$ in $(U, \tau_{R_{1,2}}(X))$. Then I is $N_{\tau_{1,2}}$-open containing F and so $I \supseteq N_{\tau_{1,2}}$-$\alpha cl(F) = F \cup N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(N_{\tau_{1,2}}$-$cl(F)))$. Since F is $N_{\tau_{1,2}}$-closed, $I \supseteq N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F))$ and hence F is $N_{(1,2)^{\star}}$-$w\pi g$-closed in $(U, \tau_{R_{1,2}}(X))$.

Theorem 4.12 If a subset F of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is both $N_{(1,2)^{\star}}$-π-open and $N_{(1,2)^{\star}}$-$w\pi g$-closed, then it is $N_{\tau_{1,2}}$-closed.

Proof. Since F is both $N_{(1,2)^{\star}}$-π-open and $N_{(1,2)^{\star}}$-$w\pi g$-closed, $F \supseteq N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) = N_{\tau_{1,2}}$-$cl(F)$ and hence F is $N_{\tau_{1,2}}$-closed in $(U, \tau_{R_{1,2}}(X))$.

Corollary 4.13 If a subset F of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is both $N_{(1,2)^{\star}}$-π-open and $N_{(1,2)^{\star}}$-$w\pi g$-closed, then it is both $N_{(1,2)^{\star}}$-regular open and $N_{(1,2)^{\star}}$-regular closed in $(U, \tau_{R_{1,2}}(X))$.

Theorem 4.14 A subset F is $N_{(1,2)^{\star}}$-$w\pi g$-closed if and only if $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) - F$ contains no non-empty $N_{(1,2)^{\star}}$-π-closed set.

Proof. $\Rightarrow$ G is $N_{(1,2)^{\star}}$-π-closed such that $G \subseteq N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) - F$. Since G^c is $N_{(1,2)^{\star}}$-π- open and $F \subseteq G^c$, from the definition of $N_{(1,2)^{\star}}$-$w\pi g$-closed set it follows that $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) \subseteq G^c$. ie. $G \subseteq (N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)))^c$. This implies that $G \subseteq (N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F))) \cap (N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)))^c = \phi$.

Sufficiency. Let $F \subseteq I$, where I is $N_{(1,2)^{\star}}$-π-open set in U. If $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F))$ is not contained in I, then $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) \cap I^c$ is a non-empty $N_{(1,2)^{\star}}$-π-closed subset of $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) - F$, we obtain a contradiction. This proves the sufficiency and hence the theorem.

Corollary 4.15 A $N_{(1,2)^{\star}}$-$w\pi g$-closed set F is $N_{(1,2)^{\star}}$-regular closed if and only if $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) - F$ is $N_{(1,2)^{\star}}$-π-closed and $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) \supseteq F$.

Proof. Necessity. Since the set F is $N_{(1,2)^{\star}}$-regular closed, $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) - F = \phi$ is $N_{(1,2)^{\star}}$-regular closed and hence $N_{(1,2)^{\star}}$-π-closed.

Sufficiency. By Theorem 4.14, $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) - F$ contains no non-empty $N_{(1,2)^{\star}}$-π-closed set. That is $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) - F = \phi$. Therefore F is $N_{(1,2)^{\star}}$-regular closed.

Theorem 4.16 Let $(U, \tau_{R_{1,2}}(X))$ be a nano bitopological space and $I \subseteq F \subseteq U$. If I is $N_{(1,2)^{\star}}$-$w\pi g$-closed set relative to F and F is both $N_{\tau_{1,2}}$-open and $N_{(1,2)^{\star}}$-$w\pi g$-closed subset of U then I is $N_{(1,2)^{\star}}$-$w\pi g$-closed set relative to U.

Proof. Let $I \subseteq G$ and G be a $N_{(1,2)^{\star}}$-π-open in $(U, \tau_{R_{1,2}}(X))$. Then $I \subseteq F \cap G$. Since I is $N_{(1,2)^{\star}}$-$w\pi g$-closed relative to F, $N_{\tau_{1,2}}$-$cl_F(N_{\tau_{1,2}}$-$int_F(I)) \subseteq F \cap G$. That is $F \cap N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(I)) \subseteq F \cap G$. We have $F \cap N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(I)) \subseteq G$ and then $(F \cap N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(I))) \cup (N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(I)))^c \subseteq G \cup (N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(I)))^c$. Since F is $N_{(1,2)^{\star}}$-$w\pi g$-closed in $(U, \tau_{R_{1,2}}(X))$, we have $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) \subseteq G \cup (N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(I)))^c$. Therefore $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(I)) \subseteq G$ since $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(I))$ is not contained in $(N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(I)))^c$. Thus I is $N_{(1,2)^{\star}}$-$w\pi g$-closed set relative to $(U, \tau_{R_{1,2}}(X))$.

Corollary 4.17 If F is both $N_{\tau_{1,2}}$-open and $N_{(1,2)^{\star}}$-$w\pi g$-closed and G is $N_{\tau_{1,2}}$-closed in a nano bitopological space $(U, \tau_{R_{1,2}}(X))$, then $F \cap G$ is $N_{(1,2)^{\star}}$-$w\pi g$-closed in $(U, \tau_{R_{1,2}}(X))$.

Proof. Since G is $N_{\tau_{1,2}}$-closed, we have $F \cap G$ is $N_{\tau_{1,2}}$-closed in F. Therefore $N_{\tau_{1,2}}$-$cl_F(F \cap G) = F \cap G$ in F. Let $F \cap G \subseteq I$, where I is $N_{(1,2)^{\star}}$-π-open in F. Then $N_{\tau_{1,2}}$-$cl_F(N_{\tau_{1,2}}$-$int_F(F \cap G)) \subseteq I$ and hence $F \cap G$ is $N_{(1,2)^{\star}}$-$w\pi g$-closed in F. By Theorem 4.16, $F \cap G$ is $N_{(1,2)^{\star}}$-$w\pi g$-closed in $(U, \tau_{R_{1,2}}(X))$.

Theorem 4.18 If F is $N_{(1,2)^{\star}}$-$w\pi g$-closed and $F \subseteq G \subseteq N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F))$, then G is $N_{(1,2)^{\star}}$-$w\pi g$-closed.

Proof. Since $F \subseteq G$, $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(G)) - G \subseteq N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) - F$. By Theorem 4.14 $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) - F$ contains no non-empty $N_{(1,2)^{\star}}$-π-closed set and so $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(G)) - G$ by Theorem 4.14, G is $N_{(1,2)^{\star}}$-$w\pi g$-closed.

Theorem 4.19 Let $(U, \tau_{R_{1,2}}(X))$ of a nano bitopological space and $F \subseteq G \subseteq U$ and G be $N_{\tau_{1,2}}$-open. If F is $N_{(1,2)^{\star}}$-$w\pi g$-closed in U, then F is $N_{(1,2)^{\star}}$-$w\pi g$-closed relative to G.

Proof. Let $F \subseteq G \cap I$ where I is $N_{(1,2)^{\star}}$-π-open in $(U, \tau_{R_{1,2}}(X))$. Since F is $N_{(1,2)^{\star}}$-$w\pi g$-closed in $(U, \tau_{R_{1,2}}(X))$, $F \subseteq I$ implies $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) \subseteq I$. That is $G \cap (N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F))) \subseteq G \cap I$ where $G \cap N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(G))$ is nano $(1,2)^{\star}$-closure of nano $(1,2)^{\star}$-interior of F in $(U, \tau_{R_{1,2}}(X))$. Thus F is $N_{(1,2)^{\star}}$-$w\pi g$-closed relative to $(U, \tau_{R_{1,2}}(X))$.

Theorem 4.20 If a subset F of a nano bitopological space $(U, \tau_{R_{1,2}}(X))$ is nano $(1,2)^{\star}$-nowhere dense, then it is $N_{(1,2)^{\star}}$-$w\pi g$-closed.

Proof. Since $N_{\tau_{1,2}}$-$int(F) \subseteq N_{\tau_{1,2}}$-$int(N_{\tau_{1,2}}$-$cl(F))$ and F is nano $(1,2)^{\star}$-nowhere dense, $N_{\tau_{1,2}}$-$int(F) = \phi$. Therefore $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F)) = \phi$ and hence F is $N_{(1,2)^{\star}}$-$w\pi g$-closed in $(U, \tau_{R_{1,2}}(X))$.

Theorem 4.21 A subset F of a nano bitopological space U is $N_{(1,2)^{\star}}$-$w\pi g$-open if $I \subseteq N_{\tau_{1,2}}$-$int(N_{\tau_{1,2}}$-$cl(F))$ whenever $I \subseteq F$ and I is $N_{(1,2)^{\star}}$-π-closed.

Proof. Let F be any $N_{(1,2)^{\star}}$-$w\pi g$-open. Then F^c is $N_{(1,2)^{\star}}$-$w\pi g$-closed. Let I be a $N_{(1,2)^{\star}}$-π-closed set contained in F. Then I^c is a $N_{(1,2)^{\star}}$-π-open set in U containing F^c. Since F^c is $N_{(1,2)^{\star}}$-$w\pi g$-closed, we have $N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F^c)) \subseteq I^c$. Therefore $I \subseteq N_{\tau_{1,2}}$-$int(N_{\tau_{1,2}}$-$cl(F))$.

Conversely, we suppose that $I \subseteq N_{\tau_{1,2}}$-$int(N_{\tau_{1,2}}$-$cl(F))$ whenever $I \subseteq F$ and I is $N_{(1,2)^{\star}}$-π-closed. Then I^c is a $N_{(1,2)^{\star}}$-π-open set containing F^c and $I^c \supseteq (N_{\tau_{1,2}}$-$int(N_{\tau_{1,2}}$-$cl(F)))^c$. It follows that $I^c \supseteq N_{\tau_{1,2}}$-$cl(N_{\tau_{1,2}}$-$int(F^c))$. Hence F^c is $N_{(1,2)^{\star}}$-$w\pi g$-closed and so F is $N_{(1,2)^{\star}}$-$w\pi g$-open.

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