

Reduced Temperature Indices

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ARTICLE INFO	ABSTRACT
Published Online: 16 April 2025	In this paper, we introduce the first and second reduced temperature indices and their corresponding polynomials of a graph. Also we compute these reduced temperature indices
Corresponding Author: V.R. Kulli	for some standard graphs, wheel graphs, friendship graphs, HC_5C_7 [p, q] nanotubes and tetrameric 1,3-adamantane.
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I. INTRODUCTION

In this paper, we consider only finite, simple, connected graphs. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$. The degree d_u of a vertex u is the number of vertices adjacent to u . For basic notations and terminologies, we refer [1].

A topological index is a numerical parameter mathematically derived from the graph structure. Many topological indices have been considered in Theoretical Chemistry and several topological indices were defined by using vertex degree concept [2]. The Zagreb, Temperature, E-Banhatti, Domination, Nirmala indices are the most degree based topological indices in Chemical Graph Theory, see [3-44]. Topological indices have their applications in various disciplines in Science and Technology [45, 46].

In [47], Fajtlowicz defined the temperature of a vertex u of a graph G as

$$T(u) = \frac{d_u}{n - d_u}, \text{ where } |V(G)| = n.$$

We define the reduced temperature of a vertex u of a graph G as

$$RT(u) = \frac{d_u - 1}{n - (d_u - 1)} = \frac{d_u - 1}{n - d_u + 1} \text{ where } |V(G)| = n.$$

The first and second reduced temperature indices of a graph G are defined as

$$RT_1(G) = \sum_{uv \in E(G)} [RT(u) + RT(v)],$$

$$RT_2(G) = \sum_{uv \in E(G)} RT(u)RT(v).$$

Considering the first and second reduced temperature indices of a graph, we define the first and second reduced temperature polynomials of a graph as

$$RT_1(G, x) = \sum_{uv \in E(G)} x^{RT(u) + RT(v)},$$

$$RT_2(G, x) = \sum_{uv \in E(G)} x^{RT(u)RT(v)}.$$

. In this paper, we compute the first and second reduced temperature indices and their corresponding polynomials for some standard graphs, wheel graphs, friendship graphs, HC_5C_7 [p, q] nanotubes and tetrameric 1,3-adamantane.

II. RESULTS FOR SOME STANDARD GRAPHS

Proposition 1. Let G be r -regular with n vertices and $r \geq 2$. Then

$$RT_1(G) = \frac{nr(r-1)}{n-r+1}.$$

Proof: Let G be an r -regular graph with n vertices and $r \geq 2$ and $\frac{nr}{2}$ edges. Then $RT(u) = \frac{r-1}{n-r+1}$.

$$\begin{aligned} RT_1(G) &= \sum_{uv \in E(G)} (RT(u) + RT(v)) \\ &= \frac{nr}{2} \left(\frac{r-1}{n-r+1} + \frac{r-1}{n-r+1} \right) \end{aligned}$$

$$= \frac{nr(r-1)}{n-r+1}.$$

Corollary 1.1. Let C_n be a cycle with $n \geq 3$ vertices. Then

$$RT_1(C_n) = \frac{2n}{n-1}.$$

Corollary 1.2. Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$RT_1(K_n) = \frac{1}{2}n(n-1)(n-2).$$

Proposition 2. Let G be r -regular with n vertices and $r \geq 2$. Then

$$RT_2(G) = \frac{nr(r-1)^2}{2(n-r+1)^2}.$$

Proof: Let G be an r -regular graph with n vertices and $r \geq 2$

and $\frac{nr}{2}$ edges. Then $RT(u) = \frac{r-1}{n-r+1}$.

$$RT_2(G) = \sum_{uv \in E(G)} RT(u)RT(v)$$

$$= \frac{nr}{2} \left(\frac{r-1}{n-r+1} \times \frac{r-1}{n-r+1} \right)$$

$$= \frac{nr(r-1)^2}{2(n-r+1)^2}$$

Corollary 2.1. Let C_n be a cycle with $n \geq 3$ vertices. Then

$$RT_2(C_n) = \frac{n}{(n-1)^2}.$$

Corollary 2.2. Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$RT_2(K_n) = \frac{n(n-2)^2}{2(n-1)^2}.$$

Proposition 3. Let P_n be a path with $n \geq 3$ vertices. Then

$$\begin{aligned} RT_1(P_n) &= 2 \left[\left(\frac{1-1}{n-1+1} \right) + \left(\frac{2-1}{n-2+1} \right) \right] \\ &\quad + (n-3) \left[\left(\frac{2-1}{n-2+1} \right) + \left(\frac{2-1}{n-2+1} \right) \right] \\ &= \frac{2(n-2)}{(n-1)}. \end{aligned}$$

Proposition 4. Let P_n be a path with $n \geq 3$ vertices. Then

$$\begin{aligned} RT_2(P_n) &= 2 \left[\left(\frac{1-1}{n-1+1} \right) \times \left(\frac{2-1}{n-2+1} \right) \right] \\ &\quad + (n-3) \left[\left(\frac{2-1}{n-2+1} \right) \times \left(\frac{2-1}{n-2+1} \right) \right] \\ &= \frac{(n-3)}{(n-1)^2}. \end{aligned}$$

Proposition 5. Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$ and $n \geq 2$. Then

$$RT_1(K_{m,n}) = \frac{mn(m^2 + n^2 - 2)}{(m+1)(n+1)}.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m+n$ vertices and mn edges such that $|V_1| = m$, $|V_2| = n$, $V(K_{r,s}) = V_1 \square V_2$ for $1 \leq m \leq n$, and $n \geq 2$. Every vertex of V_1 is incident with n edges and every vertex of V_2 is incident with m edges.

$$\begin{aligned} RT_1(G) &= \sum_{uv \in E(G)} [RT(u) + RT(v)] \\ &= mn \left[\left(\frac{m-1}{m+n-m+1} \right) + \left(\frac{n-1}{m+n-n+1} \right) \right] \\ &= \frac{mn(m^2 + n^2 - 2)}{(m+1)(n+1)}. \end{aligned}$$

Corollary 5.1. Let $K_{n,n}$ be a complete bipartite graph with $n \geq 2$. Then

$$RT_1(K_{n,n}) = \frac{2n^2(n-1)}{(n+1)}.$$

Corollary 5.2. Let $K_{1,n}$ be a star with $n \geq 2$. Then

$$RT_1(K_{1,n}) = \frac{1}{2}n(n-1).$$

Proposition 6. Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$ and $n \geq 2$. Then

$$RT_1(K_{m,n}) = \frac{mn(m^2 + n^2 - 2)}{(m+1)(n+1)}.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m+n$ vertices and mn edges such that $|V_1| = m$, $|V_2| = n$, $V(K_{r,s}) = V_1 \square V_2$ for $1 \leq m \leq n$, and $n \geq 2$. Every vertex of V_1 is incident with n edges and every vertex of V_2 is incident with m edges.

$$\begin{aligned} RT_2(G) &= \sum_{uv \in E(G)} RT(u)RT(v) \\ &= mn \left[\left(\frac{m-1}{m+n-m+1} \right) \times \left(\frac{n-1}{m+n-n+1} \right) \right] \\ &= \frac{mn(m-1)(n-1)}{(m+1)(n+1)}. \end{aligned}$$

Corollary 6.1. Let $K_{n,n}$ be a complete bipartite graph with $n \geq 2$. Then

$$RT_2(K_{n,n}) = \frac{n^2(n-1)^2}{(n+1)^2}.$$

Corollary 6.2. Let $K_{1,n}$ be a star with $n \geq 2$. Then

$$RT_2(K_{1,n}) = 0.$$

III. RESULTS FOR WHEEL GRAPHS

A wheel graph W_n is the join of C_n and K_1 . Then W_n has $n+1$ vertices and $2n$ edges. A graph W_n is presented in Figure 1.

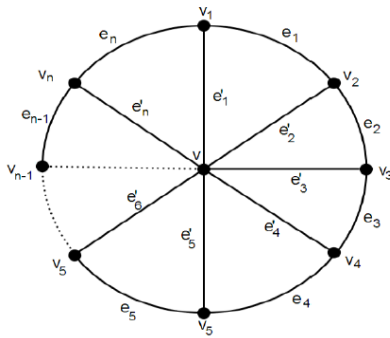


Figure 1. Wheel graph W_n

In W_n , there are two types of edges as follows:

$$E_1 = \{uv \in E(W_n) \mid d(u) = d(v) = 3\}, \quad |E_1| = n.$$

$$E_2 = \{uv \in E(W_n) \mid d(u) = 3, d(v) = n\}, \quad |E_2| = n.$$

Therefore, in W_n , there are two types of reduced temperature edges based on the reduced temperature degrees of end vertices of each edge as follows:

$$RTE_1 = \{uv \in E(W_n) \mid RT(u) = RT(v) = \frac{2}{n-1}\}, \quad |RTE_1| = n.$$

$$RTE_2 = \{uv \in E(W_n) \mid RT(u) = \frac{2}{n-1}, RT(v) = \frac{n-1}{2}\}, \quad |RTE_2| = n.$$

We calculate the first reduced temperature index and its polynomial form of a wheel graph W_n as follows:

Theorem 1. Let W_n be a wheel graph. Then

$$(i) \quad RT_1(W_n) = \frac{6n}{n-1} + \frac{n(n-1)}{2}.$$

$$(ii) \quad RT_1(W_n, x) = nx^{\frac{4}{n-1}} + nx^{\frac{4+(n-1)^2}{2(n-1)}}.$$

Proof: Applying definitions and reduced temperature edge partition of W_n , we conclude

$$\begin{aligned} RT_1(W_n) &= \sum_{uv \in E(W_n)} [RT(u) + RT(v)] \\ &= n \left[\left(\frac{2}{n-1} \right) + \left(\frac{2}{n-1} \right) \right] + n \left[\left(\frac{2}{n-1} \right) + \left(\frac{n-1}{2} \right) \right]. \end{aligned}$$

By simplifying the above equation, we get the desired result.

$$\begin{aligned} (ii) \quad RT_1(W_n, x) &= \sum_{uv \in E(W_n)} x^{RT(u)+RT(v)} \\ &= nx^{\left(\frac{2}{n-1} \right) + \left(\frac{2}{n-1} \right)} + nx^{\left(\frac{2}{n-1} \right) + \left(\frac{n-1}{2} \right)}. \end{aligned}$$

By simplifying the above equation, we obtain the desired result.

We calculate the second reduced temperature index and its polynomial form of a wheel graph W_n as follows:

Theorem 2. Let W_n be a wheel graph. Then

$$(i) \quad RT_2(W_n) = \frac{4n}{(n-1)^2} + n.$$

$$(ii) \quad RT_2(W_n, x) = nx^{\frac{4}{(n-1)^2}} + nx^1.$$

Proof: Applying definition and reduced temperature edge partition of W_n , we conclude

$$\begin{aligned} RT_2(W_n) &= \sum_{uv \in E(W_n)} RT(u)RT(v) \\ &= n \left[\left(\frac{2}{n-1} \right) \times \left(\frac{2}{n-1} \right) \right] + n \left[\left(\frac{2}{n-1} \right) \times \left(\frac{n-1}{2} \right) \right]. \end{aligned}$$

By simplifying the above equation, we derive the desired result.

$$\begin{aligned} (ii) \quad RT_2(W_n, x) &= \sum_{uv \in E(W_n)} x^{RT(u)RT(v)} \\ &= nx^{\left(\frac{2}{n-1} \right) \times \left(\frac{2}{n-1} \right)} + nx^{\left(\frac{2}{n-1} \right) \times \left(\frac{n-1}{2} \right)}. \end{aligned}$$

By simplifying the above equation, we get the desired result.

IV. RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph F_4 is shown in Figure 2. A friendship graph F_n is a graph with $2n+1$ vertices and $3n$ edges.

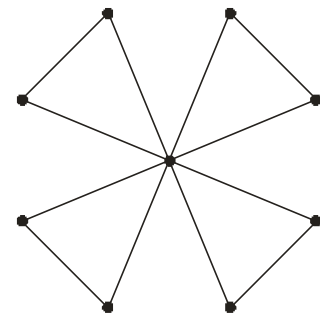


Figure 2. Friendship graph F_4

In F_n , there are two types of edges as follows:

$$E_1 = \{uv \in E(F_n) \mid d_{F_n}(u) = d_{F_n}(v) = 2\}, \quad |E_1| = n.$$

$$E_2 = \{uv \in E(F_n) \mid d_{F_n}(u) = 2, d_{F_n}(v) = 2n\}, \quad |E_2| = 2n.$$

Therefore, in F_n , we obtain that $\{RT(u), RT(v) : uv \in E(F_n)\}$ has two reduced temperature edge set partitions.

$$RTE_1 = \{uv \in E(F_n) \mid RT(u) = RT(v) = \frac{1}{2n}\},$$

$$|RTE_1| = n.$$

$$RTE_2 = \{uv \in E(F_n) \mid RT(u) = \frac{1}{2n}, RT(v) = \frac{2n-1}{2}\},$$

$$|RTE_2| = 2n.$$

We calculate the first reduced temperature index and its polynomial form of a friendship graph F_n as follows:

Theorem 3. Let F_n be a friendship graph. Then

$$(i) \quad RT_1(F_n) = 2 + n(2n-1),$$

$$(ii) \quad RT_1(F_n, x) = nx^n + 2nx^{\frac{1+n(2n-1)}{2n}}.$$

Proof: Applying definition and Banhatti edge partition of F_n , we conclude

$$(i) \quad RT_1(F_n) = \sum_{uv \in E(F_n)} [RT(u) + RT(v)]$$

$$= n \left[\left(\frac{1}{2n} \right) + \left(\frac{1}{2n} \right) \right] + 2n \left[\left(\frac{1}{2n} \right) + \left(\frac{2n-1}{2} \right) \right].$$

By simplifying the above equation, we get the desired result.

$$(ii) \quad RT_1(F_n, x) = \sum_{uv \in E(F_n)} x^{RT(u)+RT(v)}$$

$$= nx^{\frac{1}{2n} + \frac{1}{2n}} + 2nx^{\frac{1}{2n} + \frac{2n-1}{2}}.$$

By simplifying the above equation, we obtain the desired result.

We calculate the second reduced temperature index and its polynomial form of a friendship graph F_n as follows:

Theorem 4. Let F_n be a friendship graph. Then

$$(i) \quad RT_2(F_n) = \frac{1}{4n} + \frac{2n-1}{2}.$$

$$(ii) \quad RT_2(F_n, x) = nx^{\frac{1}{4n^2}} + 2nx^{\frac{2n-1}{4n}}.$$

Proof: Applying definitions and reduced temperature edge partition of F_n , we conclude

$$(i) \quad RT_2(F_n) = \sum_{uv \in E(F_n)} RT(u)RT(v)$$

$$= n \left[\left(\frac{1}{2n} \right) \times \left(\frac{1}{2n} \right) \right] + 2n \left[\left(\frac{1}{2n} \right) \times \left(\frac{2n-1}{2} \right) \right].$$

By simplifying the above equation, we get the desired result.

$$(ii) \quad RT_2(F_n, x) = \sum_{uv \in E(F_n)} x^{RT(u)RT(v)}$$

$$= nx^{\frac{1}{2n} \times \frac{1}{2n}} + 2nx^{\frac{1}{2n} \times \frac{2n-1}{2}}.$$

By simplifying the above equation, we obtain the desired result.

V. RESULTS FOR $HC_5C_7[p, q]$ NANOTUBES

In this section, we consider $HC_5C_7[p, q]$ nanotubes in which p is the number of heptagons in the first row and q rows of pentagons repeated alternately. The 2-D lattice of $HC_5C_7[8, 4]$ nanotube is presented in Figure 3.

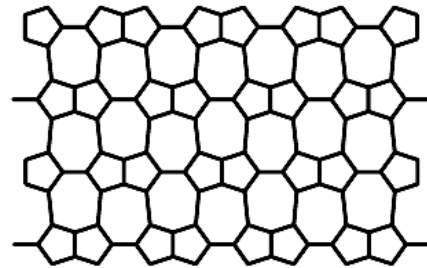


Figure 3. 2-D lattice of $HC_5C_7[8, 4]$ nanotube

Let G be a graph of a nanotube $HC_5C_7[p, q]$. By calculation, G has $4pq$ vertices and $6pq - p$ edges. Clearly, G has two types of edges based on the degree of end vertices of each edge as follows:

$$E_1 = \{uv \in E(G) \mid d_G(u)=2, d_G(v)=3\}, \quad |E_1| = 4p$$

$$E_2 = \{uv \in E(G) \mid d_G(u)=d_G(v)=3\}, \quad |E_2| = 6pq - 5p.$$

Therefore, in $TA[n]$, there are two types of edges based on the reduced temperature of end vertices of each edge as follows:

$$RTE_1 = \{uv \in E(G) \mid RT(u) = \frac{1}{4pq-1}, RT(v) = \frac{2}{4pq-2}\},$$

$$|RTE_1| = 4p.$$

$$RTE_2 = \{uv \in E(G) \mid RT(u) = \frac{2}{4pq-2}, RT(v) = \frac{2}{4pq-2}\},$$

$$|RTE_2| = 6pq - 5p.$$

Theorem 5. The first reduced temperature index of a nanotube $HC_5C_7[p, q]$ is

$$RT_1(G) = \frac{8p(3pq-1)}{(4pq-1)(2pq-1)} + \frac{2(6pq-5p)}{(2pq-1)}.$$

Proof: Let $G = HC_5C_7[p, q]$. We have

$$RT_1(G) = \sum_{uv \in E(G)} (RT(u) + RT(v))$$

$$= 4p \left(\frac{1}{4pq-1} + \frac{2}{4pq-2} \right)$$

$$+ (6pq-5p) \left(\frac{2}{4pq-2} + \frac{2}{4pq-2} \right)$$

$$= \frac{8p(3pq-1)}{(4pq-1)(2pq-1)} + \frac{2(6pq-5p)}{(2pq-1)}.$$

Theorem 6. The first reduced temperature polynomial of a nanotube $HC_5C_7[p, q]$ is

$$RT_1(G, x) = 4px^{\left(\frac{1}{4pq-1} + \frac{2}{4pq-2}\right)} + (6pq-5p)x^{\frac{2}{2pq-1}}.$$

Proof: Let $G = HC_5C_7[p, q]$. We have

$$\begin{aligned} RT_1(G, x) &= \sum_{uv \in E(G)} x^{RT(u)+RT(v)} \\ &= 4px^{\left(\frac{1}{4pq-1} + \frac{2}{4pq-2}\right)} + (6pq-5p)x^{\left(\frac{2}{4pq-2} + \frac{2}{4pq-2}\right)} \\ &= 4px^{\left(\frac{1}{4pq-1} + \frac{2}{4pq-2}\right)} + (6pq-5p)x^{\frac{2}{2pq-1}} \end{aligned}$$

Theorem 7. The second reduced temperature index of a nanotube $HC_5C_7[p, q]$ is

$$RT_2(G) = \frac{4p}{(4pq-1)(2pq-1)} + \frac{(6pq-5p)}{(2pq-1)^2}.$$

Proof: Let $G = HC_5C_7[p, q]$. We have

$$\begin{aligned} RT_2(G) &= \sum_{uv \in E(G)} RT(u)RT(v) \\ &= 4p\left(\frac{1}{4pq-1} \times \frac{2}{4pq-2}\right) \\ &\quad + (6pq-5p)\left(\frac{2}{4pq-2} \times \frac{2}{4pq-2}\right) \\ &= \frac{4p}{(4pq-1)(2pq-1)} + \frac{(6pq-5p)}{(2pq-1)^2}. \end{aligned}$$

Theorem 8. The second reduced temperature polynomial of a nanotube $HC_5C_7[p, q]$ is

$$RT_2(G, x) = 4px^{\frac{1}{(4pq-1)(2pq-1)}} + (6pq-5p)x^{\frac{2}{(2pq-1)^2}}.$$

Proof: Let $G = HC_5C_7[p, q]$. We have

$$\begin{aligned} RT_2(G, x) &= \sum_{uv \in E(G)} x^{RT(u)RT(v)} \\ &= 4px^{\left(\frac{1}{4pq-1} \times \frac{2}{4pq-2}\right)} + (6pq-5p)x^{\left(\frac{2}{4pq-2} \times \frac{2}{4pq-2}\right)} \\ &= 4px^{\frac{1}{(4pq-1)(2pq-1)}} + (6pq-5p)x^{\frac{2}{(2pq-1)^2}} \end{aligned}$$

VI. RESULTS FOR TETRAMERIC 1,3-ADAMANTANE

In Chemistry, diamondoids are variants of the carbon cage known as adamantane (C_{10}, H_{16}), the smallest unit cage structure of the diamond crystal lattice. We focus on the molecular structure of the family of tetrameric 1,3-adamantane and it is denoted by $TA[n]$. Let G be the graph

of tetrameric 1,3-adamantane $TA[n]$. The graph of tetrameric 1,3-adamantane $TA[4]$ is depicted in Figure 4.

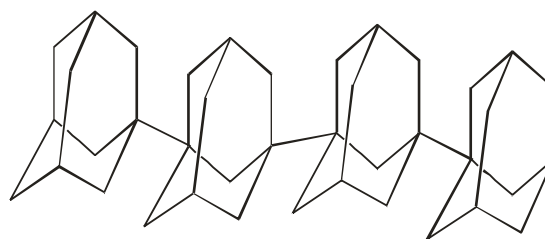


Figure 4. The graph of tetrameric 1,3-adamantane $TA[4]$

By calculation, G has $10n$ vertices and $13n - 1$ edges. Also by calculation, we obtain three edge partitions of G based on the degrees of the end vertices of each edge as follows:

$$E_1 = \{uv \in E(G) \mid d_G(u)=2, d_G(v)=3\}, |E_1| = 6n + 6.$$

$$E_2 = \{uv \in E(G) \mid d_G(u)=2, d_G(v)=4\}, |E_2| = 6n - 6.$$

$$E_3 = \{uv \in E(G) \mid d_G(u)=d_G(v)=4\}, |E_3| = n - 1.$$

Therefore, in $TA[n]$, there are three types of edges based on the reduced temperature of end vertices of each edge as follows:

$$\begin{aligned} RTE_1 &= \{uv \in E(G) \mid RT(u) = \frac{1}{10n-1}, RT(v) = \frac{2}{10n-2}\}, |RE_1| \\ &= 6n + 6. \end{aligned}$$

$$\begin{aligned} RTE_2 &= \{uv \in E(G) \mid RT(u) = \frac{1}{10n-1}, RT(v) = \frac{3}{10n-3}\}, |RE_2| \\ &= 6n - 6. \end{aligned}$$

$$RTE_3 = \{uv \in E(G) \mid RT(u)=RT(v) = \frac{3}{10n-3}\}, |RE_3| = n - 1.$$

Theorem 9. The first reduced temperature index of a tetrameric 1,3-adamantane $TA[n]$ is

$$\begin{aligned} RT_1(G) &= \left(\frac{15n-2}{(10n-1)(5n-1)}\right)(6n+6) \\ &\quad + \left(\frac{40n-6}{(10n-1)(10n-3)}\right)(6n-6) + \frac{6(n-1)}{10n-3}. \end{aligned}$$

Proof: Let $G = TA[n]$. We see that

$$\begin{aligned} RT_1(G) &= \sum_{uv \in E(G)} (RT(u) + RT(v)) \\ &= \left[\left(\frac{1}{10n-1}\right) + \left(\frac{2}{10n-2}\right)\right](6n+6) \\ &\quad + \left[\left(\frac{1}{10n-1}\right) + \left(\frac{3}{10n-3}\right)\right](6n-6) \\ &\quad + \left[\left(\frac{3}{10n-3}\right) + \left(\frac{3}{10n-3}\right)\right](n-1) \end{aligned}$$

$$= \left(\frac{15n-2}{(10n-1)(5n-1)} \right) (6n+6) \\ + \left(\frac{40n-6}{(10n-1)(10n-3)} \right) (6n-6) \\ + \frac{6(n-1)}{10n-3}.$$

Theorem 10. The first reduced temperature polynomial of a tetrameric 1,3-adamantane $TA[n]$ is

$$RT_1(G, x) = (6n+6)x^{\frac{15n-2}{(10n-1)(5n-1)}} \\ + (6n-6)x^{\frac{40n-6}{(10n-1)(10n-3)}} + (n-1)x^{\frac{6}{10n-3}}.$$

Proof: Let $G = TA[n]$. We have

$$RT_1(G, x) = \sum_{uv \in E(G)} x^{RT(u)+RT(v)} \\ = (6n+6)x^{\frac{1}{10n-1} + \frac{2}{10n-2}} + (6n-6)x^{\frac{1}{10n-1} + \frac{3}{10n-3}} \\ + (n-1)x^{\frac{3}{10n-3} + \frac{3}{10n-3}} \\ = (6n+6)x^{\frac{15n-2}{(10n-1)(5n-1)}} + (6n-6)x^{\frac{40n-6}{(10n-1)(10n-3)}} \\ + (n-1)x^{\frac{6}{10n-3}}.$$

Theorem 11. The second reduced temperature index of a tetrameric 1,3-adamantane $TA[n]$ is

$$RT_2(G) = \left(\frac{1}{(10n-1)(5n-1)} \right) (6n+6) \\ + \left(\frac{3}{(10n-1)(10n-3)} \right) (6n-6) + \frac{9(n-1)}{(10n-3)^2}.$$

Proof: Let $G = TA[n]$. We have

$$RT_2(G) = \sum_{uv \in E(G)} RT(u)RT(v) \\ = \left[\left(\frac{1}{10n-1} \right) \times \left(\frac{2}{10n-2} \right) \right] (6n+6) \\ + \left[\left(\frac{1}{10n-1} \right) \times \left(\frac{3}{10n-3} \right) \right] (6n-6) \\ + \left[\left(\frac{3}{10n-3} \right) \times \left(\frac{3}{10n-3} \right) \right] (n-1) \\ = \left(\frac{1}{(10n-1)(5n-1)} \right) (6n+6) \\ + \left(\frac{3}{(10n-1)(10n-3)} \right) (6n-6) \\ + \left(\frac{9}{(10n-3)^2} \right) (n-1).$$

$$+ \frac{9(n-1)}{(10n-3)^2}.$$

Theorem 12. The second reduced temperature polynomial of a tetrameric 1,3-adamantane $TA[n]$ is

$$RT_2(G, x) = 4px^{\frac{1}{(4pq-1)(2pq-1)}} + (6pq-5p)x^{\frac{2}{(2pq-1)^2}}.$$

Proof: Let $G = TA[n]$. We have

$$RT_2(G, x) = \sum_{uv \in E(G)} x^{RT(u)+RT(v)} \\ = (6n+6)x^{\frac{1}{10n-1} + \frac{2}{10n-2}} + (6n-6)x^{\frac{1}{10n-1} + \frac{3}{10n-3}} \\ + (n-1)x^{\frac{3}{10n-3} + \frac{3}{10n-3}} \\ = (6n+6)x^{\frac{1}{(10n-1)(5n-1)}} + (6n-6)x^{\frac{3}{(10n-1)(10n-3)}} \\ + (n-1)x^{\frac{9}{(10n-3)^2}}.$$

VII. CONCLUSION

In this paper, we have introduced the first and second reduced temperature indices of a graph. We have computed these indices for some standard graphs, wheel graphs, friendship graphs, $HC_5C_7[p, q]$ nanotubes and tetrameric 1,3-adamantane.

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