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"Fixed Point Theorems In Mengerspace Using The Notion Of Compatibility And Sub Sequentially Continuity"

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ABSTRACT

In this paper is to prove a common fixed point theorem for four mappings using the notion of compatibility and sub sequentially continuity in Menger space.

KEYWORDS

Fix point, Menger space, Compatibility, subsequently continuity.

INTRODUCTION

In 1942, Professor KarlMenger [11] has introduced the theory of probabilistic metric space in which a distributionfunction was used instead of non-negative real number as value of the metric. Thenotion of PM-space corresponds to situations when we do not know exactly the distance between two points, but we know probabilities of possible values of this distance. In 1960,Schweizer and Sklar [14]studied this concept and gave fundamental result on this space.Fixed point theory is one of the fruitful and effective tools in mathematics.

In 1986,Jungck[8] introducedthe notion of Compatible maps for a pair of self maps in metric space.In 1991,Pant [13] noticed these criteria for fixed points of contraction mappings and introduced a new continuity condition, known as reciprocal continuity and obtained a common fixed point theorem by using the compatibility in metric spaces.Healso showed that in the setting of common fixed point theorems for compatible mappings satisfying contraction conditions, the notion of reciprocal continuity is weaker than the continuity of one of the mappings.

In 1998,Jungck and Rhoades[9] introduced the concept of wealkycompatibility and showed that each pair of compatible maps is wekly compatible but the converse need not to betrue.In 2005 Singh and Jain [15]generalized the result of Mishra[12] using the concept of weak compatibility andcompatibility of pair of self maps.

In 2008 Al-Thagafi and shahzad [1] introduced the concept of occasionally weakly compatible (OWC) mappings in metric space which is the most general concept among all the commutativity concepts.In 2012,Doric et.al[6] shown that the condition of occasionally weak compatibility reduced to weak compatibility.Bouhadjera and Godet-Thobie[2] introduced two new notion namely subsequential continuity and subcompatibility which are weaker than reciprocal continuity and compatibility respectively. Further Imdad et al.[7]improved the result of Bouhadjera and Godet-Thobie[2].

The object of this paper is toprove a common fixed point theorem using the notion of compatibility and sub sequentially continuity in Menger space.

PRELIMINARY NOTES

Definition 2.1 (Schweizer and Sklar [14])A Mapping F:R→ R^+ is said to be a distribution function if it is non-decreasing and leftcontinuous with

Inf {F(t):t∈R} = 0 and Sup {F(t):t∈R} = 1

We will denote the Δ the set of all distribution function defined on [-∞,∞] while H(t) will always denote thespecific distribution function defined by

$$H(t)= \begin{cases} 0\ , \text{if } t \leq 0 \\ 1,\ \text{if } t > 0 \end{cases}$$

If X is a non-empty set, F :X×X →Δ is called a probabilistic distance on X and the value of F at (x, y) ∈X ×X is represented by $F_{x,y}$.

Definition 2.2 (Schweizer and Sklar [14])The ordered pair (X,F) is called a probabilistic metricspace(shortly PM-space) if X is nonempty set and F is a probabilistic distance satisfying the following conditions:

for all x,y,z∈X andt,s> 0

PM-1 F x,y(t) = 1 if and only if x=y

PM-2 F x,y(0) = 0

PM-3 F x,y(t) = F y,x(t)

PM-4 If F x,z(t) = 1 and F z,y(s) = 1 then F x,y(t+s)=1

the ordered triple (X,F,Δ) is called Menger space if (X,F) is PM space and Δ is a triangular norm such that for all x,y,z∈X and t,s>0

PM-5 F x,y (t+s)≥ F x,z (t) + F z,y (s)

Definition 2.3 (Schweizer and Sklar [14])A Menger space (X, F, Δ) with the continuous t-norm Tis said to be complete iff every Cauchy sequence in X converges to a point in X.

Definition 2.4 (Mishra[12]) Twoself maps A and S of a Menger Space (X,F,Δ) are said to be compatible if

$F_{ASxn,SAxn}(t)$ → 1 for all t>0 Whenever {xn} is a sequence in X such that Axn,Sxn→ z for some z∈X asn → ∞ .

Definition 2.5 (Singh and Jain [16]) Two self-maps A and S of a non-empty set X are saidto be weakly compatible (or coincidentally commuting) if they commute at theircoincidence points, i.e. if Az= Szfor some z∈ X, then ASz= SAz.

Definition.2.6 (Jungck[10]) Two self mappings A and S of non-empty set X are occasionally weakly compatible(OWC) if and only if there exist a point z∈ X which is coincidence point of A and S at which A and S commute.

Definition.2.7 (Bouhadjera and Godet-Thobie [2])A pari of self mappings(A,S) is said to be sub compatible on a Menger space(X,F,Δ)iff there exist a sequence $\{x_n\}$in X such that

$$\lim_{n\to\infty} Axn = \lim_{n\to\infty} Sxn = x \qquad \text{for some } x \in X$$

$$\text{and} \quad \lim_{n\to\infty} F_{ASx_n,SAx_n}(t) = 1 \quad \text{for all } t > 0$$

Definition.2.8A pair of self mappings(A,S) is said to be subsequentially continuous on a Menger space(X,F,Δ) if and only if there exist a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Axn = \lim_{n\to\infty} Sxn = x \qquad \text{for some } x \in X$$

$$\text{and} \quad \lim_{n\to\infty} ASxn = Ax \text{ and } \lim_{n\to\infty} SAxn = Sx$$

Lemma 2.9 Let (X,F,Δ)be a Menger space. If there exists k ∈(0, 1) such that
$F_{x,y}(kt) \geq F_{x,y}(t)$,for all x, y ∈ X and t > 0
then x = y.

MAIN RESULT

Theorem 3.1Let A, B, S and Tbe self maps on a Menger space (X, F, Δ) with continuous t-norm and if the pairs (A,S) and (B,T) are compatible and subsequentially continuous mappings then

(i) the pair (A,S) and (B,T) have a coincidence point,

(ii)there exist a constant k ∈(0, 1)such that

for all x, y ∈ X and t > 0

$$F_{Ax,By} \geq \min\{F_{Sx,Ty}(t), F_{Ax,Sx}(t), F_{By,Ty}(t), F_{Ax,Ty}(t), F_{By,Sx}(t)\}$$

Then A,B, S and T have a unique common fixed point in X.

Proof.Since the pair (A,S) and (B,T) is compatible and subsequentially continuous mappings ,then from the definition there exist a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z \qquad \text{for some } z \in X$$

$$\text{and} \quad \lim_{n\to\infty} F_{ASx_n,SAx_n}(t) = F_{Az,Sz}(t) = 1 \quad \text{for all } t > 0$$

thenAz = Sz.Hence z is a coincidence point of pair (A,S).

again, since (B,T) is compatible and subsequentially continuous mappings ,then from the definition, there exist a sequence $\{y_n\}$ in X such that

$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = w \qquad \text{for some } w \in X$$

$$\text{and} \quad \lim_{n\to\infty} F_{BTy_n,TBy_n}(t) = F_{BTy_n,TBy_n}(t) = 1 \quad \text{for all } t > 0$$

then Bw= Tw. Hence w is a coincidence point of pair(B,T).

Step1.By taking $x = x_n$and y = y_n in (ii),we have

$$\mathbf{F_{Ax_n,By_n}(kt) \geq \min\{F_{Sx_n,Ty_n}(t), F_{Ax_n,Sx_n}(t), F_{By_n,Ty_n}(t), F_{Ax_n,Ty_n}(t), F_{By_n,Sx_n}(t)\}}.$$

Taking limit as n ⟶ ∞,we get

$$F_{z,w}(kt) \geq \min\{F_{z,w}(t), F_{z,z}(t), F_{w,w}(t), F_{z,w}(t), F_{w,z}(t)\}.$$

$$F_{z,w}(kt) \geq \min\{F_{z,w}(t), 1, 1, F_{z,w}(t), F_{w,z}(t)\}.$$

$$F_{z,w}(kt) \geq F_{z,w}(t)$$

From lemma2.9 ,we have z = w

Step2. By taking $x = z$and y = y_n in (ii),we have

$$\boldsymbol{F_{Az,By_n}(kt) \geq \min\{F_{Sz,Ty_n}(t), F_{Az,Sz}(t), F_{By_n,Ty_n}(t), F_{Az,Ty_n}(t), F_{By_n,Sz}(t)\}}.$$

Taking limit as n ⟶ ∞,we get

$$F_{Az,w}(kt) \geq \min\{F_{Az,w}(t), F_{Az,Az}(t), F_{w,w}(t), F_{Az,w}(t), F_{w,Az}(t)\}.$$

$$F_{Az,w}(kt) \geq \min\{F_{Az,w}(t), 1, 1, F_{Az,w}(t), F_{w,Az}(t)\}.$$

$$F_{Az,w}(kt) \geq F_{Az,w}(t)$$

From lemma2.9, we have Az = w

Step3. By taking $x = x_n$and y = z in (ii), we have

$$\boldsymbol{F_{Ax_n,Bz}(kt) \geq \min\{F_{Sx_n,Tz}(t), F_{Ax_n,Sx_n}(t), F_{Bz,Tz}(t), F_{Ax_n,Tz}(t), F_{Bz,Sx_n}(t)\}}.$$

Taking limit as n ⟶ ∞, we get

$F_{z,Bz}(kt) \geq \min\{F_{z,Bz}(t), F_{z,z}(t), F_{Bz,Bz}(t), F_{z,Bz}(t), F_{Bz,z}(t)\}.$

$F_{z,Bz}(kt) \geq \min\{F_{z,Bz}(t), 1,1, F_{z,Bz}(t), F_{Bz,z}(t)\}.$

$F_{z,Bz}(kt) \geq F_{z,Bz}(t)$

From lemma2.9, we have z = Bz

Therefore Az=Sz=Bz=Tz=z. i.e z is a common fixed point theorem of A,B,S and T.

Step4. For uniqueness,let u (z ≠ u) is another common fixed point of A,B,S and T then Au=Su=Bu=Tu=u

By taking x= zand y = u in 3.1.2, we have

$F_{Az,Bu}(kt) \geq \min\{F_{Sz,Tu}(t), F_{Az,Sz}(t), F_{Bu,u}(t), F_{Az,Tu}(t), F_{Bu,Sz}(t)\}$

$F_{z,u}(kt) \geq \min\{F_{z,u}(t), F_{z,z}(t), F_{u,u}(t), F_{z,u}(t), F_{u,z}(t)\}$

$F_{z,u}(kt) \geq F_{z,u}(t)$

From lemma2.9, we have z = u which is contradiction of our hypothesis is z ≠ u. Hence z is unique common fixed point.

Corollary 3.2Let A and S be self maps on a Menger space (X, F, Δ) with continuous t-norm and if the pairs (A,S) and (B,T) are compatible and subsequentially continuous mappings then

(i) the pair (A,S) has a coincidence point,

(ii)there exist a constant k ∈(0, 1)such that

for all x, y ∈ X and t > 0

$F_{Ax,Ay} \geq \min\{F_{Sx,Sy}(t), F_{Ax,Sx}(t), F_{Ay,Sy}(t), F_{Ax,Sy}(t), F_{Ay,Sx}(t)\}$

Then A and S have a unique common fixed point in X.

Example 3.3Let X=[0,∞) and d be the usual metric on X and for each t∈[0, 1] define

$$F_{x,y}(t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0 \\ 0, & \text{if } t = 0 \end{cases} \text{ for all } x,y \in X$$

Clearly (X, F, Δ) be a Menger space where t-normΔ is defined by Δ(a,b)=min{a,b} for all a,b∈[0, 1].

We define self maps A and S on X

$$A(X) = \begin{cases} \frac{x}{4}, & \text{if } x \in [0,1] \\ 5x-4, & \text{if } x \in (1,\infty) \end{cases} \quad S(X) = \begin{cases} \frac{x}{5}, & \text{if } x \in [0,1] \\ 4x-3, & \text{if } x \in (1,\infty) \end{cases}$$

Consider a sequence $\{x_n\} = \{\frac{1}{n}\}$ in X. Then

$$\lim_{n\to\infty} A(x_n) = \lim_{n\to\infty}\left(\frac{1}{4n}\right) = 0 = \lim_{n\to\infty}\left(\frac{1}{5n}\right) = \lim_{n\to\infty} S(x_n)$$

$$\text{now , } \lim_{n\to\infty} AS(x_n) = \lim_{n\to\infty} A\left(\frac{1}{5n}\right) = \lim_{n\to\infty}\left(\frac{1}{20n}\right) = 0 = A(0)$$

$$\lim_{n\to\infty} SA(x_n) = \lim_{n\to\infty} S\left(\frac{1}{4n}\right) = \lim_{n\to\infty}\left(\frac{1}{20n}\right) = 0 = S(0)$$

$$\text{and } \lim_{n\to\infty} F_{ASx_n,SAx_n}(t) = 1 \quad \text{for all } t > 0$$

Consider another sequence $\{x_n\} = \{1 + \frac{1}{n}\}$ in X. Then

$$\lim_{n\to\infty} A(x_n) = \lim_{n\to\infty}(5 + \frac{5}{n} - 4) = 1 = \lim_{n\to\infty}(4 + \frac{4}{n} - 3) = \lim_{n\to\infty} S(x_n)$$

$$now\ ,\ \lim_{n\to\infty} AS(x_n) = \lim_{n\to\infty} A(1 + \frac{4}{n}) = \lim_{n\to\infty}(5 + \frac{20}{n} - 4) = 1 \neq A(1)$$

$$\lim_{n\to\infty} SA(x_n) = \lim_{n\to\infty} S(1 + \frac{5}{n}) = \lim_{n\to\infty}(4 + \frac{20}{n} - 3) = 1 \neq S(1)$$

but $\lim_{n\to\infty} F_{ASx_n,SAx_n}(t) = 1$ for all $t > 0$

Thus the pair(A,S) compatible and subsequentially continuous.

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