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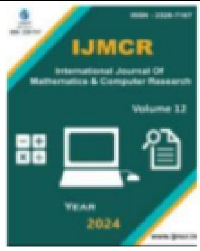

A NOTE ON PRABHAKAR DERIVATIVE

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Abstract. Prabhakar's derivative is an important milestone in the history of fractional calculus. By introducing the generalized Mittag-Leffler function and showing how it can solve singular integral equations, Prabhakar provided a powerful mathematical tool. This contribution remains highly valuable and continues to influence modern science and engineering, ensuring its importance for many years ahead. This survey explores the mathematical foundation, properties, and applications of the Prabhakar fractional derivative in diverse scientific and engineering fields. The paper provides an overview of its recent historical development, mathematical formulation, key properties, and potential research directions.

Key words and Phrases: Hilfer-Prabhakar derivatives, Prabhakar integrals, Mittag-Leffler functions, Fractional derivative
MSC Classification 2020: 26A33, 33E12, 34A08

1. INTRODUCTION

In 1971, T.R. Prabhakar [1] extended the Mittag-Leffler function by introducing a three-parameter generalization, now known as the Prabhakar function. This advancement enabled the additional flexibility and adaptability required for modeling complex systems with memory and hereditary effects. This derivative quickly gained prominence for its ability to bridge gaps in existing fractional calculus models, finding applications in viscoelasticity, anomalous diffusion, relaxation phenomena, and other fields. The Prabhakar fractional derivative, named after T.R. Prabhakar, generalizes the Riemann-Liouville and Caputo derivatives. It finds applications in viscoelasticity, anomalous diffusion, and relaxation phenomena. In 2014 Garra, et al. [2], generalized Hilfer-Prabhakar derivatives were proposed, demonstrating utility in equations governing anomalous diffusion and stochastic processes. Building on this, the Sumudu and Laplace transforms of Prabhakar derivatives were developed by Panchal, et al. [3] , leading to solutions for problems in physics involving heat transfer and fractional kinetics. Studies in 2016 by 2016 Eshaghi and Ansart [5] also established Green's functions for boundary value problems and explored stability in fractional systems using Prabhakar derivatives. Key contributions by Garrappa and Maione [9] included numerical methods for solving differential equations, extensions to q-calculus, and applications in viscoelasticity and dielectrics, exemplified by the Havriliak-Negami model. By 2020 [17][18], the focus shifted towards generalized fractional operators and their applications in renewal processes, nonlocal systems, and heat equations. Recent works in 2022 by Pachpatte [22] and 2023 by Mohd Khalid and Subhash Alha [26] have introduced discrete-time counterparts and analyzed their role in complex systems.

2. PRELIMINARIES

Definition 2.1. *Let $f \in L^1_{loc}(a,b)$, where $-\infty \leq a < t < b \leq \infty$, be a locally integrable real-valued function. Define the power-law kernel as:* $K_\alpha(t) = \frac{t^{\alpha-1}}{\Gamma(\alpha)}, \quad \alpha > 0,$ *where $\Gamma(\alpha)$ is the Gamma function. The Riemann–Liouville integral of order α is defined as:* $(I^\alpha_{a+}f)(t) = \frac{1}{\Gamma(\alpha)}\int_a^t \frac{f(u)}{(t-u)^{1-\alpha}}du = (f * K_\alpha)(t),\ \alpha > 0.$

Definition 2.2. *Let $f \in L^1(a,b)$, where $-\infty \leq a < t < b \leq \infty$, and $f * K_{m-\alpha} \in W^{m,1}(a,b)$, with $m = \lceil \alpha \rceil$, $\alpha > 0$, where, $W^{m,1}(a,b)$ is the Sobolev space defined as: $W^{m,1}(a,b) = \left\{ f \in L^1(a,b) : \frac{d^m}{dt^m} f \in L^1(a,b) \right\}$. The Riemann-Liouville derivative of order α is defined as: $(D^\alpha_{a+}f)(t) = \frac{d^m}{dt^m}\left(I^{m-\alpha}_{a+}f\right)(t)$, where $I^{m-\alpha}_{a+}f$ is the Riemann–Liouville integral of order $m-\alpha$. This can be written explicitly as:* $(D^\alpha_{a+}f)(t) = \frac{1}{\Gamma(m-\alpha)}\frac{d^m}{dt^m}\int_a^t (t-s)^{m-1-\alpha} f(s)\,ds.$
We denote by $AC^n(a,b)$, $n \in \mathbb{N}$, the space of real-valued functions $f(t)$ with continuous derivatives up to order $n-1$ on (a,b), such that $f^{(n-1)}(t)$ belongs to the space of absolutely continuous functions $AC(a,b)$. that is
$AC^n(a,b) = \left\{ f : (a,b) \to \mathbb{R} : \frac{d^{n-1}}{dx^{n-1}} f(x) \in AC(a,b) \right\}.$

Definition 2.3. *Let the parameter $\alpha > 0$, $m = \lceil \alpha \rceil$, and $f \in AC^m(a,b)$, where $AC^m(a,b)$ is the space of functions with absolutely continuous derivatives up to order $m-1$. The Caputo derivative (also known as the regularized Riemann–Liouville derivative) of order $\alpha > 0$ is defined as: $({}^C D^\alpha_{a+}f)(t) = \left(I^{m-\alpha}_{a^+}\frac{d^m}{dt^m}f\right)(t)$, where $I^{m-\alpha}_{a+}$ is the Riemann–Liouville integral of order $m-\alpha$. Explicitly, this can be written as:* $({}^C D^\alpha_{a^+}f)(t) = \frac{1}{\Gamma(m-\alpha)}\int_a^t (t-s)^{m-1-\alpha}\frac{d^m}{ds^m}f(s)\,ds.$

Definition 2.4. *The Gamma function, denoted as $\Gamma(z)$, is defined as:*
$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}\,dt, \quad \Re(z) > 0.$

Definition 2.5. *The one-parameter Mittag-Leffler function is defined as:*
$E_\alpha(z) = \sum_{n=0}^\infty \frac{z^n}{\Gamma(\alpha n+1)}.$

Definition 2.6. *The two-parameter Mittag-Leffler function is defined as:*
$E_{\alpha,\beta}(z) = \sum_{n=0}^\infty \frac{z^n}{\Gamma(\alpha n+\beta)}, \quad \Re(\alpha) > 0.$

Definition 2.7. *The three-parameter Mittag-Leffler function, also known as the Prabhakar function, is defined as:*
$E^\gamma_{\alpha,\beta}(z) = \sum_{n=0}^\infty \frac{(\gamma)_n}{n!\,\Gamma(\alpha n+\beta)}z^n, \quad \Re(\alpha) > 0,$ *where $(\gamma)_n$ is the Pochhammer symbol, given by: $(\gamma)_0 = 1$, $(\gamma)_n = \gamma(\gamma+1)(\gamma+2)\cdots(\gamma+n-1)$, for $n = 1, 2, \ldots$*

Definition 2.8. *Let* $f \in L[0,b],\ 0 < t < b \leq \infty$. *The Prabhakar integral can be written as:* $E^{\gamma}_{\rho,\mu,\omega,0^+} f(t) = \int_0^t (t-y)^{\mu-1} E^{\gamma}_{\rho,\mu}\big(\omega(t-y)^{\rho}\big) f(y)\, dy = \big(f * e^{\gamma}_{\rho,\mu,\omega}\big)(t)$, *where* $\rho, \mu, \omega, \gamma \in \mathbb{C}$, *with* $\Re(\rho), \Re(\mu) > 0$.

3. LITERATURE SURVEY

In 1971 Prabhakar [1] extended the Mittag-Leffler function to a three-parameter form, now widely known as the Prabhakar function. This generalization provided a versatile kernel for modeling processes exhibiting complex memory and hereditary characteristics. Prabhakar's work centered on solving a singular integral equation with the Prabhakar function in the kernel. This equation is expressed as: $\int_a^x (x-t)^{\beta-1} E^{\rho}_{\alpha,\beta}\left(\lambda(x-t)^{\alpha}\right) f(t)\, dt = g(x)$, where $Re(\beta) > 0$, $E^{\rho}_{\alpha,\beta}(z)$ is the three-parameter Mittag-Leffler function: $E^{\rho}_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\rho)_n z^n}{\Gamma(\alpha n+\beta) n!}, \quad Re(\alpha) > 0$. Prabhakar defined a linear operator $\mathfrak{E}(\alpha,\ \beta,\ \rho,\ \lambda)$ on a space L of functions by the integral in $\int_a^x (x-t)^{\beta-1} E^{\rho}_{\alpha,\beta}\left(\lambda(x-t)^{\alpha}\right) f(t)\, dt = g(x)$, where $Re(\beta) > 0$ and employ an operator of fractional integration $I^{\mu} : L \to L$ to prove results on $\mathfrak{E}(\alpha,\ \beta,\ \rho,\ \lambda)$; these results are used to find out the solutions of $\int_a^x (x-t)^{\beta-1} E^{\rho}_{\alpha,\beta}\left(\lambda(x-t)^{\alpha}\right) f(t)\, dt = g(x)$, where $Re(\beta) > 0$. The technique used can be applied to obtain analogous results on the integral equation $\mathfrak{E}^*(\alpha,\ \beta,\ \rho,\ \lambda) f(x) \equiv \int_x^b (t-x)^{\beta-1} E^{\rho}_{\alpha,\beta}\left(\lambda(t-x)^{\alpha}\right) f(t)\, dt = g(x)$, where $Re(\beta) > 0$.

In 2014 Garra, et al. [2] presented a generalization of Hilfer derivatives in which Riemann-Liouville integrals are replaced by more general Prabhakar integrals. They analyzed and discuss its properties. Furthermore, they also showed some applications of these generalized Hilfer-Prabhakar derivatives in classical equations of mathematical physics such as the heat and the free electron laser equations, and in difference-differential equations governing the dynamics of generalized renewal stochastic processes.

In 2016 Panchal, et al. [3] obtained the Sumudu transforms of Hilfer-Prabhakar fractional derivative and regularized version of Hilfer-Prabhakar fractional derivative. These results are used to obtain relation between them involving Mittag-Leffler function. Also these results are applied to solve some problems in physics. They obtained the solutions of problems involving Hilfer-Prabhakar fractional derivative and regularized version of Hilfer-Prabhakar fractional derivative by using Fourier and Sumudu transform techniques. In 2016 Panchal, et al. [4] defined the regularized version of k-Prabhakar fractional derivative, k-Hilfer-Prabhakar fractional derivative, regularized version of k-Hilfer-Prabhakar fractional derivative and they also find their Laplace and Sumudu transforms. Using these results, the relation between k-Prabhakar fractional derivative and its regularized version involving k-Mittag-Leffler function is obtained. Similarly the relation between k-Hilfer-Prabhakar fractional derivative and its regularized version is also obtained. Further, they find the solutions of some problems in physics in which k-Hilfer-Prabhakar fractional derivative and its regularized version are involved.

In 2016 Shiva Eshaghi and Alireza Ansart [5] studied a fractional boundary value problem including the Prabhakar fractional derivative. They obtained associated Green function for this fractional boundary value problem and got a Lyapunov-type inequality for it. In 2016 Derakhshan, et al. [6] presented the stability regions for asymptotic stability of fractional differential systems. They alos gave a brief comparison with the stability aspects of fractional differential systems in the sense of Riemann-Liouville fractional derivatives. In 2016 [7] Polito and Tomovski studied some properties of the Prabhakar integrals and derivatives and of some of their extensions such as the regularized Prabhakar derivative or the Hilfer-Prabhakar derivative. They derived some Opial- and Hardy-type inequalities, and also some relationships with probability theory.

In 2016 Mainardi and Garrappa [8] they have established some properties of the three parameters Mittag-Leffler function. In particular, they have established the conditions on the parameters α, β and γ for which the function turns out locally integrable and completely monotonic. These conditions are essential in order to suitably model relaxation phenomena of non-Debye type, such as anomalous polarization processes in dielectrics. In particular the classical Havriliak-Negami model is extended to a wider range of the parameters. Moreover, some numerical methods have been discussed and compared with the aim of identifying suitable techniques for the accurate and efficient numerical computation of the three parameters Mittag-Leffler function. An approach based on the inversion of the LT, which appears as the most reliable, has been used for validating the theoretical findings and providing some graphical representations of the function.

In 2017 Roberto Garrappa and Guido Maione [9] discussed the problem of numerically solving differential equations with Prabhakar derivatives, problems of this kind arise in the simulation of anomalous relaxation properties in Havriliak-Negami models. Fractional integrals and derivatives based on the Prabhakar function are useful to describe anomalous dielectric properties of materials whose behaviour obeys to the Havriliak-Negami model. They have described and investigated some formulas for defining these operators. They have devised a product-integration rule with weights expressed in terms of the Prabhakar function and studied the convergence properties. By means of some numerical experiments the effectiveness of the proposed approach has been illustrated.

In 2017 Pachpatte, et al. [10] developed Lyapunov type inequality for hybrid fractional boundary value problem involving the prabhakar fractional derivative. They have obtained Lyapunov type inequalities in two different cases:

(1) $h_i(t, y(t)) = 0,\ i = 1, 2, ..., n$ and
(2) $h_i(t, y(t)) \neq 0,\ i = 1, 2, ..., n$.

In 2017 Bulavatsky [11] constructed a generalized mathematical model to describe the fractional differential dynamics of filtration processes in fractured porous media, based on the use of the concept of Hilfer-Prabhakar fractional derivative. Within the framework of this model, he obtained a number of closed form solutions to boundary-value problems of filtration theory for modeling the dynamics of pressures at launch of wells in case of plane-radial filtration, as well as by activity of galleries under plane-parallel filtration.

In 2018 Garra and Garrappa [12] studied fractional integral and derivative operators of fractional-order based on the Prabhakar function are presented together with their applications to dielectric models of Havriliak-Negami type. They reviewed some of the main properties of the function, the asymptotic expansion for large arguments is investigated in the whole complex plane and, with major emphasis, along the negative semi-axis. Fractional integral and derivative operators of Prabahar type are hence considered and some nonlinear heat conduction equations with memory involving Prabhakar derivatives.

In 2018 A. Giusti [13] provided a series expansion of the Prabhakar integral in terms of Riemann-Liouville integrals of variable order. Then, by using this last result they finally argue that the operator introduced by Caputo and Fabrizio cannot be regarded as fractional. Besides, they also observed that the one suggested by Atangana and Baleanu is indeed fractional, but it is ultimately related to the ordinary Riemann-Liouville and Caputo fractional operators. All these statements are then further supported by a precise analysis of differential equations involving the aforementioned operators. To further strengthen their narrative, they also showed that these new operators do not add any new insight to the linear theory of viscoelasticity when employed in the constitutive equation of the Scott-Blair model.

In 2019 Zhao and Sun [14] studied fractional derivative and the initial value condition. They gave a Caputo type derivative with a Prabhakar-like kernel, and then the fading memory principle and the compatibility principle [15] of this kernel are verified. In other words, this derivative is in the framework of general Caputo fractional derivative [15]. They further proved that this derivative satisfies the initial value condition. They also studied physical model. The corresponding anomalous relaxation model with this derivative is discussed, and its solution is expressed by Prabhakar functions. It shows their model contains the Debye model and the fractional relaxation model as particular cases, and it is a direct extension of the Cole-Cole model.

In 2020 Giusti, et al. [16] they highlighted discovery and modern development of Prabhakar function, they discuss how the latter allows one to introduce an enhanced scheme for fractional calculus. They summarized the progress in the application of this new general framework to physics and renewal processes. They also provided a collection of results on the numerical evaluation of the Prabhakar function. In 2020 Giusti [17] discussed the very nature of the Prabhakar derivative by framing it in Kochubeis general fractional calculus. He discussed to what extent Prabhakar calculus can be framed within this general scheme. He also investigated the behavior of the Prabhakar kernel $e^{-\gamma}_{\alpha,m-\beta}(\lambda;t) = t^{m-\beta-1}E^{-\gamma}_{\alpha,m-\beta}(\lambda t^{\alpha})$, that if $\Re(\beta) > 0$ and $\beta \notin \mathbb{N}$ then $e^{-\gamma}_{\alpha,m-\beta}(\lambda;t) = t^{m-\beta-1}E^{-\gamma}_{\alpha,m-\beta}(\lambda t^{\alpha})$ is always weakly singular. On the other hand, if we take the integer limit for β, the Prabhakar kernel $e^{-\gamma}_{\alpha,m-\beta}(\lambda;t) = t^{m-\beta-1}E^{-\gamma}_{\alpha,m-\beta}(\lambda t^{\alpha})$ leaves $L^1_{loc}(\mathbb{R}^+)$, entering the realm of distributions. This implies that, in this limit, the Prabhakar derivative can be split into an ordinary derivative and a series of linear Volterra-like integro-differential operators, with either regular ($\Re(\alpha) \geq 1$) or singular ($0 < \Re(\alpha) < 1$) kernels.

In 2020 Samraiz, Perveen, et al. [18] introduced the (k, s)-Hilfer-Prabhakar fractional derivative and discussed its properties. They generalized Laplace transform of this newly proposed operator. As an application, they developed the generalized fractional model of the free-electron laser equation, the generalized time-fractional heat equation, and the generalized fractional kinetic equation using the (k, s)-Hilfer-Prabhakar derivative. In 2020 Garrappa and Kaslik [19] studied the asymptotic stability of systems of differential equations with the Prabhakar derivative, providing an exact characterization of the corresponding stability region. Asymptotic expansions (for small and large arguments) of the solution of linear differential equations of Prabhakar type and a numerical method for nonlinear systems are derived. Numerical experiments are hence presented to validate theoretical findings.

In 2021 Michelitsch, et al. [20] analyzed discrete-time counterparts: Renewal processes with integer IID interarrival times which converge in well-scaled continuous-time limits to the Prabhakar-generalized fractional Poisson process. These processes exhibit non-Markovian features and long-time memory effects. They recovered for special choices of parameters the discrete-time versions of classical cases, such as the fractional Bernoulli process and the standard Bernoulli process as discretetime approximations of the fractional Poisson and the standard Poisson process, respectively. They derived difference equations of generalized fractional type that govern these discrete timeprocesses where in well-scaled continuous-time limits known evolution equations of generalized fractional Prabhakar type are recovered. They also developed in Montroll-Weiss fashion the Prabhakar Discrete-time random walk (DTRW) as a random walk on a graph time-changed with a discrete-time version of Prabhakar renewal process. They also derived the generalized fractional discrete-time Kolmogorov-Feller difference equations governing the resulting stochastic motion. Prabhakar-discrete-time processes open a promising field capturing several aspects in the dynamics of complex systems.

In 2021 Eshaghi, et al. [21] presented several criteria for the generalized Mittag- Leffler stability and the asymptotic stability of this system by using the Lyapunov direct method. Further, they provided two test cases to illustrate the effectiveness of results. Also applied the numerical method to solve the generalized fractional system with the regularized Prabhakar fractional systems and reveal asymptotic stability behavior of the presented systems by employing numerical simulation.

In 2022 Pachpatte, et al. [22] considered a nonlocal fractional boundary value problem with Prabhakar derivative and obtained a Hartman-Wintner type inequality for it. They gave result for following problem assume that $2 < \mu \leq 3$ and $x \in C[a, b]$. If the nonlocal fractional boundary value problem has a unique nontrivial solution, then it satisfies: $x(t) =$
$\int_a^b G(t,s)x(s)\,ds + \frac{\beta(t-a)^{\mu-1}E^{\gamma}_{\rho,\mu}(\omega(t-a)^{\rho})}{(b-a)^{\mu-2}E^{\gamma}_{\rho,\mu}(\omega(b-a)^{\rho}) - \beta(\xi-a)^{\mu-1}E^{\gamma}_{\rho,\mu}(\omega(\xi-a)^{\rho})} \int_a^b G(\xi,s)x(s)\,ds,$

where the Green's function is defined as: $G(t,s) =$

$$\begin{cases} \frac{(t-a)^{\mu-1}E^{\gamma}_{\rho,\mu}(\omega(t-a)^{\rho})(b-s)^{\mu-2}E^{\gamma}_{\rho,\mu-1}(\omega(b-s)^{\rho})}{(b-a)^{\mu-2}E^{\gamma}_{\rho,\mu-1}(\omega(b-a)^{\rho})} - (t-s)^{\mu-1}E^{\gamma}_{\rho,\mu}(\omega(t-s)^{\rho}), \\ \qquad a \le s \le t \le b, \\ \frac{(t-a)^{\mu-1}E^{\gamma}_{\rho,\mu}(\omega(t-a)^{\rho})(b-s)^{\mu-2}E^{\gamma}_{\rho,\mu-1}(\omega(b-s)^{\rho})}{(b-a)^{\mu-2}E^{\gamma}_{\rho,\mu-1}(\omega(b-a)^{\rho})}, \\ \qquad a \le t \le s \le b. \end{cases}$$

In 2022 Fernandez, et al. [23] solved the linear differential equations with variable coefficients and Prabhakar-type operators featuring Mittag-Leffler kernels. In each case, the unique solution is constructed explicitly as a convergent infinite series involving compositions of Prabhakar fractional integrals. They also extended these results to Prabhakar operators with respect to functions. As an important illustrative example, they considered the case of constant coefficients, and gave the solutions in a more closed form by using multivariate Mittag-Leffler functions. In 2022 Shatmardan, et al. [24] introduced the Prabhakar fractional q-integral and q-differential operators. They studied the semi-group property of the Prabhakar fractional q-integral operator, after which introduced the corresponding q-differential operator. They also presented formulas for compositions of q-integral and q-differential operators. They showed the boundedness of the Prabhakar fractional q-integral operator in the class of q-integrable functions.

In 2023 Gorska et al. [25] studied the exact solution of two kinds of generalized Fokker-Planck equations in which the integral kernels are given either by the distributed order function $k_1(t) = \int_0^1 t^{-\mu}/\Gamma(1-\mu)d\mu$ or the distributed order Prabhakar function $k_2(\alpha,\gamma;\lambda;t) = \int_0^1 e^{-\gamma}_{\alpha,1-\mu}(\lambda;t)d\mu$, where the Prabhakar function is denoted as $e^{-\gamma}_{\alpha,1-\mu}(\lambda;t)$. Both of these integral kernels can be called the fading memory functions and are the Stieltjes functions. It is also shown that their Stieltjes character is enough to ensure the non-negativity of the mean square values and higher even moments. The odd moments vanished. Thus, the solution of generalized Fokker-Planck equations can be called the probability density functions. They introduced also the Volterra-Prabhakar function and its generalization which are involved in the definition of $k_2(\alpha,\gamma;\lambda;t)$ and generated by it the probability density function $p_2(x,t)$.

In 2023 Khalid and Alha [26] described the fractional derivatives of Ψ-Prabhakar, Ψ-Hilfer-Prabhakar, and its regularized form in terms of Ψ-Mittage-Leffler type functions. These are then used to solve a number of Cauchy type equations involving Ψ-Hilfer-Prabhakar fractional derivatives and their regularized form, including the generalized fractional free electron laser equation.

In 2024 Sachin, at al. [27] investigated new fractional derivatives in the sense of Ψ-fractional calculus to find their generalized transforms called Ψ-Laplace and Ψ-Sumudu transforms. These derivatives are more generalization of fractional derivatives and effectively applicable for various applications like cauchy problems, heat transfer problem. In order to explain the obtained results, some examples were illustrated. It is noted that since generalized derivatives are global and contain a wide class of fractional derivatives.

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