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Degree Equitable Complete Cototal domination number of graph

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ABSTRACT

Let $G = (V, E)$ be any connected graph A dominating set D is said to be a complete cototal dominating set if the induced subgraph $< V - D >$ has no isolated vertices.. A complete cototal dominating set D is said to be degree equitable complete cototal dominating set if for every vertex $v \in < D >$ there exist a vertex $u \in V - D$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$ provided $< D >$ and $< V - D >$ contains no isolated vertices. The minimum cardinality of degree equitable complete cototal dominating set is called degree equitable complete cototal domination number of a graph and it is denoted by $\gamma^e_{ccl}(G)$. In this paper, we have obtained the $\gamma^e_{ccl}(G)$ of some standard class of graphs and further established some bounds for $\gamma^e_{ccl}(G)$.

Keywords: Domination number, total domination number, complete cototal domination number, degree equitable complete cototal domination number.

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1. INTRODUCTION

All graphs considered here are simple, finite, connected and nontrivial. Let $G = (V(G), E(G))$ be a graph, where *V(G)* is the vertex set and *E(G)* be the edge set of *G*.

A subset $D \subseteq V$ is said to be a *dominating set* of G if every vertex $V - D$ is adjacent to at least one vertex in D. The minimum cardinality of a minimal dominating set is called the *domination number* of G [4].

A subset D of V is called an *equitable dominating set* if for every $v \in V - D$ there exist a vertex $u \in D$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$, where *deg(u) and deg(v)* denotes the degree of a vertex *u* and

v respectively. The minimum cardinality of such a vertex u and v respectively. The minimum cardinality of such a dominating set is denoted by γ^e and is called the *equitable domination number* [10].

Let $G = (V, E)$ be any connected graph. A subset D of V is called a *connected dominating set* of G if every vertex $v \in V - D$ is adjacent to some vertex in D and $< D >$ is connected [9].

A set $D \subseteq V$ of a graph $G = (V, E)$ is called a *total dominating set* if the induced subgraph $< D >$ has no isolated vertices. The total domination number of G is the minimum cardinality of a total dominating set of G [5].

A dominating set D is said to be a *cototal dominating set* if the induced subgraph $< V - D >$ has no isolated vertices. The cototal domination number $\gamma_{cl}(G)$ of G is the minimum cardinality of a cototal dominating set of G [8].

A total dominating set D is said to be a complete *cototal dominating set* if the induced subgraph $< V - D >$ has no isolated vertices. The *complete cototal domination number* $\gamma_{cc}\ (G)$ of G is the minimum cardinality of a complete cototal dominating set of G [3].

Analogously, we introduce new concept degree equitable complete cototal domination as follows.

Definition 1.

A complete cototal dominating set D is said to be a *degree equitable complete cototal dominating set*, if every vertex $v \in D\ there\ exist\ a\ vertex\ u \in V - D$ such that $uv \in E(G)\ \ and\ \ |\deg(u) - \deg(v)| \leq 1$ .The minimum cardinality of degree equitable complete cototal dominating set is called *degree equitable complete cototal domination number* of a graph and it is denoted by $\gamma^e_{ccl}(G)$.

Example:

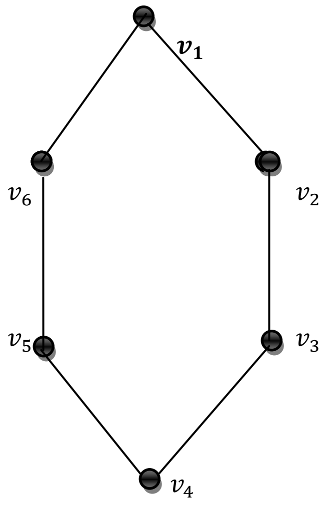

Figure 1.

Dominating set $D = \{v_{1,} v_4\}$

Total dominating set $D = \{v_{2,} v_{3,} v_{6,} v_5\}$

Complete cototal dominating set $D = \{v_{2,} v_{3,} v_{4,} v_5\}$

Degree equitable dominating set $D = \{v_{1,} v_4\}$

Degree equitable complete cototal dominating set $D = \{\{v_{5,} v_{6,} v_{1,} v_2\}$

From the above figure 1, we can observe that, domination number and degree equitable domination number is same and total domination number and complete cototal domination number same. Further complete cototal domination number and degree equitable complete cototal domination number is same.

2. RESULTS

We need the following auxiliary results which will be helpful in proving our results.

Theorem A[4]: For any graph G without isolated vertices, $\gamma(G) \leq \frac{n}{2}$

Theorem B[5]: For any graph G without isolates, $\gamma_t(G) \leq \frac{2n}{3}$

Theorem C[9]: For any tree T, $\gamma_c(T) = n - \xi$, where ξ is the set of all pendant vertices in T.

Theorem D[3]:

i) For any path P_p, $\gamma_{cc}(P_p) = p - 2\lfloor\frac{p-2}{4}\rfloor$

ii) For any cycle C_p, $\gamma_{cc}(C_p) = p - 2\lfloor\frac{p}{4}\rfloor$

iii) For any complete graph $G = K_p$,

$$\gamma_{cc}(K_p) = \begin{cases} p & \text{if } P = 2,3 \\ 2 & \text{otherwise} \end{cases}$$

OBSERVATIONS:

For any graph G without isolated vertices is

a. $\gamma(G) \leq \gamma^e(G)$

b. $\gamma_t(G) = \gamma_{cc}(G)$

c. $\gamma_{cc}(G) \geq \gamma^e_{ccl}(G)$

Firstly, we obtain the degree equitable complete cototal domination number $\gamma^e_{ccl}(G)$ of some standard class of graphs. Which are listed in the following propositions.

Proposition 1:

i) For any Path $G = P_n$, n≥ 6 , $\gamma^e_{ccl}(P_n) = n - 2\lfloor\frac{n-2}{4}\rfloor$

ii) For any cycle graph $G = C_n$, $n \geq 4$, $\gamma^e_{ccl}(C_n) = n - 2\lfloor\frac{n}{4}\rfloor$

iii) For any complete graph $G = K_n$, $n \geq 4$, $\gamma^e_{ccl}(K_n) = 2$

iv) For any complete bipartite graph $G = K_{m.n}$,

$$\gamma^e_{ccl}(K_{m,n}) = \begin{cases} 2 & \text{if } |m-n| \leq 1 \\ m+n & \text{otherwise} \end{cases}$$

Proof:

i) Let G be a path P_n, $n \geq 6$

Let D be any complete cototal dominating set of P_n i.e $< D >$ and $< V - D >$ contains no isolated vertices. Further note that for every vertex $u \in V - D \,\exists\, vertex\ v \in D$ such that $|\deg(u) - \deg(v)| \leq 1$. Hence D is a degree equitable complete cototal dominating set of P_n. Therefore by Theorem D a result follows.

(ii) and (iii)

By the same argument in Proposition (i) and Theorem D results follows.

iv). Let $G = K_{m,n}$ be a complete bipartite graph with partite 6 and partition m & n. Let D be any complete cototal dominating set of G. If $|m - n| \leq 1$ then D will be degree equitable complete cototal dominating set of G and by definition $< D >$ and $< V - D >$ contains no isolated vertices. Hence $|D|$ must be 2.

Therefore $\gamma^{e}_{ccl}(G) = 2$

Suppose $|m - n| > 1$ then entire vertex set of G will act as degree equitable complete cototal dominating set of G.

Hence $\gamma^{e}_{ccl}(G) = |D| = m + n$.

Theorem 1: Every graph without isolated vertices as degree equitable complete cototal dominating set and hence a degree equitable complete cototal domination number.

Proof: Let $G = (V, E)$ be any graph without isolated vertices. For simplification and without loss of generality. Let us assume that G be any connected graph then by default v itself a degree equitable complete cototal dominating set and since G as no isolates each vertex therefore $v \in V(G)$ is adjacent to some vertex u in G and we can see that every vertex $V - D$ is adjacent to at least one vertex in D and $|\deg(u) - \deg(v)| \leq 1$. This will give us a degree equitable complete cototal dominating set of G. If D contains no proper subsets which are a degree equitable complete cototal dominating set then D will be a minimal degree equitable dominating set.

Hence every graph with no isolated vertices as a degree equitable complete cototal dominating set and hence degree equitable complete cototal domination number.

Theorem 2: For any graph G of order at least 4, without isolated vertices, $2 \leq \gamma^{e}_{ccl}(G) \leq n$. The equality of lower bound holds if and only if G contains an edge $e = uv$ such that $\deg(u) = n - 1$ for every vertex $w \in V - D$, either $|\deg(u) - \deg(w)| \leq 1 \; or \; |\deg(v) - \deg(w)| \leq 1$ and equality of upper bound holds good if every pendent vertex is incident with a support vertex of degree at least 3.

Proof: The lower bound follows from Proposition (i) and the upper bound follows from Theorem (1). Now we consider the equality case of lower bound.

Suppose G contains an edge $E = uv$ such that $\deg(u) = n - 1$ and for every vertex $w = V - D$ either $|\deg(u) - \deg(w)| \leq 1 \; or |\deg(v) - \deg(w)| \leq 1$ then $D = \{u, v\}$ will form degree equitable complete cototal dominating set of G. Hence

$$\gamma^{e}_{ccl}(G) = |D| = |\{u, v\}| = 2$$

Conversely,

Suppose $\gamma_{ccl}^{e}(G) = 2$ and D does not satisfy the above condition then there exist a vertex $z \in V - D$ which is not dominated by any other vertex in D. Hence to dominated z we need a vertex in its neighborhood which is degree equitable to z.

Since by definition we have to select at least two vertices in neighborhood of z said $x, y \in N(z)$.

Hence $D = \{u, v, x, y\}$ which implies $\gamma_{ccl}^{e}(G) = |D| = |\{u, v, x, y\}| = 4$ a contradiction.

Now let us consider the equality case of upper bound.

Let D be a degree equitable complete cototal dominating set of G. If every pendant vertex of G is incident with a support vertex of degree at least 3 then by the definition of degree equitable complete cototal dominating set every pendant vertices and its support vertex is belongs to D. Hence $|D| \leq n$. Further by Theorem (1) $n \leq |D|$. Therefore $|D| = n$.

Hence $\gamma_{ccl}^{e}(G) = |D| = n$

Theorem 3: Let G be any graph of order at least 4, then $\gamma_{ccl}^{e}(G) \leq 2m - n + 2$. Further equality holds if and only if G is a tree.

Proof: Let G be any graph of order n and size m and contains no isolated vertices.

Obviously, $\gamma_{ccl}^{e}(G) \leq n \leq 2m - n + 2$.

Since G contains no isolated vertices therefore $m \geq n - 1$ thus $\gamma_{ccl}^{e}(G) \leq 2m - n + 2$ if and only if every edge of a tree incident with a support vertex.

Conversely,

Let $\gamma_{ccl}^{e}(G) = 2m - n + 2$ then $2m - n + 2 \leq n$ which implies $m \leq n - 1$. So G is a tree with $\gamma_{ccl}^{e}(G) = n$ therefore by Theorem (2) the result follows.

Theorem 4: For any graph G, $\frac{2n}{\Delta(G)+1} \leq \gamma_{ccl}^{e}(G)$, further equality holds if $\gamma_{ccl}^{e}(G) = 2$.

Proof: Let G be a any graph of order n and size m containing no isolates. Let D be a any degree equitable complete cototal dominating set of G. Since every vertex dominates at most itself and $\Delta(G)$ of other vertices. Therefore by definition of degree equitable complete cototal dominating set containing pair wise vertices which dominates itself and its neighborhood and every vertex in complement of D must be degree equitable with at least one vertex in D.

Hence , $\frac{2n}{\Delta(G)+1} \leq \gamma_{ccl}^{e}(G)$,

For equality, suppose, $\gamma_{ccl}^{e}(G) = 2$, then by Theorem (2), $\Delta(G) = n-1$

Hence , $\frac{2n}{n-1+1} = 2$, Equality holds.

Theorem 5: For any tree T of order at least 2, $\gamma_{ccl}^{e}(T) \geq \Delta(T) + 1$ further equality holds if $T = K_{1,n-1}$.

Proof: Let $G = T$ be a tree of order atleast 2 with maximum degree Δ.

Let v be a vertex of maximum degree i.e. $\deg(v) = \Delta(T)$

Let D be a degree equitable complete cototal dominating set of G by default v and it's at least one neighbor say $w \in D$

i.e. $\gamma_{ccl}^{e}(T) \geq \{v, w\} \geq \Delta(T) + 1$

For equality,

Suppose $T = K_{1,n-1}$ then D contains entire vertex set of T.

i.e. $\gamma_{ccl}^{e}(T) = |D| = n$

Further note that $\Delta(K_{1,n-1}) = n-1$

By putting these we get, $\gamma_{ccl}^{e}(T) = n-1+1 = n$.

Theorem 6: For any Tree T,$diam(T) + 1 \leq \gamma_{ccl}^{e}(T)$, equality holds if $G = K_{1,n-1}$.

Proof: Let T be any nontrivial tree from Theorem (2)

$$\gamma_{ccl}^{e}(G) \leq n$$

Further for any tree $daim(T) = n-1$

Hence by Theorem (5) $daim(T) + 1 \leq \gamma_{ccl}^{e}(T)$

Equality follows from Theorem (5).

3. Compression with other domination parameters.

Theorem 7: For any *r-regular* graph G,$\gamma_{ccl}^{e}(G) = \gamma_{cc}(G)$

Proof: suppose G is the regular graph. Then every vertex as the same degree r. Let D be a minimum complete cototal dominating set of G, then $|D| = \gamma_{cc}(G)$. Let $u \in V - D$ then as D is a complete co-total dominating set, there exist a vertex $v \in D$ and $uv \in E(G)$. Also deg(u) $= \deg(v) = r$. $Therefore$ | $\deg(u)$ – $\deg(v)$ | $= 0 < 1$. Hence D is degree equitable complete cototal dominating set of G, so that $\gamma_{ccl}^{e}(G) \leq |D| \leq \gamma_{cc}(G)$. But $\gamma_{cc}(G) \leq \gamma_{ccl}^{e}(G)$.
Hence $\gamma_{cc}(G) = \gamma_{ccl}^{e}(G)$.

Theorem 8: For any connected graph G without isolated vertices $\gamma(G) + \gamma_{ccl}^{e} \leq \frac{3n}{2}$

Proof: Let G be any nontrivial connected graph of order n and size m. Then by Theorem A, $\gamma(G) \leq \frac{n}{2}$ and by theorem (2) $\gamma_{ccl}^{e}(G) \leq n$.

Hence, $\gamma_{ccl}^{e}(G) + \gamma(G) \leq n + \frac{n}{2} \leq \frac{3n}{2}$.

Theorem 9: For any tree, $\gamma_c(T) + 1 \leq \gamma_{ccl}^{e}(T) \leq 2n - \xi$, where ξ is number of pendant vertices in T.

Proof: Let G be any nontrivial tree then by Theorem C, $\gamma_c(T) = n - \xi$, where ξ is set of all pendant vertices in tree T. Then by Theorem (2) $\gamma_{ccl}^{e}(T) \leq n$.

Hence, $\gamma_{ccl}^{e}(T) + \gamma_c(T) \leq n + n - \xi \leq 2n - \xi$.

Theorem 10: For any graph G without isolated vertices $\gamma_t(G) + \gamma_{ccl}^{e}(G) \leq \frac{5n}{3}$

Proof: Let G be any graph without isolated vertices, then by Theorem B, $\gamma_t(G) \leq \frac{2n}{3}$

Also by theorem (2), $\gamma_{ccl}^{e}(G) \leq n$

Hence, $\gamma_{ccl}^{e}(G) + \gamma_t(G) \leq n + \frac{2n}{3} \leq \frac{5n}{3}$.

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