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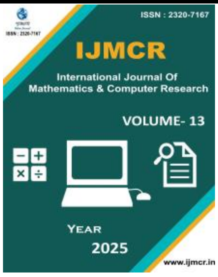

Axial Force Influence on Transverse Displacement and Rotation Under Moving Load of Elastically Supported Damped Shear Beam

AJIJOLA Olawale Olaonipekun

Department of Mathematical Sciences, Faculty of Science,Adekunle Ajasin University, Akungba-Akoko, P.M.B. 001, Ondo State, Nigeria

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Corresponding Author:
AJIJOLA Olawale Olaonipekun

ABSTRACT

This research investigates the influence of axial force on the transverse displacement and rotation of a damped shear beam resting on an elastic foundation and subjected to moving load traveling at a constant velocity. The governing equations are coupled second-order partial differential equations. To simplify these equations, the finite Fourier series method was employed, transforming the coupled second-order partial differential equations into a set of coupled second-order ordinary differential equations. The simplified equations that describe the motion of the beam-load system were subsequently solved using Laplace transformation in conjunction with convolution theory to obtain the solutions. Comprehensive analyses are conducted to investigate the effects of axial force on the transverse displacement and rotation of damped shear beams of different length sizes when subjected to the moving load traversing the beam at different velocities respectively. Additionally, the study examines the effect of axial force on the critical velocities of the vibrating system. The findings reveal that the transverse displacement and rotation of the beam are noticeably reduced with increasing axial force. It is also found that as the value of axial force increases, the critical velocity increases indicating a safer dynamical system. From a practical perspective, this clearly indicates that axial force significantly enhances the dynamic stability of the beam when subjected to the moving load.

KEYWORDS: Axial Force, Moving Load, Shear Beam, Critical velocity, Resonance

I. INTRODUCTION

In modern structural design and analysis, the vibration analysis of elastic structures is a significant area of research within structural and mechanical engineering, having been extensively examined to understand the dynamic behavior of continuous systems. Specifically, the response of elastic structures, such as beams subjected to various types of moving loads, has garnered considerable interest from numerous researchers [1-7]. A fundamental understanding of the complex dynamic interactions between structural components and the loads acting upon them is crucial, as it aids in managing structural vibrations and enhances the operational efficiency of such dynamic systems. Moving loads can produce substantial vibrations in elastic structures they traverse, especially at high speeds. Unlike stationary loads or subsystems that produce constant stresses and deformations, moving loads result in effects that vary according to the load's position, which is characteristically time-dependent [8]. As a result, a substantial body of literature has been dedicated to addressing moving load problems. The vibration response of beams under moving loads has been thoroughly investigated in [9-13].

Moreover, beams supported by elastic foundations are frequently encountered in the analysis of various structures, including buildings, geotechnical highways, railway systems, and numerous other related structures. The study of the vibrations of beam-like structures due to moving loads on elastic foundations holds significant technological relevance. Notably, Ogunbamike [14] investigated the dynamic behavior of a simply supported Timoshenko beam resting on a Winkler foundation and subjected to a moving uniformly distributed load. Also, Clastornik et al. [15] performed a study on the dynamic analysis of elastic beams that are supported by a variable Winkler elastic foundation In a similar vein, the dynamic response to traveling load of elastic structure resting on bi-parametric elastic foundations have been extensively studied. Ogunbamike and Oni [16] explored the dynamic characteristics of a non-prismatic Rayleigh beam with general boundary conditions, supported by a Vlasov elastic foundation and subjected to partially distributed moving masses with varying velocities. They utilized Generalized Galerkin method to derive closed-form solutions for this class of dynamical problems. Rajib et al. [17] investigated the dynamic response of a beam when under both moving loads and moving masses, supported by a Pasternak foundation. Additionally, Jimoh and Awelewa [18] investigated the dynamic behavior of a non-uniform elastic structure resting on an exponentially decaying Vlasov foundation, subjected to repeated rolling concentrated loads.

In the majority of the previously mentioned studies, the influence of axial force was not considered; however, such effects can have a considerable impact on the dynamic response of structures. Thus, engineers commonly introduce artificial stresses into structures before applying loads to ensure that the stresses experienced during loading are more favorable than they would typically be. These artificial stresses can exist as forces acting either axially or in various other directions. When these forces are aligned axially, they are termed axial forces. This technique of inducing artificial stresses is referred to as pre-stressing. The main objective of pre-stressed structures is to minimize tensile stresses, thereby reducing the risk of flexural cracking or bending under operational conditions. As a result, extensive research has been conducted on the vibrations of pre-stressed beams subjected to moving loads. Jimoh, Oni and Ajijola [19] examined the effect of variable axial force on the deflection of thick beams under a distributed moving load. They calculated the transverse displacement for both moving force and moving distributed mass models of the dynamic problem for various time intervals and conducted an analysis. Jimoh, Ogunbamike, and Ajijola [20] explored the dynamic response of non-uniformly prestressed thick beams under distributed moving load traveling at different velocities. They employed a technique based on Galerkin’s method, utilizing a series representation of the Heaviside function to transform the equations, which were then simplified using Struble's asymptotic method and solved through Laplace transformation techniques in conjunction with convolution theory. Their findings indicated that the moving distributed force does not serve as an upper bound for accurately solving the moving distributed mass problem. Additionally, they discovered that increasing certain pertinent structural parameters leads to a reduction in the response amplitudes of non-uniformly prestressed thick beams under moving distributed loads. Ogunbamike [21] also investigated the dynamic response of a Timoshenko beam supported by an elastic foundation and subjected to a harmonic moving load.

The theory of shear beams plays a crucial role in structural engineering, emphasizing beams where shear deformation is significant. In contrast to the conventional Euler-Bernoulli beam theory, which presumes that plane sections remain unchanged and perpendicular to the neutral axis, shear beam theory incorporates shear deformations, thereby proving essential for the analysis of short and deep beams.

It is important to remark at this juncture that extensive research has been undertaken on dynamic problems related to Bernoulli-Euler beams and various other beam types subjected to both lumped and distributed loads [22-25]. However, there is a scarcity of research focusing on shear beams under moving loads. The literature addressing shear beams models remains limited until recently when landmark feat was achieved by [26] who studied the dynamic behaviour of damped shear beam resting on bi-parametric elastic foundation when traversed by moving load travelling at constant velocity. Similarly, Ajijola [27] analyzed the transverse displacement and rotation of an axially prestressed damped shear beam supported by a Vlasov foundation when subjected to a moving load.

Thus, this paper presents the influence of axial force on the transverse displacement and rotation of a damped shear beam when subjected to moving load travelling at a constant velocity. Furthermore, the effect of axial force on the critical velocity of a simply supported damped shear beam traversed by a moving load is presented in this research.

II. MATHEMATICAL MODEL

The governing equations of motion describing the transverse displacement $V(x,t)$ and rotation $\phi(x,t)$ of a shear beam when subjected to a moving load traveling at a constant velocity are formulated as coupled second order partial differential equations given by

$$M\frac{\partial^2 V(x,t)}{\partial t^2}+\frac{\partial}{\partial x}\left[K^*G^*A\left(\phi(x,t)-\frac{\partial V(x,t)}{\partial x}\right)\right] + F(x,t) = P(x,t) \tag{1}$$

$$\frac{\partial}{\partial x}\left(EI\frac{\partial\phi(x,t)}{\partial x}\right)-K^*G^*A\left(\phi(x,t)-\frac{\partial V(x,t)}{\partial x}\right)=0. \tag{2}$$

where M is the mass per unit length of the beam, K^* is the shear correction factor, G^* is the shear parameter of the beam, A is the cross-sectional area of the beam, E is the Young modulus of elasticity of the beam material, I is the moment of inertia, EI is the flexural stiffness / rigidity, x is the spatial coordinate, t is the time coordinate, $F(x,t)$ is the foundation reaction and $P(x,t)$ is the moving load acting on the beam per unit length.

The relationship between the foundation reaction $F(x,t)$ and lateral deflection $V(x,t)$ is given by

$$F(x,t)=KV(x,t)-G\frac{\partial^2 V(x,t)}{\partial x^2} \tag{3}$$

where K and G are two parameters of the foundation model. Specifically, K is the Foundation Stiffness and G is the Shear Modulus.

In this study, it is assumed that the load function $P(x,t)$ is given in the form

$$P(x,t)=P_0\delta(x-ct). \tag{4}$$

$\delta(\cdot)$ is the well-known Dirac delta function with the property.

$$\int_b^a \partial(x-ct)f(x)dx = \{\ 0,\ \ for\ ct\ < a\ <\ b,$$
$f(ct),\ for\ a\ < ct < b,\ 1,\ for\ a\ < b < ct.$ (5)

It is remarked here that the beam under consideration is assumed to have simple support at both ends $x\ =\ 0$ and $x\ =\ L$. Thus, boundary conditions are given as

$V(0,t) = V(L,t) = 0,\ \frac{\partial V(0,t)}{\partial x} = \frac{\partial V(L,t)}{\partial x} = 0$
(6)

$\phi(0,t) = \phi(L,t) = 0,\ \frac{\partial \phi(0,t)}{\partial x} = \frac{\partial \phi(L,t)}{\partial x} = 0$
(7)

and the initial conditions are given as

$$V(0,x) = 0 = \frac{\partial V(x,0)}{\partial t},\quad \phi(0,x) = \ 0 = \frac{\partial \phi(x,0)}{\partial t} \tag{8}$$

Now, introducing damping and axially prestressed parameters and in view of (3) and (4) after some simplifications and re-arrangements, equation (1) and (2) become

$$\frac{\partial}{\partial x}\left[K^*G^*A\left(\phi(x,t) - \frac{\partial V(x,t)}{\partial x}\right)\right] + M\frac{\partial^2 V(x,t)}{\partial t^2}$$
$$-N_0\frac{\partial^2 V(x,t)}{\partial x^2} - C_0\frac{\partial V(x,t)}{\partial t} + KV(x,t) - G\frac{\partial^2 V(x,t)}{\partial x^2}$$
$$= P_0\delta(x-ct) \tag{9}$$

$$\frac{\partial}{\partial x}\left(EI\frac{\partial \phi(x,t)}{\partial x}\right) - K^*G^*A\left(\phi(x,t)\ - \frac{\partial V(x,t)}{\partial x}\right) = 0 \tag{10}$$

Where N_0 is the axial force and C_0 is the coefficient of viscous damping per unit length of the beam.

Hence, (9) and (10) are the second order partial differential equations governing the flexural motion of an elastically supported damped shear beam when subjected to a moving load traveling at a constant velocity.

III. SOLUTION PROCEDURE

The shear beam examined in this study is both finite and uniform. To derive the analytical solution for the initial boundary value problem presented in equations (9) and (10), we utilize the finite Fourier transformation method in conjunction with the Laplace Transform techniques. Consequently, we present the following definitions.

Definition 1: The finite Fourier sine transform $\omega_0(n,t)$ of a function $\omega(x,t)$ is defined as

$$\omega_0(n,t) = \int_0^l \omega(x,t)\ sin\ sin\ \frac{n\pi x}{l}\ dx \tag{11}$$

and the inverse transform is

$$\omega(x,t) = \frac{2}{l}\sum_{n=1}^{\infty}\omega_0(n,t)\ sin\ sin\ \frac{n\pi x}{l}\ dx. \tag{12}$$

Definition 2: The finite Fourier cosine transform $\gamma_0(n,t)$ of a function $\gamma(x,t)$ is defined as

$$\gamma_0(n,t) = \int_0^l \gamma(x,t)\ cos\ cos\ \frac{n\pi x}{l}\ dx \tag{13}$$

and the inverse transform is

$$\gamma(x,t) = \frac{2}{l}\sum_{n=1}^{\infty}\gamma_0(n,t) cos\ \frac{n\pi x}{l}\ dx. \tag{14}$$

Thus, applying (11) and (13) to the governing equations (9) and (10) respectively, in conjunction with the Dirac delta function property in (5), we obtain

$$\frac{\partial^2 V(n,t)}{\partial t^2} + \alpha_1\frac{\partial V(n,t)}{\partial t} + \alpha_2 V(n,t) - \alpha_3\frac{\partial \phi(n,t)}{\partial x}$$
$$= Q_1\ sin\ \theta_n\ t \tag{15}$$

and

$$\phi(n,t) = \ \alpha_0 V(n,t) \tag{16}$$

where

$\alpha_1 = -\frac{C_0}{M},\ \ \alpha_2 = \left(\frac{n\pi}{ML}\right)^2(N_0+G) - \frac{K}{M},$

$\alpha_3 = \left(\frac{n\pi}{ML}\right)K^*G^*A,$

$Q_1 = \frac{P_0}{M},\quad \theta_n = \frac{n\pi c}{L},\quad \alpha_0 = \frac{\frac{n\pi}{L}K^*G^*A}{EI\left(\frac{n\pi}{L}\right)^2 + K^*G^*A}$

Now putting (16) into (15), we have

$$\frac{\partial^2 V(n,t)}{\partial t^2} + \alpha_1\frac{\partial V(n,t)}{\partial t} + \alpha_2 V(n,t)$$
$$-\alpha_3\frac{\partial}{\partial x}\left(\alpha_0 V(n,t)\right) = \ Q_1\ sin\ \theta_n\ t \tag{17}$$

The term involving the derivative with respect to x in (17) vanishes as $V(n,t)$ is a function of t alone and after some simplifications and re-arrangements, we obtain

$$V_{tt}(n,t) + \alpha_1 V_t(n,t) + \alpha_4 V(n,t) = Q_1\ sin\ \theta_n\ t \tag{18}$$

where

$\alpha_4 = \alpha_2 - \alpha_0\alpha_3$

Next, we subject (18) to Laplace transformation

$$\mathcal{L}(f(t)) = \mathrm{F(s)} = \int_0^{\infty} f(t)\ e^{-st}dt \tag{19}$$

where s is the Laplace parameter. In view of (19), (18) becomes

$$s^2\tilde{V}(n,s) + \alpha_1 s\tilde{V}(n,s) + \alpha_4\tilde{V}(n,s) = Q_1\left[\frac{\theta_n}{s^2+\theta_n{}^2}\right]. \tag{20}$$

After simplification and rearrangement, we obtain the simple algebraic equation given by

$$\tilde{V}(n,s) = Q_1\left[\frac{1}{s^2+\alpha_1 s+\alpha_4}\right]\left[\frac{\theta_n}{s^2+\theta_n{}^2}\right] \tag{21}$$

which is further simplified to give

$$\tilde{V}(n,s) = Q_1\left[\frac{1}{(s+\alpha_5)^2+\beta^2}\right]\left[\frac{\theta_n}{s^2+\theta_n{}^2}\right] \tag{22}$$

Where

$\beta^2 = \alpha_4 - (\alpha_5)^2,\quad \alpha_5 = \left(\frac{\alpha_1}{2}\right)$ (12)

(13)

At this juncture, in order to obtain the Laplace inversions of (22), we let

$$F(s) = \left[\frac{1}{(s+\alpha_5)^2+\beta^2}\right] \tag{23}$$

and

$$G(s) = \left[\frac{\theta_n}{s^2+\theta_n{}^2}\right] \tag{24}$$

so that the Laplace inversion of (22) is the convolution of (23) and (24) defined by

$$F(s) * G(s) = \int_0^t f(t-u)g(u)du. \tag{25}$$

Noting that

$$\mathcal{L}^{-1}[F(s)] = \frac{1}{p} \exp\ (-\alpha_5 t)\sin(\beta t) \tag{26}$$

and

$$\mathcal{L}^{-1}[G(s)] = \sin(\theta_n t) \tag{27}$$

Now using (23) and (24) in (25), (22) becomes

$$V(n,t) = \frac{Q_1 e^{-\alpha_5 t}}{\beta(\varphi_1-\varphi_0)(\varphi_2-\varphi_0)}\{\varphi_2[\beta e^{\alpha_5 t} \sin\theta_n t - \theta_n \sin\beta t] + \varphi_0[\beta e^{\alpha_5 t}\sin\theta_n t + \theta_n \sin\beta t] - \alpha_1\beta\theta_n[e^{\alpha_5 t}\cos\theta_n t - \cos\beta t]\} \tag{28}$$

where,

$$\varphi_1 = (\beta+\theta_n)^2, \quad \varphi_2 = (\beta-\theta_n)^2, \quad \varphi_0 = -(\alpha_5)^2$$

Thus, in view of (12), one obtains

$$V(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} \frac{Q_1 e^{-\alpha_5 t}}{\beta(\varphi_1-\varphi_0)(\varphi_2-\varphi_0)}\{\varphi_2[\beta e^{\alpha_5 t}\sin\theta_n t - \theta_n \sin\beta t] + \varphi_0[\beta e^{\alpha_5 t}\sin\theta_n t + \theta_n \sin\beta t] - \alpha_1\beta\theta_n[e^{\alpha_5 t}\cos\theta_n t - \cos\beta t]\}\sin\frac{n\pi x}{l} \tag{29}$$

which represents the transverse displacement of an elastically supported damped shear beam when subjected to moving load traveling at a constant velocity.

Now, using (28) in (16), we have

$$\phi(n,t) = \frac{\alpha_0 Q_1 e^{-\alpha_5 t}}{\beta(\varphi_1-\varphi_0)(\varphi_2-\varphi_0)}\{\varphi_2[\beta e^{\alpha_5 t}\sin\theta_n t - \theta_n \sin\beta t] + \varphi_0[\beta e^{\alpha_5 t}\sin\theta_n t + \theta_n \sin\beta t] - \alpha_1\beta\theta_n[e^{\alpha_5 t}\cos\theta_n t - \cos\beta t]\}. \tag{30}$$

Similarly, in view of (14), one obtains

$$\phi(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} \frac{\alpha_0 Q_1 e^{-\alpha_5 t}}{\beta(\varphi_1-\varphi_0)(\varphi_2-\varphi_0)}\{\varphi_2[e^{\alpha_5 t}\sin\theta_n t - \theta_n \sin\beta t] + \varphi_0[\beta e^{\alpha_5 t}\sin\theta_n t + \theta_n \sin\beta t] - \alpha_1\beta\theta_n[e^{\alpha_5 t}\cos\theta_n t - \cos\beta t]\}\cos\frac{n\pi x}{l} \tag{31}$$

which represents the rotation of an elastically supported damped shear beam when subjected to a moving load traveling at a constant velocity.

IV. Numerical Simulation and Discussion of Result

The uniform damped shear beam of lengths (L) = 8.5m, 10.5m, 12.5m and 14.5m respectively are considered in order to illustrate the analysis presented in this study. Similarly, the load is assumed to travel along the beam with different speeds (c) = 50 m/s, 75 m/s, 100 m/s and 125 m/s respectively. The Young modulus of elasticity $E = 2.10924 \times 10^9 Kg/m$, moment of inertia I = 2.87698×10^{-3}, $\pi = 22/7$, the damping coefficient $Co = 300000$ and the mass per unit length of the beam $M = 2758.291 kg/m$. The values of axial force N are varied between $0\ N$ *and* $3000000\ N$.

In this present study, three special cases of effect of axial force on dynamic response of a simply supported damped shear beam under the action of moving load were investigated. The cases are termed;

1. the effect of axial force N on transverse displacement and rotation of a damped shear beam when the lengths of the beam (L) are 8.5m, 10.5m, 12.5m and 14.5m respectively,
2. the effect of axial force N on transverse displacement and rotation of a damped shear beam when the load speeds (c) are 50 m/s, 75 m/s, 100 m/s and 125 m/s respectively and
3. the effect of axial force N on critical velocity.

The transverse displacement V and rotation ϕ of the beam are calculated and plotted against time t for various values of axial force N. The results are shown on the various graphs given below.

Figures 1 to 8 describe the transverse displacement and rotation of a simply supported damped shear beam under the action of moving load travelling at constant velocity for various values of axial force N when the lengths of the beam (L) are 8.5m, 10.5m, 12.5m and 14.5m respectively and for the fixed values of other parameters. It is clearly seen from the figures 1 to 8 that as the value of axial force N increases, the transverse displacement and rotation of the beam decrease noticeably. Consequently, increase in the value of axial force N stiffens the beam, counteracts the bending effect and thus significantly reduces the transverse displacement and rotation of the vibrating beam. Hence, the presence of axial force N increases the overall rigidity of the beam system.

Similarly, the response amplitude profile of a simply supported uniform damped shear beam subjected to moving load for various values of axial force N when the load speeds (c) are 50 m/s, 75 m/s, 100 m/s and 125 m/s respectively and for the fixed values of other parameters are presented in Figures 9 to 16. From the graphs, similar result is obtained. It is observed that increase in the value of axial force N reduces the transverse displacement and rotation of the vibrating beam considerably. In actual practices, this implies that as the value of axial force N increases, the tensile stresses present in the vibrating beam reduce significantly. This makes the beam to become more inflexible and stable to resist the lateral deflection and intensive vibration, allowing it to carry larger transverse loads even at high velocities without buckling. Thus, the likelihood of flexural cracking or bending of the beam system is minimized.

Finally, the effect of axial force N on the critical velocity of a simply supported damped shear beam traversed by moving load is presented in figure 17. It is evident from the graph that for various values of the axial force N and for the fixed values of other parameters, the higher the value of the axial force N, the higher the critical velocity of the beam. Practically speaking, this indicates that increase in the value of axial force N makes the beam more unsusceptible to buckling and vibrations. It enhances the dynamic stability of the beam system, thereby minimizing the risk of resonance and ensuring the safety of the structure's occupants.

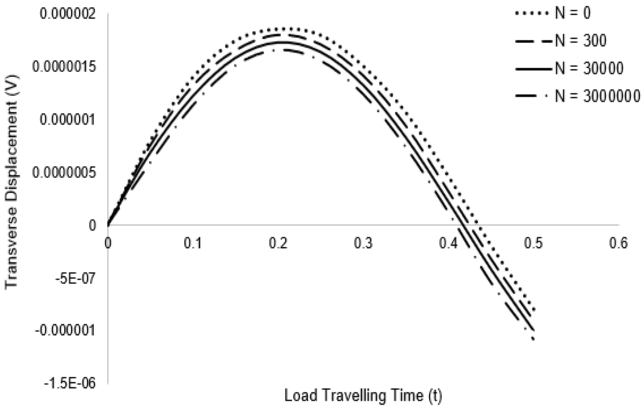

Figure 1: Transverse displacement of a simply supported damped shear beam under the action of moving load for various values of N when the beam length (L) = 8.5 and for fixed values of $\mathbf{K = 40000000}$, $\mathbf{G = 40000000}$ and $\mathbf{Co = 300000}$

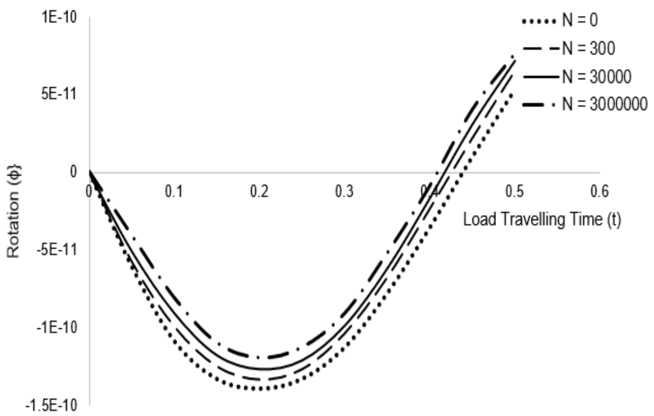

Figure 2: Rotation of a simply supported damped shear beam under the action of moving load for various values of N when the beam length (L) = 8.5 and for fixed values of $\boldsymbol{K = 40000000}$, $\boldsymbol{G = 40000000}$ and $\boldsymbol{Co = 300000}$.

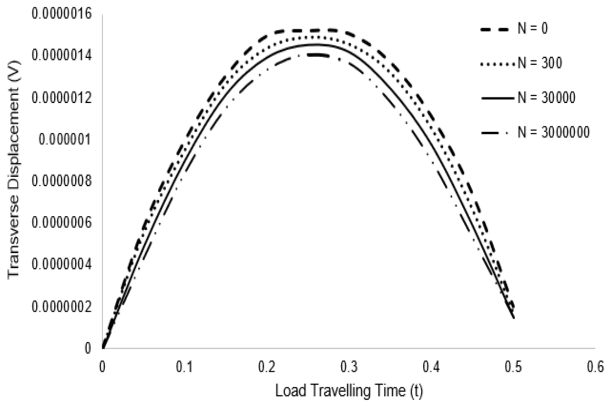

Figure 3: Transverse displacement of a simply supported damped shear beam under the action of moving load for various values of N when the beam length (L) = 10.5 and for fixed values of $\boldsymbol{K = 40000000}$, $\boldsymbol{G = 40000000}$ and $\boldsymbol{Co = 300000}$.

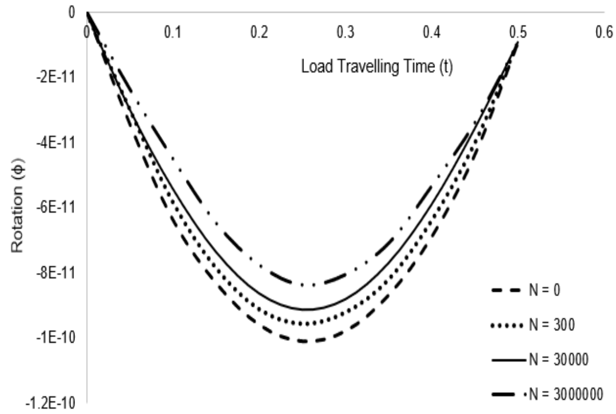

Figure 4: Rotation of a simply supported damped shear beam under the action of moving load for various values of N when the beam length (L) = 10.5 and for fixed values of $\boldsymbol{K = 40000000}$, $\boldsymbol{G = 40000000}$ and $\boldsymbol{Co = 300000}$.

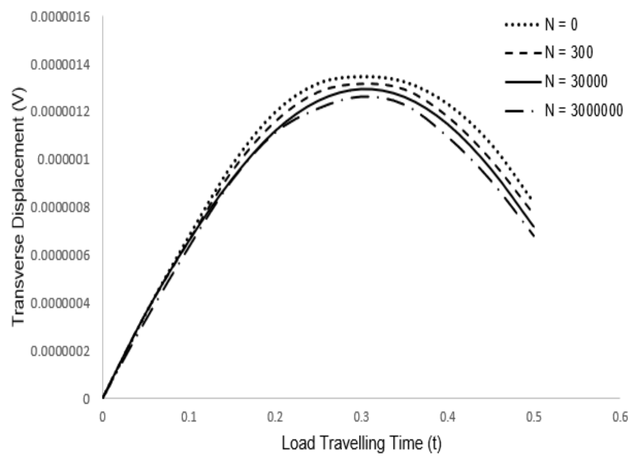

Figure 5: Transverse displacement of a simply supported damped shear beam under the action of moving load for various values of N when the beam length (L) = 12.5 and for fixed values of $\boldsymbol{K = 40000000}$, $\boldsymbol{G = 40000000}$ and $\boldsymbol{Co = 300000}$.

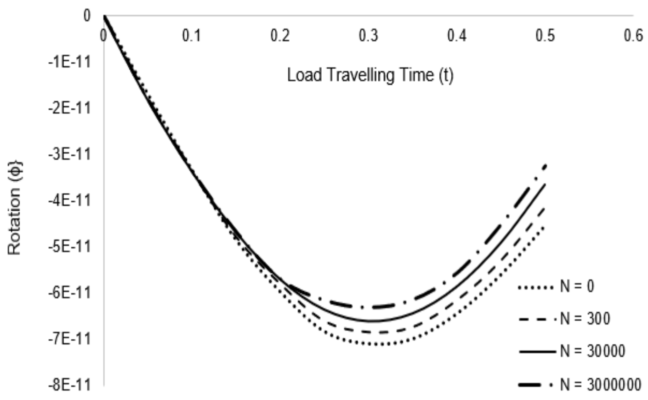

Figure 6: Rotation of a simply supported damped shear beam under the action of moving load for various values of N when the beam length (L) = 12.5 and for fixed values of $K = 40000000$, $G = 40000000$ and $Co = 300000$.

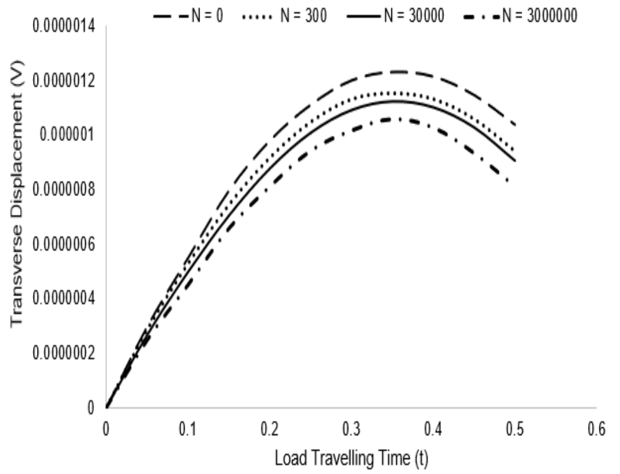

Figure 7: Transverse displacement of a simply supported damped shear beam under the action of moving load for various values of N when the beam length (L) = 14.5 and for fixed values of $K = 40000000$, $G = 40000000$ and $Co = 300000$

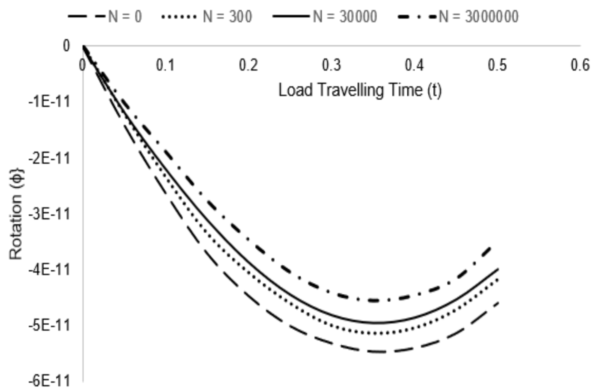

Figure 8: Rotation of a simply supported damped shear beam under the action of moving load for various values of N when the beam length (L) = 14.5 and for fixed values of $K = 40000000$, $G = 40000000$ and $Co = 300000$.

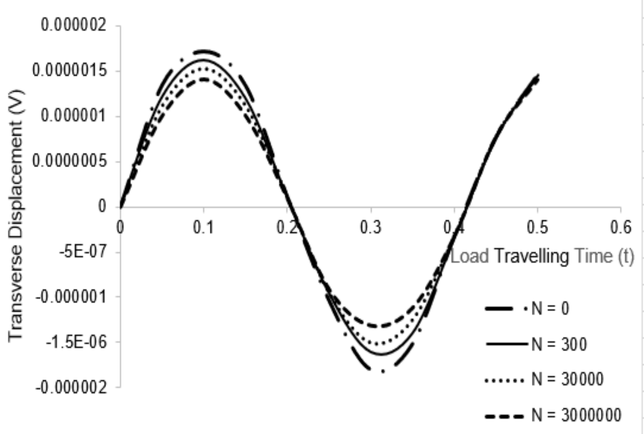

Figure 9: Transverse displacement of a simply supported damped shear beam under the action of moving load for various values of N when the load speed (c) = 50 and for fixed values of $K = 40000000$, $G = 40000000$ and $Co = 300000$

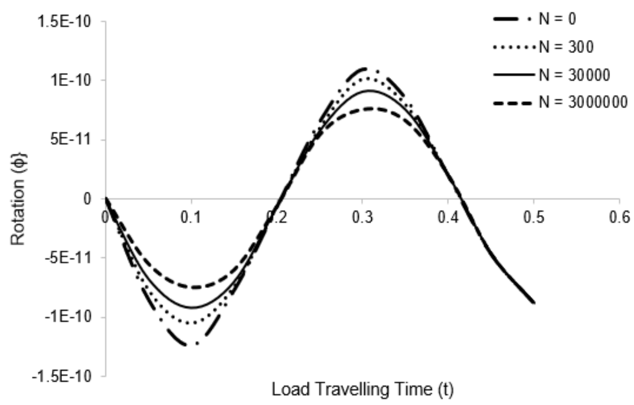

Figure 10: Rotation of a simply supported damped shear beam under the action of moving load for various values of N when the load speed (c) = 50 and for fixed values of $K = 40000000$, $G = 40000000$ and $Co = 300000$.

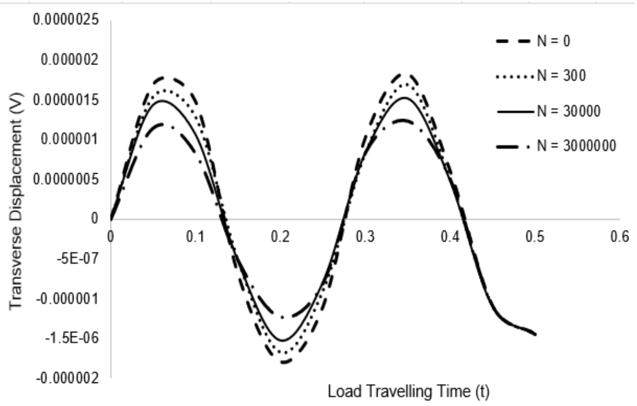

Figure 11: Transverse displacement of a simply supported damped shear beam under the action of moving load for various values of N when the load speed (c) = 75 and for fixed values of $K = 40000000$, $G = 40000000$ and $Co = 300000$.

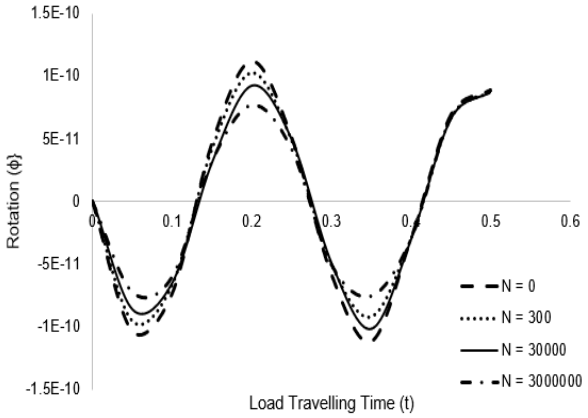

Figure 12: Rotation of a simply supported damped shear beam under the action of moving load for various values of N when the load speed (c) = 75 and for fixed values of $K = 40000000$, $G = 40000000$ and $Co = 300000$.

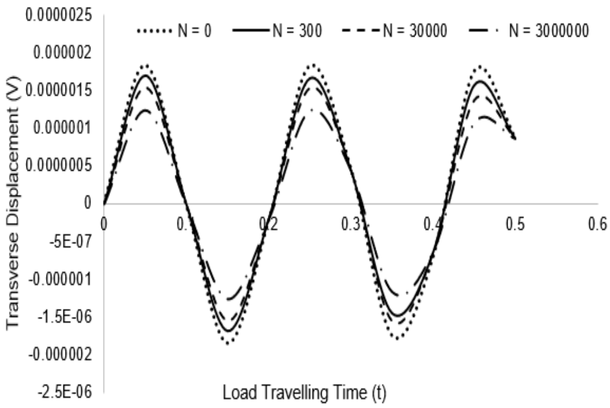

Figure 13: Transverse displacement of a simply supported damped shear beam under the action of moving load for various values of N when the load speed (c) = 100 and for fixed values of $\mathrm{K} = 40000000$, $\mathrm{G} = 40000000$ and $\mathrm{Co} = 300000$.

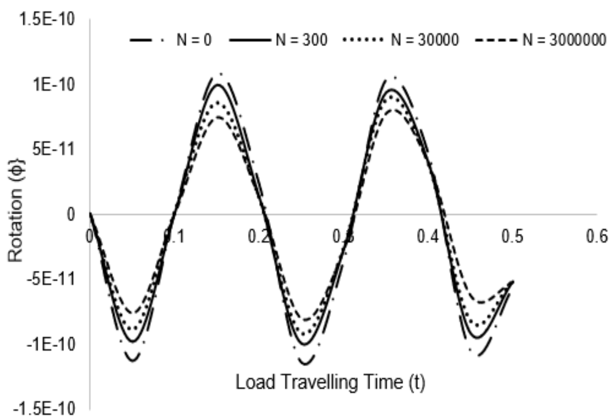

Figure 14: Rotation of a simply supported damped shear beam under the action of moving load for various values of N when the load speed (c) = 100 and for fixed values of $K = 40000000$, $G = 40000000$ and $Co = 300000$.

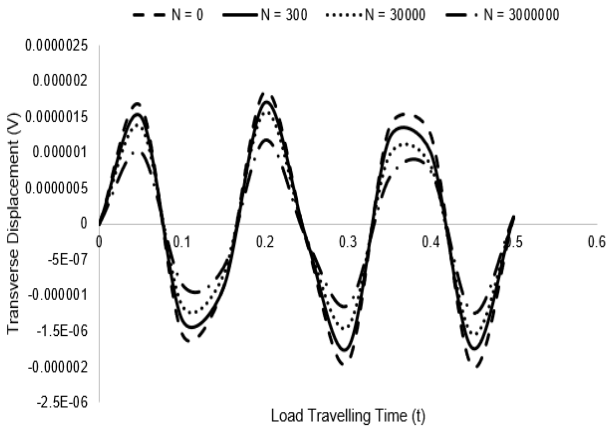

Figure 15: Transverse displacement of a simply supported damped shear beam under the action of moving load for various values of N when the load speed (c) = 125 and for fixed values of $K = 40000000$, $G = 40000000$ and $Co = 300000$.

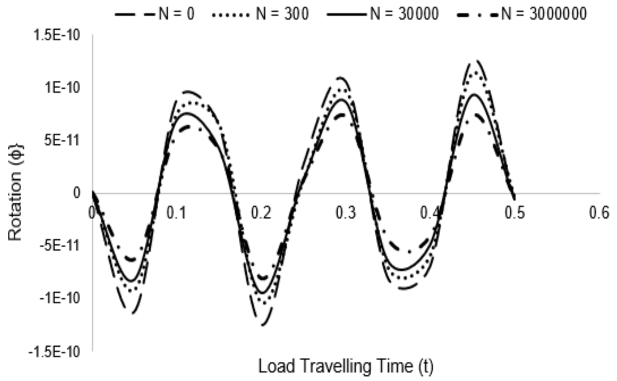

Figure 16: Rotation of a simply supported damped shear beam under the action of moving load for various values of N when the load speed (c) = 125 and for fixed values of $K = 40000000$, $G = 40000000$ and $Co = 300000$.

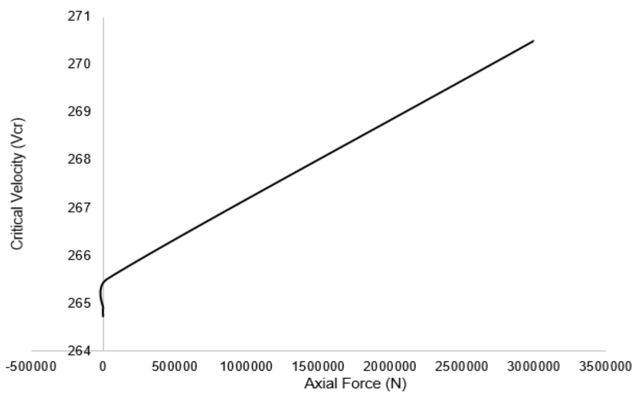

Figure 17: Variation of the critical velocity against axial force N

IV. CONCLUSION

This paper presents axial force influence on transverse displacement and rotation under moving of a damped shear beam supported by an elastic foundation. A solution methodology involving finite Fourier transform techniques and Laplace transformation, along with convolution theory, is employed to obtain the solution for the coupled second-order partial differential equations that characterize the motion of the beam-load system. Thorough analyses are performed to explore the influence of axial force on the transverse displacement and rotation of damped shear beams of different length sizes when subjected to a moving load traversing at different velocities. Furthermore, the research investigates how axial force influences the critical velocities of the vibrating system. The plotted graphs distinctly demonstrate that axial force considerably improves the stability of the beam under the moving load. The results indicate that both the transverse displacement and rotation of the beam decrease significantly as axial force increases. Additionally, it is observed that higher axial force values correspond to increased critical velocities, suggesting a more secure dynamic system. Consequently, in the design of engineering structures such as bridges, pipelines, railway tracks, aerospace components, railway bridges, overhead cranes, cableways and tunnels, the profound impact of axial force on the critical velocity and dynamic stability of the structures should be put into considerations to guarantee the safety, reliability and efficiency of the design.

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