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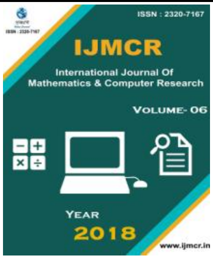

Intuitionistic Fuzzy Rings with Operators

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ARTICLE INFO	ABSTRACT
Published Online: 7 February 2018 Corresponding Author: **Poonam Kumar Sharma[2]**	In this paper, we further study the theory of intuitionistic fuzzy rings and give some new concepts such as intuitionistic fuzzy ring with operators, intuitionistic fuzzy ideal with opera- tors, intuitionistic fuzzy quotient ring with operators, etc. while their some elementary properties are discussed.

KEYWORDS: Intuitionistic fuzzy ring; Intuitionistic fuzzy ring with operators; Intuitionistic fuzzy ideals with operators; Intuitionistic fuzzy quotient ring with operators; Homomorphism.

AMS Classification: 03F55, 06E20.

1. Introduction

Atanassov [1], [2] and [3] introduced and developed the theory of intuitionistic fuzzy sets. Using the Atanassov' *s*idea Biswas [6] established the intuitionistic fuzzification of the concept of sub- group of a group and introduced the notion of intuitionistic fuzzy subgroups. Hur, Kang and Song

[7] introduced the concept of intuitionistic fuzzy ring. Banerjee and Basnet [4] further studied this concept and introduced the notion of intuitionistic fuzzy subring and ideal of a ring. The no- tion of intuitionistic nil radical, Semiprime intuitionistic fuzzy ideal and Euclidean intuitionistic fuzzy ideal were defined and studied by Jun, Qzturk and Park [15]. Meena and Thomas [9] and [10] extend these concepts into lattice setting and introduced intuitionistic L-fuzzy ring and intu- itionistic L-fuzzy ideals. In [11] Sharma introduced the notion of t- intuitionistic fuzzy set and developed the concept of t- intuitionistic fuzzy quotient ring. In this paper we further study the theory of intuitionistic fuzzy ring and give some new concepts such as intuitionistic fuzzy ring with operators, intuitionistic fuzzy ideal with operators, intuitionistic fuzzy quotient ring with operators, etc., while their some elementary properties are discussed. Some results in references [8], [12] and [13] are extended.

2. Preliminaries

For the sake of convenience we set out the former concepts which will be used in this paper. For elementary concepts and notions on intuitionistic fuzzy ring theory, we refer to [5].

Definition 2.1 ([1])**.** Let X be a non-empty set. An intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, with functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$. For each $x \in X$, $\mu_A(x)$ define the degree of membership and $\nu_A(x)$ define the degree of non-membership of the element x to the set A, with the condition that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Remark 2.2. (i) When $\mu_A(x) + \nu_A(x) = 1$ i.e., $\nu_A(x) = 1 - \mu_A(x), \forall x \in X$, then A is called a fuzzy set.

(ii) For simplicity, we shall use the symbol $A = (\mu_A, \nu_A)$ for the intuitionistic fuzzy set $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$.

Definition 2.3 ([4],[5])**.** An intuitionistic fuzzy set (IFS) $A = (\mu_A, \nu_A)$ of a ring R is said to be an intuitionistic fuzzy subring (IFSR) if

(i) $\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y)$

(ii) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$

$(iii) \nu_A(x-y) \le \nu_A(x) \vee \nu_A(y)$

$(iv) \nu_A(xy) \le \nu_A(x) \vee \nu_A(y), \forall x,y \in R.$

Definition 2.4 ([4],[5])**.** An IFS $A = (\mu_A, \nu_A)$ of a ring R is said to be an intuitionistic fuzzy normal subring (IFNSR) if A is an IFSR of R and

$\mu_A(xy) = \mu_A(yx)$

$\nu_A(xy) = \nu_A(yx), \forall x,y \in R.$

Definition 2.5 ([4],[5])**.** An IFS $A = (\mu_A, \nu_A)$ of a ring R is said to be an intuitionistic fuzzy left ideal (IFLI) if

$(i) \mu_A(x-y) \ge \mu_A(x) \wedge \mu_A(y)$

$(ii) \mu_A(xy) \ge \mu_A(x)$

$(iii) \nu_A(x-y) \le \nu_A(x) \vee \nu_A(y)$

$(iv) \nu_A(xy) \le \nu_A(x), \forall x,y \in R.$

Definition 2.6 ([4],[5])**.** An IFS $A = (\mu_A, \nu_A)$ of a ring R is said to be an intuitionistic fuzzy right ideal (IFRI) if

$(i) \mu_A(x-y) \ge \mu_A(x) \wedge \mu_A(y)$

$(ii) \mu_A(xy) \ge \mu_A(y)$

$(iii) \nu_A(x-y) \le \nu_A(x) \vee \nu_A(y)$

$(iv) \nu_A(xy) \le \nu_A(y), \forall x,y \in R.$

Definition 2.7 ([4],[5])**.** An IFS $A = (\mu_A, \nu_A)$ of a ring R is said to be an intuitionistic fuzzy ideal (IFI) if

$(i) \mu_A(x-y) \ge \mu_A(x) \wedge \mu_A(y)$

$(ii) \mu_A(xy) \ge \mu_A(x) \wedge \mu_A(y)$

$(iii) \nu_A(x-y) \le \nu_A(x) \vee \nu_A(y)$

$(iv) \nu_A(xy) \le \nu_A(x) \vee \nu_A(y), \forall x,y \in R.$

Theorem 2.8 ([5])**.** *Let A and B be two IFSRs (IFIs) of a ring R. Then $A \cap B$ is also an IFSR (IFI) of R*

Theorem 2.9 ([5])**.** *Let A be an IFS of a ring R, then A is an IFSR(IFI) of R if and only if either $C_{(\alpha,\beta)}(A) = \phi$ or $C_{(\alpha,\beta)}(A)$, for all $\alpha, \beta \in [0, 1]$ such that $\alpha + \beta \le 1$, is a subring (ideal) of R, where $C_{(\alpha,\beta)}(A) = \{x \in R : \mu_A(x) \ge \alpha$ and $\nu_A(x) \le \beta\}$, is a (α, β)-level cut set of an IFS A.*

Theorem 2.10 ([5])**.** *Let $f: R_1 \to R_2$ be a ring homomorphism and A and B be respectively the IFSR (IFI) of R_1 and R_2. Then $f(A)$ and $f^{-1}(B)$ be respectively be IFSR(IFI) of R_2 and R_1.*

3. Intuitionistic fuzzy rings with operators

Throughout this paper, R will be a commutative ring with unity and M be a non-empty set.

Definition 3.1 ([14])**.** A ring with operators is an algebraic system consisting of a ring R, a set M and a function defined in the product set $M \times R$ and having values in R such that, if mx denotes the element in R determined by the element x of R and the element m of M, which satisfies the following:

(i) $m(x+y) = mx + my, \forall x,y \in R$ and $m \in M$;

(ii) $m(xy) = (mx)y = x(my), \forall x,y \in R$ and $m \in M$.

Then m is said to be a (left) operator of R, M is said to be (left) operator set of R. R is said to be ring with operators. We use phrase R is an M-ring instead of a ring with operators. If a subring of M-ring R is also M-ring, then it is said to be a M-subring of R. An ideal of M-ring R is called an M-ideal of R.

Definition 3.2 ([14],[8])**.** Let R_1 and R_2 both be M-rings. If $\Phi : R_1 \to R_1$ be a homomorphism from R_1 into R_2. If $\Phi(mx) = m\Phi(x) \forall x \in R_1, m \in M$, then Φ is called a M-homomorphism.

Definition 3.3. An intuitionistic fuzzy subring A of an M-ring R is said to be an intuitionistic fuzzy subring of R with operators if $\mu_A(mx) \geq \mu_A(x)$ and $\nu_A(mx) \leq \nu_A(x), \forall x \in R$ and $\forall m \in M$.

We use the phrase A is an M-intuitionistic fuzzy subring (M-IFSR) of R instead of A an intuition- istic fuzzy subring of R with operators.

Example 3.4. Consider $R = Z$ = Set of integers, and let $M = \{0, i\}$ be the set of endomorphism^r s on Z. Then R is an M-ring.

Define an IFS A = (μA, νA) on R defined by

$$\mu_A(X) = \begin{cases} 1 & \text{if x is even integer} \\ 0.5 & \text{if x is odd integer} \end{cases} \qquad \nu_A(X) = \begin{cases} 0 & \text{if x is even integer} \\ 0.25 & \text{if x is odd integer} \end{cases}$$

Then it can be easily verified that A is an M-IFSR of R.

Example 3.5. Let R be an M-ring while S is a non-empty subset of R. If χ_S is the characteristic function of S, then S is an M-subring of R if and only if IFS $A = (\chi_S, \overline{\chi_S})$ is an M-IFSR of R where $\overline{\chi_S}(x)\ \ 1 - \chi_S(x)$ for all $x \in R$.

Theorem 3.6. *Let A, B are M-IFSR of an M-ring R. Then $A \cap B$ is an M-IFSR of R.*

Proof. It is clear that $A \cap B$ is an IFSR of R.

For any $x, y \in R$, $m \in M$ we have

$\mu_{A\cap B}(mx) = \mu_A(mx) \wedge \mu_B(mx) \geq \mu_A(x) \wedge \mu_B(x) = \mu_{A\cap B}(x)$.

Similarly, $\nu_{A\cap B}(mx) = \nu_A(mx) \vee \nu_B(mx) \leq \nu_A(x) \vee \nu_B(x) = \nu_{A\cap B}(x)$.

Hence $A \cap B$ is an M-IFSR of R.

Corollary 3.7. *The intersection of any family of M-IFSRs of R is an M-IFSR of R.*

Definition 3.8. An IFS A of an M-ring R is said to be an M-intuitionistic fuzzy normal subring (M-IFNSR) of R, if A is not only an M-IFSR of R but also an IFNSR of R.

Theorem 3.9. *Let A, B are M-IFNSRs of an M-ring R, then $A \cap B$ is an M-IFNSR of R.*

Proof. By Theorem (3.6) we see that $A \cap B$ is an M-IFSR of R.

Also, $\mu_{A\cap B}(xy) = \mu_A(xy) \wedge \mu_B(xy) = \mu_A(yx) \wedge \mu_B(yx) = \mu_{A\cap B}(yx)$.

Similarly, we can show that $\nu_{A\cap B}(xy) = \nu_{A\cap B}(yx)$,for all x, yR. This shows that $A \cap B$ is an IFSR of R, hence $A \cap B$ is an M-IFNSR of R.

Corollary 3.10. *The intersection of any family of M-IFNSRs of R is an M-IFNSR of R.*

Definition 3.11. An IFSR A of an M-ring R is said to be an M-intuitionistic fuzzy ideal (M-IFI) of R if A is not only an M-IFSR of R, but also an IFI of R.

Theorem 3.12. *Let A and B are M-IFIs of an M-ring R, then $A \cap B$ is an M-IFI of R.*

Proof. Since both A and B are M-IFIs of R. Therefore, A and B are M-IFSR of R and so by Theorem (3.9) $A \cap B$ is an M-IFSR of R. Moreover, A and B are also IFIs of R implies that $A \cap B$ is also an IFI of R. Hence $A \cap B$ is an M-IFI of R.

Theorem 3.13. *An IFSR A of an M-ring R is an M-IFSR of R if and only if for any $\alpha, \beta \in [0, 1]$ such that $\alpha + \beta \leq 1$, $C_{(\alpha,\beta)}(A)$ is an M-subring of R, where $C_{(\alpha,\beta)}(A) \neq \phi$*

Proof. Firstly, let $x, y \in C_{(\alpha,\beta)}(A)$ and $m \in M$ be any elements, then we have

$\mu_A(x) \geq \alpha$, $\mu_A(y) \geq \alpha$ and $\nu_A(x) \leq \beta$, $\nu_A(y) \leq \beta$.

Now, $\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y) \geq \alpha$ and $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y) \geq \alpha$ and

$\nu_A(x - y) \leq \nu_A(x) \vee \nu_A(y) \leq \beta$ and $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y) \leq \beta$ and

$\mu_A(mx) \geq \mu_A(x) \geq \alpha$ and $\nu_A(mx) \leq \nu_A(x) \leq \beta \Rightarrow x - y, xy, mx \in C_{(\alpha,\beta)}(A)$.

Hence $C_{(\alpha,\beta)}(A)$ is a M-subring of R.

Conversely, let A be an IFSR of M-ring R such that $C_{(\alpha,\beta)}(A)\ f \neq \phi$ is a M-subring of R for all $\alpha, \beta \in [0, 1]$ such that $\alpha + \beta \leq 1$.

Let $x, y \in C_{(\alpha,\beta)}(A)$ and $m \in M$ be any elements such that $\mu_A(x) = \alpha_1$ and $\nu_A(x) = \beta_1$;

$\mu_A(y) = \alpha_2$ and $\nu_A(y) = \beta_2$; $\mu_A(mx) = \alpha_3$ and $\nu_A(mx) = \beta_3$, where $\alpha_i, \beta_i \in [0, 1]$ such that $\alpha_i + \beta_i \leq 1$ for all i = 1, 2, 3.

Let $\alpha = \alpha_1 \wedge \alpha_2$, $\beta = \beta_1 \vee \beta_2$ and $\alpha^r = \alpha_1 \vee \alpha_3$, $\beta^r = \beta_1 \wedge \beta_3$.

As $C_{(\alpha,\beta)}(A)$ and $C_{(\alpha^r,\beta^r)}(A)$ are M - subring of R, therefore, $x - y, xy, mx \in C_{(\alpha,\beta)}(A)$ and $x - y, xy, mx \in C_{(\alpha^r,\beta^r)}(A)$.

Now, $\mu_A(x - y) \geq \alpha = \alpha_1 \wedge \alpha_2 = \mu_A(x) \wedge \mu_A(y)$ implies $\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y)$.

Also, $\mu_A(xy) \geq \alpha = \alpha_1 \wedge \alpha_2 = \mu_A(x) \wedge \mu_A(y)$ implies $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$ and $\mu_A(mx) \geq \alpha^r = \alpha_1 \vee \alpha_3 \geq \alpha_1 = \mu_A(x)$.

Similarly, we can show that $\nu_A(x - y) \leq \nu_A(x) \vee \nu_A(y)$ and $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$ and $\nu_A(mx) \leq \nu_A(x)$. Hence A is an M- IFSR of R.

Theorem 3.14. *An IFSR A of an M-ring R is an M-IFI of R if and only if for any $\alpha, \beta \in [0, 1]$ such that $\alpha + \beta \leq 1$, $C_{(\alpha,\beta)}(A)$ is an M-ideal of R, where $C_{(\alpha,\beta)}(A) \neq \phi$*

Proof. It follows from Theorem (2.9) and Theorem (3.13).

Theorem 3.15. *Any M-subring S of a M-ring R can be realized as a (α, β)- level cut M-subring of some M-IFSR of R.*

Proof. Let A be an IFS on R defined by

$$\mu_A(X) = \begin{cases} \alpha & \text{if } x \in S, 0 < \alpha < 1 \\ 0 & \text{if } x \notin S \end{cases} \qquad \nu_A(X) = \begin{cases} \beta & \text{if } x \in S, 0 < \beta < 1 \\ 1 & \text{if } x \notin S \end{cases}$$

where $\alpha + \beta \leq 1$, S is M-subring of M-ring R. (Note that here $C_{(\alpha,\beta)}(A) = S$)

We claim that A is an M-IFSR of R.

Let x, y be any two elements of R and $m \in M$ be any element. Then

Case(i) When both x, y are in S, then $m(x + y)$ and $m(xy)$ are in S.

So, $\mu_A(m(x + y)) = \mu_A(m(xy)) = \mu_A(x) = \mu_A(y) = \alpha$ and $\nu_A(m(x + y)) = \nu_A(m(xy)) = \nu_A(x) = \nu_A(y) = \beta$.

Thus, $\mu_A(m(x + y)) \geq \mu_A(x) \wedge \mu_A(y)$ and $\mu_A(m(xy)) \geq \mu_A(x) \wedge \mu_A(y)$.

Similarly, we have $\nu_A(m(x + y)) \leq \nu_A(x) \vee \nu_A(y)$ and $\nu_A(m(xy)) \geq \nu_A(x) \vee \mu_A(y)$.

Case(ii) When $x \in S$ and $y \notin S$, then $m(x + y) \notin S$ and $m(xy) \notin S$.

So, $\mu_A(m(x + y)) = \mu_A(m(xy)) = 0$ and $\mu_A(x) = \alpha$, $\mu_A(y) = 0$ and $\nu_A(m(x + y)) = \nu_A(m(xy)) = 1$ and $\nu_A(x) = \beta$, $\nu_A(y) = 1$.

Thus, $\mu_A(m(x + y)) \geq \mu_A(x) \wedge \mu_A(y)$ and $\mu_A(m(xy)) \geq \mu_A(x) \wedge \mu_A(y)$.

Similarly, we have $\nu_A(m(x + y)) \leq \nu_A(x) \vee \nu_A(y)$ and $\nu_A(m(xy)) \geq \nu_A(x) \vee \mu_A(y)$.

Case(iii) When $x \notin S$ and $y \notin S$, then $m(x+y)$ and $m(xy)$ may or may not be in S. In both the cases we can see that $\mu_A(m(x + y)) \geq \mu_A(x) \wedge \mu_A(y)$ and $\mu_A(m(xy)) \geq \mu_A(x) \wedge \mu_A(y)$.

and $\nu_A(m(x + y)) \leq \nu_A(x) \vee \nu_A(y)$ and $\nu_A(m(xy)) \geq \nu_A(x) \vee \mu_A(y)$.

Combining all the cases, we see that A is an M-IFSR of R.

Theorem 3.16. *Any M-ideal I of a M-ring R can be realized as a (α, β)- level cut M-ideal of some M-IFI of R.*

Proof. This follows from above Theorem (3.15), by replacing M-subring S with the M-ideal I of R.

Theorem 3.17. *Let Φ be an M- homomorphism from the M-ring R_1 to the M-ring R_2, then*

(i) If B is an M-IFSR of R_2, then $\Phi^{-1}(B)$ is an M-IFSR of R_1.

(ii) If B is an M-IFNSR of R_2, then $\Phi^{-1}(B)$ is an M-IFNSR of R_1.

(iii) If B is an M-IFI of R_2, then $\Phi^{-1}(B)$ is an M-IFI of R_1.

Proof.

(i) Since $\Phi^{-1}(B)$ is an IFSR of R_1, then

$\Phi^{-1}(B)(mx) = (\mu_{\Phi^{-1}(B)}(mx), \nu_{\Phi^{-1}(B)}(mx))$, for all $m \in M$ and $x \in R_1$, where

$\mu_{\Phi^{-1}(B)}(mx) = \mu_B(\Phi(mx)) = \mu_B(m\Phi(x)) \geq \mu_B(\Phi(x)) = \mu_{\Phi^{-1}(B)}(x)$ and

$\nu_{\Phi^{-1}(B)}(mx) = \nu_B(\Phi(mx)) = \nu_B(m\Phi(x)) \leq \nu_B(\Phi(x)) = \nu_{\Phi^{-1}(B)}(x)$.

Hence $\Phi^{-1}(B)$ is an M-IFSR of R_1.

(ii) By part (i) $\Phi^{-1}(B)$ is an M-IFSR of R_1.

It remain only to show that $\Phi^{-1}(B)$ is an IFNSR of R_1.

Now, $\mu_{\Phi^{-1}(B)}(xy) = \mu_B(\Phi(xy)) = \mu_B(\Phi(yx)) = \mu_{\Phi^{-1}(B)}(yx)$ and

$\nu\Phi^{-1}(B)(xy) = \nu B(\Phi(xy)) = \nu B(\Phi(yx)) = \nu\Phi^{-1}(B)(yx)$.

Hence $\Phi^{-1}(B)$ is an M-IFNSR of R_1. This complete the proof.

(iii) It follows immediately from part (ii) and Theorem (2.10).

Theorem 3.18. *Let Φ be an M- homomorphism from the M-ring R_1 to the M-ring R_2, then*

(i) If A is an M-IFSR of R_1, then $\Phi(A)$ is an M-IFSR of R_2.

(ii) If A is an M-IFNSR of R_1, then $\Phi(A)$ is an M-IFNSR of R_2.

(iii) If A is an M-IFI of R_1, then $\Phi(A)$ is an M-IFI of R_2.

Proof.

(i) Since $\Phi(A)$ is an IFSR of R_2, then

$\Phi(A)(my) = (\mu\Phi(A)(my), \nu\Phi(A)(my))$, for all $m \in M$ and $y \in R_2$, where

$$\begin{aligned}\mu\Phi(A)(my) &= Sup\{\mu_A(mx) : \Phi(mx) = my, x \in R_1, s.t., \Phi(x) = y\}\\ &= Sup\{\mu_A(mx) : m\Phi(x) = my\}\\ &= Sup\{\mu_A(mx) : \Phi(x) = y\}\\ &\geq Sup\{\mu_A(x) : \Phi(x) = y\}\\ &= \mu\Phi(A)(y).\end{aligned}$$

Similarly, we can show that $\nu\Phi(A)(my) \leq \nu\Phi(A)(y)$, $\forall m \in M, y \in R_2$. Hence $\Phi(A)$ is an M-IFSR of R_2.

(ii) Let $y_1, y_2 \in R_2$ be any two elements. Then $\exists^r s\, x_1, x_2 \in R_1$ such that $\Phi(x_1) = y_1, \Phi(x_2) = y_2$.

$\mu\Phi(A)(y_1y_2) = \mu\Phi(A)(\Phi(x_1)\Phi(x_2)) = \mu\Phi(A)(\Phi(x_2)\Phi(x_1)) = \mu\Phi(A)(\Phi(x_2x_1)) = \mu\Phi(A)(y_2y_1)$.

Similarly, we can show that $\nu\Phi(A)(y_1y_2) = \nu\Phi(A)(y_2y_1)$. By using part (i), $\Phi(A)$ is an M-IFNSR of R_2.

(iii) It follows immediately from part (ii) and Theorem (2.10).

4. M-Intuitionistic Fuzzy Quotient Ring

Let R be a ring, A be an intuitionistic fuzzy ideal of R. Sharma [11] had proved that the set $R/A = \{x + A : x \in R\}$ form a ring under the operations

$$(x + A) + (y + A) = (x + y) + A \text{ and } (x + A)(y + A) = (xy) + A$$

called the intuitionistic fuzzy quotient ring of R with respect to A.

Theorem 4.1. *Let A be an M-IFI of an M-ring R. Then R/A is an M-ring.*

Proof. For all $m \in M, (x + A), (y + A) \in R/A$, we define $m(x + A) = mx + A$ so that

$$\begin{aligned}m((x + A) + (y + A)) &= m((x + y + A))\\ &= m(x + y) + A\\ &= mx + my + A\\ &= (mx + A) + (my + A)\\ &= m(x + A) + m(y + A).\end{aligned}$$

And

$$\begin{aligned}m((x + A)(y + A)) &= M((xy + A))\\ &= m(xy) + A\\ &= (mx)y + A\\ &= (mx + A)(y + A)\\ &= (m(x + A))(y + A)\\ &= (x + A)(m(y + A))\end{aligned}$$

So, R/A is an M-ring.

Remark 4.2. The above M-ring R/A is called the M-intuitionistic fuzzy quotient ring of R with respect to A.

We now define an IFS on R/A. Let B be any M-intuitionistic fuzzy ring of R, B/A be an IFS of R/A defined as follows:

$(B/A)(a + A) = (\mu_{B/A}(a + A), \nu_{B/A}(a + A))$, where

$\mu_{B/A}(a+A) = Sup\{\mu_B(x) : x+A = a+A; x \in R\}$ and $\nu_{B/A}(a+A) = Inf\{\nu_B(x) : x+A = a + A; x \in R\}$, for all $a + A \in R/A$.

Theorem 4.3. *The above intuitionistic fuzzy subset B/A is an M-intuitionistic fuzzy subring of M-ring R/A.*

Proof. Let $C = B/A$, $c = a + A$ and $d = b + A$, for every $a, b \in R$. Then

$$\mu_C(c-d) = \mu_C((a+A)-(b+A))$$

$$= \mu_C(a-b+A)$$
$$= Sup\{\mu_B(x) : x+A = a-b+A\}$$
$$= Sup\{\mu_B(y-z) : y+A = a+A, z+A = b+A\}$$
$$\geq Sup\{\mu_B(y) \wedge \mu_B(z) : y+A = a+A, z+A = b+A\}$$
$$= Sup\{\mu_B(y) : y+A = a+A\} \wedge Sup\{\mu_B(z) : z+A = b+A\}$$
$$= \mu_C(c) \wedge \mu_C(d).$$

Thus, we get $\mu_{B/A}((a+A)-(b+A)) \geq \mu_{B/A}(a+A) \wedge \mu_{B/A}(b+A)$.

Similarly, we can show that $\nu_{B/A}((a+A)-(b+A)) \leq \nu_{B/A}(a+A) \vee \nu_{B/A}(b+A)$.

$$\mu_C(cd) = \mu_C((a+A)(b+A))$$
$$= \mu_C(ab+A)$$
$$= Sup\{\mu_B(x) : x+A = ab+A\}$$
$$= Sup\{\mu_B(yz) : y+A = a+A, z+A = b+A\}$$
$$\geq Sup\{\mu_B(y) \wedge \mu_B(z) : y+A = a+A, z+A = b+A\}$$
$$= Sup\{\mu_B(y) : y+A = a+A\} \wedge Sup\{\mu_B(z) : z+A = b+A\}$$
$$= \mu_C(c) \wedge \mu_C(d).$$

Thus, we get $\mu_{B/A}((a+A)(b+A)) \geq \mu_{B/A}(a+A) \wedge \mu_{B/A}(b+A)$.

Similarly, we can show that $\nu_{B/A}((a+A)(b+A)) \leq \nu_{B/A}(a+A) \vee \nu_{B/A}(b+A)$.

$$\mu_{B/A}(m(a+A)) = \mu_{B/A}(ma+A)$$
$$= Sup\{\mu_B(x) : x+A = ma+A\}$$
$$\geq Sup\{\mu_B(mc) : mc+A = ma+A\}$$
$$\geq Sup\{\mu_B(mc) : c+A = a+A\}$$
$$\geq Sup\{\mu_B(c) : c+A = a+A\}$$
$$= \mu_{B/A}(a+A).$$

Similarly, we can show that $\nu_{B/A}(m(a+A)) \leq \nu_{B/A}((a+A))$. Hence B/A is an M-intuitionistic fuzzy subring of M-ring R/A.

Definition 4.4. The above intuitionistic fuzzy ring B/A is called the M-intuitionistic fuzzy ring of B with respect to A or the intuitionistic fuzzy quotient ring with operators with respect to A.

Theorem 4.5. *Let R be an M-ring, A be an M-IFI of R, B be any M-IFSR of R and $f : R \rightarrow R/A$ defined by $f(x) = x+A$, for all $x \in R$. Then f is an M-homomorphism from R onto R/A and $f(B) = B/A$.*

Proof. It is clear that f is a natural homomorphism from R onto R/A.
For M-homorphism: Let $x \in R$ and $m \in M$ be any elements, then
$f(mx) = mx + A = m(x + A) = mf(x)$.
For any $a + A \in R/A$, we have
$f(B)(a+A) = (\mu_{f(B)}(a+A), \nu_{f(B)}(a+A))$, where
$\mu_{f(B)}(a+A) = Sup\{\mu_B(x) : f(x) = a+A\}$ and $\nu_{f(B)}(a+A) = Inf\{\mu_B(x) : f(x) = a+A\}$. Thus f is a M-homomorphism from R onto R/A and $f(B) = B/A$.

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