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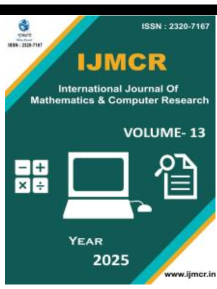

Downhill Nirmala Indices of Graphs

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ARTICLE INFO	ABSTRACT
Published Online: **26 April 2025** **Corresponding Author:** **V. R. Kulli**	In this study, we introduce the downhill Nirmala and modified downhill Nirmala indices and their corresponding exponentials of a graph. Furthermore, we compute these indices for some standard graphs, wheel graphs, gear graphs and helm graphs.
KEYWORDS: downhill Nirmala index, modified downhill Nirmala index, graph.	

I. INTRODUCTION

In this study, G denotes a finite, simple, connected graph, $V(G)$ and $E(G)$ denote the vertex set and edge set of G. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. Any undefined terminologies and notations may be found in [1].

A topological index is a numerical parameter mathematically derived from the graph structure. In Chemical Graph Theory, concerning the definition of the topological index on the molecular graph and concerning chemical properties of drugs can be studied by the graph index calculation. Several topological indices have been considered in Theoretical Chemistry and many topological indices were defined by using vertex degree concept [2]. The Zagreb, Gourava, Revan, temperature, Sombor indices are the most degree based topological indices in Chemical Graph Theory, see [3-29]. Topological indices have their applications in various disciplines in Science and Technology [30, 31, 32].

A u-v path P in G is a sequence of vertices in G, starting with u and ending at v, such that consecutive vertices in P are adjacent, and no vertex is repeated. A path $\pi = v_1, v_2, \ldots v_{k+1}$ in G is a downhill path if for every i, $1 \le i \le k$, $d_G(v_i) \ge d_G(v_{i+1})$.

A vertex v is downhill dominates a vertex u if there exists a downhill path originated from u to v. The downhill neighborhood of a vertex v is denoted by $N_{dn}(v)$ and defined as: $N_{dn}(v) = \{u\text{: } v \text{ downhill dominates } u\}$. The downhill degree $d_{dn}(v)$ of a vertex v is the number of downhill neighbors of v [33].

Recently, some downhill indices were studied in [34, 35].

The Nirmala index was introduced in [36] and it is defined as

$$N(G) = \sum_{uv \in E(G)} \sqrt{d_G(u) + d_G(v)}.$$

Motivated by the definition of Nirmala index, we introduce the downhill Nirmala index of a graph and it is defined as

$$DWN(G) = \sum_{uv \in E(G)} \sqrt{d_{dn}(u) + d_{dn}(v)}.$$

Considering the downhill Nirmala index, we introduce the downhill Nirmala exponential of a graph G and defined it as

$$DWN(G, x) = \sum_{uv \in E(G)} x^{\sqrt{d_{dn}(u) + d_{dn}(v)}}.$$

We define the modified downhill Nirmala index of a graph G as

$$^{m}DWN(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_{dn}(u) + d_{dn}(v)}}.$$

Considering the modified downhill Nirmala index, we introduce the modified downhill Nirmala exponential of a graph G and defined it as

$$^{m}DWN(G, x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{d_{dn}(u) + d_{dn}(v)}}}.$$

Recently, some Nirmala indices were studied in [37-50].

In this paper, the downhill Nirmala index, modified downhill Nirmala index and their corresponding exponentials of certain graphs are computed.

II. RESULTS FOR SOME STANDARD GRAPHS

Proposition 1. Let G be r-regular with n vertices and $r\geq 2$. Then

$$DWN(G)=\frac{nr\sqrt{(n-1)}}{\sqrt{2}}.$$

Proof: Let G be an r-regular graph with n vertices and $r\geq 2$ and $\frac{nr}{2}$ edges. Then $d_{dn}(v)=n-1$ for every v in G.

$$DWN(G)=\sum_{uv\in E(G)}\sqrt{d_{dn}(u)+d_{dn}(v)}$$

$$=\frac{nr}{2}\sqrt{(n-1)+(n-1)}=\frac{nr\sqrt{(n-1)}}{\sqrt{2}}.$$

Corollary 1.1. Let C_n be a cycle with $n\geq 3$ vertices. Then

$$DWN(C_n)=\sqrt{2}n\sqrt{(n-1)}.$$

Corollary 1.2. Let K_n be a complete graph with $n\geq 3$ vertices. Then

$$DWN(K_n)=\frac{n(n-1)\sqrt{(n-1)}}{\sqrt{2}}.$$

Proposition 2. Let P be a path with $n\geq 3$ vertices. Then

$$DWN(P)=2\sqrt{n-1}+(n-3)\sqrt{2(n-1)}.$$

Proof: Let P be a path with $n\geq 3$ vertices. Then there are two types of the downhill degree of edges as follows:

$E_1 = \{uv \in E(P) \mid d_{dn}(u)=0,\ d_{dn}(v)=n-1\}$, $|E_1| = 2$.

$E_2 = \{uv \in E(P) \mid d_{dn}(u)= d_{dn}(v)=n-1\}$, $|E_2| = n-3$.

Then

$$DWN(P)=\sum_{uv\in E(P)}\sqrt{d_{dn}(u)+d_{dn}(v)}$$

$$=2\sqrt{0+(n-1)}+(n-3)\sqrt{(n-1)+(n-1)}$$

$$=2\sqrt{n-1}+(n-3)\sqrt{2(n-1)}.$$

Proposition 3. Let $K_{m,n}$ be a complete bipartite graph with $m< n$. Then

$$DWN(K_{m,n})=mn\sqrt{n}.$$

Proof: We obtain the partition of the edge set of $K_{m,n}$ as follows:

$E_1 = \{uv \in E(K_{m,n}) \mid d_{dn}(u)=n,\ d_{dn}(v)=0\}$, $|E_1| = mn$.

We get

$$DWN(K_{m,n})=mn\sqrt{n+0}=mn\sqrt{n}.$$

Corollary 3.1. Let $K_{1,n}$ be a star with $n\geq 3$ vertices. Then

$$DWN(K_{1,n})=n\sqrt{n}.$$

Corollary 3.2. Let K_n be a complete graph with $n\geq 3$ vertices. Then

$$DWN(K_{n,n})=n^2\sqrt{n}.$$

III. RESULTS FOR WHEELS

The wheel W_n is the join of C_n and K_1. Clearly W_n has n+1vertices and $2n$ edges. The vertex K_1 is called apex and the vertices of C_n are called rim vertices.

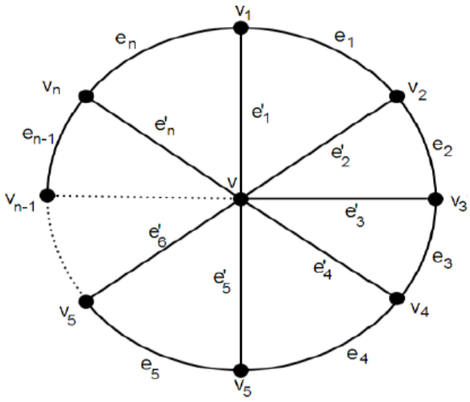

Figure 1. Wheel W_n

Lemma 1. Let W_n be a wheel with n+1vertices, $n\geq 4$. Then there are two types of the downhill degree of vertices as given below:

$V_1 = \{u \in V(W_n) \mid d_{dn}(u) = n\}$, $|V_1| = 1$.

$V_2 = \{u \in V(W_n) \mid d_{2dn}(u) = n-1\}$, $|V_2| = n$.

Lemma 2. Let W_n be a wheel with n+1vertices, $n\geq 4$. Then there are two types of the downhill degree of edges as follows:

$E_1 = \{uv \in E(W_n) \mid d_{dn}(u) = n, d_{dn}(v) = n-1\}$, $|E_1| = n$.

$E_2 = \{uv \in E(W_n) \mid d_{dn}(u) = d_{dn}(v) = n-1\}$, $|E_2| = n$.

Theorem 1. Let W_n be a wheel with n+1vertices, $n\geq 4$. Then the downhill Nirmala index of W_n is

$$DWN(W_n)=n\sqrt{2n-1}+n\sqrt{2}\sqrt{n-1}.$$

Proof: From definition and by Lemma 2, we deduce

$$DWN(W_n)=\sum_{uv\in E(W_n)}\sqrt{d_{dn}(u)+d_{dn}(v)}$$

$$=n\sqrt{n+(n-1)}+n\sqrt{(n-1)+(n-1)}$$

$$=n\sqrt{2n-1}+n\sqrt{2}\sqrt{n-1}.$$

Theorem 2. Let W_n be a wheel with n+1vertices, $n\geq 4$. Then the downhill Nirmala exponential of W_n is

$$DWN(W_n,x)=nx^{\sqrt{2n-1}}+nx^{\sqrt{2n-2}}.$$

Proof: From definition and by Lemma 2, we deduce

$$DWN(W_n,x)=\sum_{uv\in E(W_n)}x^{\sqrt{d_{dn}(u)+d_{dn}(v)}}$$

$$=nx^{\sqrt{n+(n-1)}}+nx^{\sqrt{(n-1)+(n-1)}}$$

$$=nx^{\sqrt{2n-1}}+nx^{\sqrt{2n-2}}.$$

Theorem 3. Let W_n be a wheel with n+1vertices, n≥4. Then the modified downhill Nirmala index of W_n is

$$^{m}DWN(W_n)=\frac{n}{\sqrt{2n-1}}+\frac{n}{\sqrt{2(n-1)}}.$$

Proof: From definition and by Lemma 2, we deduce

$$^{m}DWN(W_n)=\sum_{uv\in E(W_n)}\frac{1}{\sqrt{d_{dn}(u)+d_{dn}(v)}}$$

$$=\frac{n}{\sqrt{n+(n-1)}}+\frac{n}{\sqrt{(n-1)+(n-1)}}$$

$$=\frac{n}{\sqrt{2n-1}}+\frac{n}{\sqrt{2(n-1)}}.$$

Theorem 4. Let W_n be a wheel with n+1vertices, n≥4. Then the modified downhill Nirmala exponential of W_n is

$$^{m}DWN(W_n,x)=nx^{\frac{1}{\sqrt{2n-1}}}+nx^{\frac{1}{\sqrt{2(n-1)}}}.$$

Proof: From definition and by Lemma 2, we deduce

$$^{m}DWN(W_n,x)=\sum_{uv\in E(W_n)}x^{\frac{1}{\sqrt{d_{dn}(u)+d_{dn}(v)}}}$$

$$=nx^{\frac{1}{\sqrt{n+(n-1)}}}+nx^{\frac{1}{\sqrt{(n-1)+(n-1)}}}$$

$$=nx^{\frac{1}{\sqrt{2n-1}}}+nx^{\frac{1}{\sqrt{2(n-1)}}}.$$

IV. RESULTS FOR GEAR GRAPHS

A bipartite wheel graph is a graph obtained from W_n with n+1 vertices adding a vertex between each pair of adjacent rim vertices and this graph is denoted by G_n and also called as a gear graph. Clearly, $|V(G_n)|$ = 2n+1 and $|E(G_n)|$ = 3n. A gear graph G_n is depicted in Figure 2.

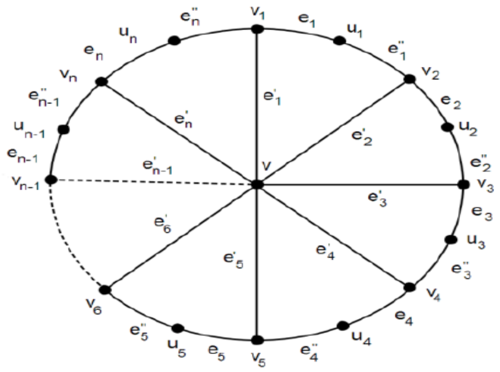

Figure 2. Gear graph G_n

Lemma 3. Let G_n be a gear graph with 2n+1vertices, 3n edges, n≥4. Then G_n has three types of the downhill degree of vertices as follows:

$V_1 = \{u\in V(G_n) \mid d_{dn}(u) = 2n\}$. $\quad |V_1| = 1$.

$V_2 = \{u \in V(G_n) \mid d_{dn}(u) = 2\}$. $\quad |V_2| = n$.

$V_3 = \{u \in V(G_n) \mid d_{dn}(u) = 0\}$. $\quad |V_3| = n$.

Lemma 4. Let G_n be a gear graph with 2n+1vertices, 3n edges, n≥4. Then G_n has two types of the downhill degree of edges as follows:

$E_1 = \{u \in E(G_n) \mid d_{dn}(u) = 2n,\ d_{dn}(v) = 2\}$, $|E_1| = n$.

$E_2 = \{u \in E(G_n) \mid d_{dn}(u) = 2,\ d_{dn}(v) = 0\}$, $|E_2| = 2n$.

Theorem 5. Let G_n be a gear graph with 2n+1vertices, 3n edges, n≥4. Then the downhill Nirmala index of G_n is

$$DWN(G_n)=n\sqrt{2n+2}+n2\sqrt{2}.$$

Proof: From definition and by Lemma 4, we deduce

$$DWN(G_n)=\sum_{uv\in E(G_n)}\sqrt{d_{dn}(u)+d_{dn}(v)}$$

$$=n\sqrt{2n+2}+2n\sqrt{2+0}$$

$$=n\sqrt{2n+2}+n2\sqrt{2}.$$

Theorem 6. Let G_n be a gear graph with 2n+1vertices, 3n edges, n≥4. Then the downhill Nirmala exponential of G_n is

$$DWN(G_n,x)=nx^{\sqrt{2n+2}}+2nx^{\sqrt{2}}.$$

Proof: From definition and by Lemma 4, we deduce

$$DWN(G_n,x)=\sum_{uv\in E(G_n)}x^{\sqrt{d_{dn}(u)+d_{dn}(v)}}$$

$$=nx^{\sqrt{2n+2}}+2nx^{\sqrt{2+0}}$$

$$=nx^{\sqrt{2n+2}}+2nx^{\sqrt{2}}.$$

Theorem 7. Let G_n be a gear graph with 2n+1vertices, 3n edges, n≥4. Then the modified downhill Nirmala index of G_n is

$$^{m}DWN(G_n)=\frac{n}{\sqrt{2n+2}}+\frac{2n}{\sqrt{2}}.$$

Proof: From definition and by Lemma 4, we deduce

$$^{m}DWN(G_n)=\sum_{uv\in E(G_n)}\frac{1}{\sqrt{d_{dn}(u)+d_{dn}(v)}}$$

$$=\frac{n}{\sqrt{2n+2}}+\frac{2n}{\sqrt{2+0}}$$

$$=\frac{n}{\sqrt{2n+2}}+\frac{2n}{\sqrt{2}}.$$

Theorem 8. Let G_n be a gear graph with 2n+1vertices, 3n edges, n≥4. Then the modified downhill Nirmala exponential of G_n is

$$^{m}DWN(G_n,x)=nx^{\frac{1}{\sqrt{2n+2}}}+2nx^{\frac{1}{\sqrt{2}}}.$$

Proof: From definition and by Lemma 4, we deduce

$$ {}^{m}DWN\left(G_n,x\right)=\sum_{uv\in E(G_n)} x^{\frac{1}{\sqrt{d_{dn}(u)+d_{dn}(v)}}} $$

$$ = nx^{\frac{1}{\sqrt{2n+2}}} + 2nx^{\frac{1}{\sqrt{2+0}}} $$

$$ = nx^{\frac{1}{\sqrt{2n+2}}} + 2nx^{\frac{1}{\sqrt{2}}}. $$

V. RESULTS FOR HELM GRAPHS

The helm graph H_n is a graph obtained from W_n (with n+1 vertices) by attaching an end edge to each rim vertex of W_n. Clearly, $|V(H_n)| = 2n+1$ and $|E(H_n)| = 3n$. A graph H_n is shown in Figure 3.

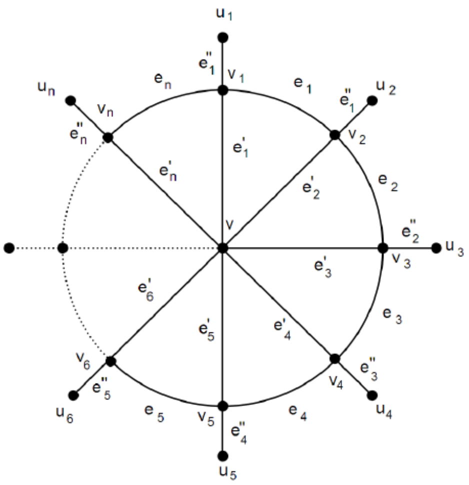

Figure 3. Helm graph H_n

Lemma 5. Let H_n be a helm graph with 2n+1 vertices, n≥5. Then H_n has three types of the downhill degree of vertices as given below:

$V_1 = \{u \in V(H_n) \mid d_{dn}(u) = 2n\}$,

$V_2 = \{u \in V(H_n) \mid d_{dn}(u) = 2n - 1\}$,

$V_3 = \{u \in V(H_n) \mid d_{dn}(u) = 0\}$,

Lemma 6. Let H_n be a helm graph with 3n edges, n≥3. Then H_n has three types of the downhill degree of edges as follows:

$E_1 = \{uv \in E(H_n) \mid d_{dn}(u) = 2n,\ d_{dn}(v) = 2n - 1\}$.

$E_2 = \{uv \in E(H_n) \mid d_{dn}(u) = d_{dn}(v) = 2n - 1\}$.

$E_3 = \{uv \in E(H_n) \mid d_{dn}(u) = 2n - 1,\ d_{dn}(v) = 0\}$.

Theorem 9. Let H_n be a helm graph with 2n+1 vertices, n≥3. Then the downhill Nirmala index of H_n is

$$ DWN\left(H_n\right)= n\sqrt{4n-1}+\left(\sqrt{2}+1\right)n\sqrt{2n-1}. $$

Proof: By using definition and by Lemma 6, we obtain

$$ DWN\left(H_n\right)=\sum_{uv\in E(H_n)}\sqrt{d_{dn}(u)+d_{dn}(v)} $$

$$ = n\sqrt{2n+(2n-1)}+n\sqrt{(2n-1)+(2n-1)} $$

$$ +n\sqrt{(2n-1)+0} $$

$$ = n\sqrt{4n-1}+\left(\sqrt{2}+1\right)n\sqrt{2n-1}. $$

Theorem 10. Let H_n be a helm graph with 2n+1vertices, 3n edges, n≥4. Then the downhill Nirmala exponential of H_n is

$$ DWN\left(H_n,x\right)= nx^{\sqrt{4n-1}}+nx^{\sqrt{2(2n-1)}}+nx^{\sqrt{2n-1}}. $$

Proof: From definition and by Lemma 6, we deduce

$$ DWN\left(H_n,x\right)=\sum_{uv\in E(H_n)} x^{\sqrt{d_{dn}(u)+d_{dn}(v)}} $$

$$ = nx^{\sqrt{2n+(2n-1)}}+nx^{\sqrt{(2n-1)+(2n-1)}}+nx^{\sqrt{(2n-1)+0}} $$

$$ = nx^{\sqrt{4n-1}}+nx^{\sqrt{2(2n-1)}}+nx^{\sqrt{2n-1}}. $$

Theorem 11. Let H_n be a helm graph with 2n+1 vertices, n≥3. Then the modified downhill Nirmala index of H_n is

$$ {}^{m}DWN\left(H_n\right)=\frac{n}{\sqrt{4n-1}}+\frac{n}{\sqrt{2(2n-1)}}+\frac{n}{\sqrt{(2n-1)}}. $$

Proof: By using definition and by Lemma 6, we obtain

$$ {}^{m}DWN\left(H_n\right)=\sum_{uv\in E(H_n)}\frac{1}{\sqrt{d_{dn}(u)+d_{dn}(v)}} $$

$$ =\frac{n}{\sqrt{2n+(2n-1)}}+\frac{n}{\sqrt{(2n-1)+(2n-1)}} $$

$$ +\frac{n}{\sqrt{(2n-1)+0}} $$

$$ =\frac{n}{\sqrt{4n-1}}+\frac{n}{\sqrt{2(2n-1)}}+\frac{n}{\sqrt{(2n-1)}}. $$

$|V_1| = 1$.

$|V_2| = n$.

$|V_3| = n$.

Theorem 12. Let H_n be a helm graph with 2n+1 vertices, n≥3. Then the modified downhill Nirmala index of H_n

Proof: By using definition and by Lemma 6, we obtain

$|E_1| = n$.

$|E_2| = n$.

$|E_3| = n$.

$$ {}^{m}DWN\left(H_n,x\right)=\sum_{uv\in E(H_n)} x^{\frac{1}{\sqrt{d_{dn}(u)+d_{dn}(v)}}} $$

$$ = nx^{\frac{1}{\sqrt{2n+(2n-1)}}}+nx^{\frac{1}{\sqrt{(2n-1)+(2n-1)}}}+nx^{\frac{1}{\sqrt{(2n-1)+0}}} $$

$$ = nx^{\frac{1}{\sqrt{4n-1}}}+nx^{\frac{1}{\sqrt{2(2n-1)}}}+nx^{\frac{1}{\sqrt{(2n-1)}}}. $$

VI. CONCLUSION

In this paper, a novel invariant is considered which is the downhill Nirmala index. The downhill Nirmala index, modified downhill Nirmala idex and their corresponding exponentials for some graphs are determined.

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